ality of nodal curves.

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Natural Suppression of Strong P- and T-Invariance Violations and Calculable Mixing Angles in SU(2)⊗U(1)

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Permutation symmetry is imposed on an $SU(2)_L \otimes U(1) \otimes CP$-invariant Lagrangian with six quarks. The mixing angles of the b and t are determined: b decays predominantly to u and a meson containing b should have a lifetime $\tau_B \approx 10^{-11}$ sec or less. A phase in the gauge couplings of b and t causes nonconservation of CP in K_L decays that is naturally small. No violations of P and T invariance are induced into the strong interactions by $\phi = \arg(\mathrm{Det}M)$ at the tree level. One-loop corrections yield an upper bound of $\phi < 10^{-10}(m_s/m_b)(m_t/m_b)^2$.

Although CP is not conserved in $K_L \to 2\pi$ decay it was generally assumed to be an exact symmetry of the strong interactions. Indeed, one of the arguments in favor of the color gauge theory of strong interactions was the automatic absence of large C-, P-, and T-nonconserving interactions.[1] The discovery of instantons and vacuum tunneling has complicated this picture because the interaction

$$\mathcal{L}_{\mathrm{eff}} = \frac{\bar{\theta}}{64\pi^2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}, \tag{1}$$

where F is the gluon field strength, preserves C but does not conserve either P or T and hence does not conserve CP. This term links strong and weak interactions for if the spontaneous breaking of the weak-interaction gauge group produces a quark mass matrix with phase

$$\phi = \arg[(\mathrm{Det}M_n)(\mathrm{Det}M_p)], \tag{2}$$

then $\bar{\theta} = \theta + \phi$, where θ is the strength of the bare coupling before spontaneous symmetry breaking. It is possible, but not natural, to choose $\bar{\theta} = 0$. There are three natural explanations of why $\bar{\theta}$ is so small.[2] One is that the u quark is massless; another is that there is a very light pseudoscalar meson, the axion.[2,3] Both of these possibilities appear to be ruled out by experiment.[4] The third solution, which we shall pursue, is that CP is an exact symmetry of the Lagrangian (i.e., $\theta=0$ initially) and the spontaneous breaking of the weak-interaction gauge group induces a phase ϕ, which is calculable and small.[5]

The problem of guaranteeing that the induced ϕ be very small is quite delicate. CP-invariance of the Lagrangian requires not only that $\theta=0$ but also that the Yukawa couplings and Higgs couplings be real.[6] A single Higgs doublet trivially gives $\phi=0$, but also gives no CP-invariance violation in K_L decay. The solution we report on here employs two Higgs doublets and a discrete permutation symmetry in $SU(2)_L\otimes U(1)$.

We work in $SU(2)_L\otimes U(1)$ with three quark doublets, i.e., six quarks. The gauge couplings of the quarks are clearly invariant under premutation of the three doublets and we shall extend this invariance to the entire Lagrangian. To do so we define the usual doublets as $\psi_\mu=(p_{\mu L},n_{\mu L})$, where $\mu=a,b,c$ label the three doublets. The Combination $\psi_0\equiv\frac{1}{3}\sqrt{3}(\psi_a+\psi_b+\psi_c)$ is invariant under permutation of a,b,c. The orthogonal combinations may be defined in a variety of ways. We find it most convenient to define $\psi_+=\frac{1}{3}\sqrt{3}(\psi_a+\omega\psi_b+\omega^2\psi_c)$ and $\psi_-=\frac{1}{3}\sqrt{3}(\psi_a+\overline{\omega}\psi_b+\overline{\omega}^2\psi_c)$, where $\omega\equiv e^{i2\pi/3}$ is the cube root of unity. When a,b,c are permuted, ψ_+ goes into a multiple of ψ_-. Thus (ψ_+,ψ_-) transform irreducibly under the symmetric group S_3 and the $3!=6$ matrices of the representation may be easily constructed.[7]

This fixes the S_3 properties of the left-handed quarks we shall make analogous assignments: (p_{+R},p_{-R}) and p_{0R} form a doublet and singlet of S_3; (n_{+R},n_{-R}) and n_{0R} form a doublet and singlet of S_3. As already mentioned, a single Higgs field will not give CP nonconservation. We therefore use two Higgs fields; both are SU(2) doublets and they form an S_3 doublet (φ_+,φ_-). All of these S_3 doublets transform just the same as (ψ_+,ψ_-).

One dividend of this discrete symmetry is that it greatly restricts the number of possible couplings. The most general $SU(2)_L\otimes U(1)\otimes S_3\otimes CP$-invariant Yukawa interaction is

$$\mathfrak{L}_n^{\text{Yuk}}=f(\overline{\psi}_-\varphi_+n_{+R}+\overline{\psi}_+\varphi_-n_{-R})+g(\overline{\psi}_-\varphi_-+\overline{\psi}_+\varphi_+)n_{0R}+h\overline{\psi}_0(\varphi_-n_{+R}+\varphi_+n_{-R}),\tag{3}$$

for the n_R quarks where f,g,h are real and positive. [Note that it is the f coupling that restricts the invariance to the discrete group S_3 rather than the continuous group O(2).] The most general interaction of the p_R quarks is

$$\mathfrak{L}_p^{\text{Yuk}}=g'(\overline{\psi}_-\tilde{\varphi}_-p_{+R}+\overline{\psi}_+\tilde{\varphi}_+p_{-R})+f'(\overline{\psi}_-\tilde{\varphi}_++\overline{\psi}_+\tilde{\varphi}_-)p_{0R}+h'\overline{\psi}_0(\tilde{\varphi}_+p_{+R}+\tilde{\varphi}_-p_{-R}).\tag{4}$$

(The interchange $f\to g'$, $g\to f'$ is for later convenience.) It is easy to assay the predictive power of this scheme. There are six Yukawa constants which we shall use to fit the six quark masses. There are two vacuum expectation values, whose ratio will be used to fit the Cabibbo angle. Once these parameters are fixed, the phase ϕ of the quark mass matrix is determined. In addition, the b and t mixing angles will be predicted in terms of the quark masses and the Cabibbo angle.

Let $v_+=\langle\varphi_+^{(0)}\rangle$ and $v_-=\langle\varphi_-^{(0)}\rangle$ be the vacuum expectation values of the neutral Higgs fields. The elements of the mass matrices M_n and M_p for the negative and positive quarks, respectively, may be read off from

$$(\overline{n}_-,\overline{n}_+,\overline{n}_0)_L\begin{pmatrix} fv_+ & 0 & gv_- \\ 0 & fv_- & gv_+ \\ hv_- & hv_+ & 0\end{pmatrix}\begin{pmatrix} n_+ \\ n_- \\ n_0\end{pmatrix}_R,\tag{5}$$

$$(\overline{p}_-,\overline{p}_+,\overline{p}_0)_L\begin{pmatrix} g'\overline{v}_- & 0 & f'\overline{v}_+ \\ 0 & g'\overline{v}_+ & f'\overline{v}_- \\ h'\overline{v}_+ & h'\overline{v}_- & 0\end{pmatrix}\begin{pmatrix} p_+ \\ p_- \\ p_0\end{pmatrix}_R.\tag{6}$$

These matrices have some remarkable properties. First note that

$$\begin{aligned}\mathrm{Det}M_n&=-fgh[(v_+)^3+(v_-)^3],\\ \mathrm{Det}M_p&=-f'g'h'[(\overline{v}_+)^3+(\overline{v}_-)^3].\end{aligned}\tag{7}$$

$\mathrm{Det}M_n$ and $\mathrm{Det}M_p$ have equal and opposite phases and hence $\phi=0$ in tree approximation regardless of the vacuum expectation values. This is already a nontrivial result and works only because the Yukawa constants factor out of both determinants.[8] Generally one-loop corrections to the mass matrices would be expected to give $\phi\approx G_Fm^2/4\pi^2$. When m is m_b or m_t this gives $\phi\approx10^{-4}$ or 10^{-5}, which gives much too large a value for the electric dipole moment of the neutron. In this model there is, however, an additional suppression. Before discussing this effect we will first discuss the model in tree approximation.

Symmetry breaking.—The Higgs potential will be invariant under $SU(2)\otimes U(1)S_3\otimes CP$ except that we allow explicit soft breaking of S_3:

$$V=4\lambda_1[(\bar{\varphi}_+\varphi_+)(\bar{\varphi}_-\varphi_-)-(\bar{\varphi}_-\varphi_+)(\bar{\varphi}_+\varphi_-)]+\lambda_2[\bar{\varphi}_+\varphi_++\bar{\varphi}_-\varphi_--v^2]^2$$
$$+\lambda_3[\bar{\varphi}_-\varphi_++\bar{\varphi}_+\varphi_--v^2\sin\rho\cos\delta]^2+\lambda_3[i(\bar{\varphi}_-\varphi_+-\bar{\varphi}_+\varphi_-)-v^2\sin\rho\sin\delta]^2. \tag{8}$$

The two quadratic terms proportional to $\sin\rho$ explicitly break S_3.[9] We take all $\lambda_i>0$. Minimization is trivial and gives

$$v_+=v\cos\tfrac{1}{2}\rho,\quad v_-=ve^{i\delta}\sin\tfrac{1}{2}\rho. \tag{9}$$

Had we imposed S_3 on the quadratic terms in (8) then v_- would be zero.[10] We will be interested in the case $v_+\gg v_-\neq 0$ and will choose v_+ positive real.

Quark masses and mixing angles.—The quark mass hierarchy will require that $f\gg h\gg g$ and $f'\gg h'\gg g'$. It is easiest to diagonalize (5) and (6) in the limit $g=g'=0$ (i.e., $m_d=m_u=0$) and then perturb in g and g'. For the positive quarks the diagonalization yields $m_u=g'v_+$, $m_c=h'v_+$, and $m_t=f'v_+$. The negative quarks require more care because the combination $(f/h)|v_-/v_+|$ arises in the diagonalization. (Recall that $f\gg h$ but $v_-\ll v_+$.) It is useful to define $\tan\theta=(f/h)|v_-/v_+|$. Then we have $m_d=gv_+\cos\theta$, $m_s=hv_+/\cos\theta$, and $m_b=fv_+$. The charged current coupled to physical quarks is then

$$J_-^{\ \mu}=(\bar{u}\,\bar{c}\,\bar{t})_L\gamma^\mu A\begin{pmatrix}d\\ s\\ b\end{pmatrix}_L,\ \text{with}\quad A=\begin{pmatrix}\cos\theta & \sin\theta & -(h/f)\tan\theta\\ -\sin\theta & \cos\theta & (h/f)\Delta^*\\ (h/f)\sin\theta(1+\Delta) & (h/f)\cos\theta(\tan^2\theta-\Delta) & 1\end{pmatrix},$$

where $\Delta\equiv e^{-3i\delta}\tan\theta(h/f)$.[11] Note that $h/f=\cos\theta(m_s/m_b)$ from above so that the only free parameter is the phase angle δ.

Let us now make a few observations about A: (i) Obviously Cabibbo universality holds and $\theta=\theta_C$. This is a nontrivial result. (ii) The b quark decays predominantly into u with a mixing angle $m_s\sin\theta_C/m_b$. A meson containing b should have a lifetime

$$\tau_B=\tfrac{1}{9}(m_b/m_s\sin\theta_C)^2\tau_\mu(m_\mu/m_B)^5.$$

This is well below the experimental upper bound[12] of 5×10^{-8} sec for m_B in the range 4 to 10 GeV. For example, with $m_b/m_s=25$ this gives $\tau_B=2\times10^{-11}$ sec for $m_B=4$ GeV and considerably smaller for larger m_B. (iii) CP nonconservation in the K_1K_2 mass matrix arises from the phase on the $t\to s$ gauge coupling. Letting M_{12} be the off-diagonal term in the mass matrix, one finds[13]

$$\epsilon\equiv\frac{\mathrm{Im}M_{12}}{\sqrt{2}\,|\Delta M|}=\sqrt{2}\left(\frac{m_s}{m_b}\right)^3\sin\theta_C\sin3\delta\left[\ln\left(\frac{m_t}{m_c}\right)^2-1+\left(\frac{m_sm_t}{m_bm_c}\right)^2\right].$$

The experimental value of ϵ is 2×10^{-3}. Obviously this result depends sensitively on the mass of b; in particular $m_b/m_s=10$ gives good agreement. Note that we refer always to the masses in the Lagrangian, not to the constituent masses. There may be considerable uncertainty in the value of m_b/m_s, e.g., estimates of m_c/m_s range from 3.5 to 7.7.[14]

$\Delta S=2$ via Higgs exchanges.—Since there are two Higgs doublets one must check for strangeness-changing neutral Higgs exchanges. Rewriting (3) in terms of the physical quarks gives

$$\mathcal{L}_{|\Delta S|=1}^{\ \mathrm{Yuk}}=\bar{d}_Ls_Rf\ (e^{-i\delta}\varphi_-^{(0)}-|v_-/v_+|\,\varphi_+^{(0)})\ +\bar{s}_Ld_Rg\sin\theta_C\varphi_+^{(0)}.$$

To obtain the effective $\Delta S=2$ Hamiltonian, we must diagonalize the Higgs-meson mass matrix to find the eigenstates. The end result, to leading order, is

$$H_{|\Delta S|=2}^{\ \mathrm{eff}}\approx(\bar{d}_Ls_R)^2\left[\frac{G_Fm_s^{\,2}\tan^2\theta_C}{M_{\mathrm{H}_1}^{\ 2}}R_1\right]+(\bar{s}_Ld_R)^2\left[\frac{G_Fm_d^{\,2}\tan^2\theta_C}{M_{\mathrm{H}_2}^{\ 2}}R_2\right]$$

where $M_{\mathrm{H}_{1,2}}$ are Higgs meson masses and $R_{1,2}$ are products of ratios of Higgs quartic couplings, and are typically $O(1)$. Two features of the above expression are worthy of note: The first is that there is an overall $|v_-/v_+|^2$ suppression factor and the second is that $H_{\Delta S=2}^{\ \mathrm{eff}}$ is real so there is no contribution to the imaginary part of the K_1K_2 mass matrix and hence no CP-invariance violation due to Higgs exchange.

One-loop corrections to ϕ.—We are now equipped to estimate radiative corrections to the quark mass matrix. This is most easily done by considering corrections to (5) and (6). First there are single loops involving γ or the Z boson. These involve no phases and are just a common multiplicative renormalization of f, g, and h. There are no one-loop diagrams containing $W^{\pm}$ since the right-handed quarks are all SU(2) singlets.

Thus only single Higgs loops can modify ϕ. These may be analyzed according to whether they correct the v_+ entries, the v_- entries, or the zero entries in (5) and (6). Because of the interplay of SU(2) $\otimes$ U(1) and S_3 invariance the corrected M_n has the form[15]

$$M_n=\begin{pmatrix} fv_+(1+a_+^{(1)}) & -f\bar{v}_-a_0^{(1)} & gv_-(1+a_-^{(1)}) \\ -f\bar{v}_-a_0^{(2)} & fv_-(1+a_-^{(2)}) & gv_+(1+a_+^{(2)}) \\ hv_-(1+a_-^{(3)}) & hv_+(1+a_+^{(3)}) & -(gh/f)\bar{v}_-a_0^{(3)} \end{pmatrix}.$$

The $a^{(j)}$ are all real constants whose magnitude is typically

$$a^{(j)}\approx \pm(f^2+f'^2)\lambda v_+^2/16\pi^2 M_H^2\approx \pm(f^2+f'^2)/128\pi^2,$$

where the occurrence of f or f' depends on whether positive or neutral Higgs scalars are propagated. The corrections to the positive quark masses are analogous but with $j=4,5,6$. Remarkably, the leading contribution to ϕ is then

$$\phi=\mathrm{Im}[(v_+\bar{v}_-)^3\sum_1^6 a_+^{(j)}+(v_-\bar{v}_+)^3\sum_1^6 a_-^{(j)}]\,|v_+|^{-6}.$$

Even if there are no cancellations at all among the twelve constants $a_{\pm}^{(j)}$, this gives

$$\phi\leqslant\frac{12}{128\pi^2}(f^2+f'^2)\left|\frac{v_-}{v_+}\right|^3\sin 3\delta.$$

The additional suppression $|v_-/v_+|^3$ yields quite an acceptable value of ϕ:

$$\phi\leqslant\frac{3G_F}{8\pi^2}(m_b^2+m_t^2)\left(\frac{m_s}{m_b}\sin\theta_C\right)^3\approx 10^{-10}\left(\frac{m_s}{m_b}\right)\left(\frac{m_t}{m_b}\right)^2.$$

Note that we have consistently made pessimistic estimates, i.e., no cancellations, and have retained even the Higgs mesons that are gauged away.

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Path-Integral Measure for Gauge-Invariant Fermion Theories

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It is shown that the path-integral measure for gauge-invariant fermion theories is not invariant under the γ_5 transformation and the Jacobian gives rise to an extra phase factor corresponding to the Adler-Bell-Jackiw anomaly. The derivation of "anomalous" Ward-Takahashi identities by means of the variational derivative can thus be made consistent in the path-integral formalism.

The derivation of the "anomalous" chiral Ward-Takahashi (W-T) identities[1] in the path-integral formalism has not been transparent in the past, as the "anomaly" term, which is seen only after the one-loop renormalization, had to be added to the action by hand. We here show that the path-integral measure for gauge-invariant fermion theories is not invariant under the γ_5 transformation and it gives rise to an extra phase factor corresponding to the anomaly. The derivation of W-T identities can thus be made consistent in the path-integral formalism.

We start with the gauge-invariant Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + (\tfrac{1}{2}g^2)\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} \tag{1}$$

suitably continued to Euclidean space. The operator $\not{D} \equiv \gamma^\mu(\partial_\mu + A_\mu)$ after the Wick rotation $x^0 \to -ix^4$ and $A_0 \to iA_4$ becomes a Hermitian operator

$$\not{D} = i\gamma^0 D_4 + \gamma^k D_k \equiv \gamma^4 D_4 + \gamma^k D_k. \tag{2}$$

We consider the fermions in the n-dimensional representation of the gauge group SU(n):

$$iA_\mu \equiv gA_\mu{}^a T^a \tag{3}$$

with

$$[T^a, T^b] = if_{abc}T^c, \quad \mathrm{Tr}(T^aT^b) = (\tfrac{1}{2})\delta^{ab},$$

and

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu].$$

The variation of (1) under the γ_5 transformation

$$\begin{aligned} \psi(x) &\to \exp[i\alpha(x)\gamma_5]\psi(x), \\ \bar{\psi}(x) &\to \bar{\psi}(x)\exp[i\alpha(x)\gamma_5] \end{aligned} \tag{4}$$

gives rise to

$$\mathcal{L} \to \mathcal{L} - \partial_\mu\alpha(x)\bar{\psi}\gamma^\mu\gamma_5\psi - 2mi\alpha(x)\bar{\psi}\gamma_5\psi \tag{5}$$

for an infinitesimal parameter $\alpha(x)$.

To define the functional integral precisely, we first expand $\psi(x)$ and $\bar{\psi}(x)$ as

$$\begin{aligned} \psi(x) &= \textstyle\sum_n a_n \varphi_n(x), \\ \bar{\psi}(x) &= \textstyle\sum_n \varphi_n(x)^\dagger \bar{b}_n \end{aligned} \tag{6}$$

in terms of a complete set of eigenfunctions of the Hermitian operator $\not{D}$, (2), in Euclidean space:

$$\begin{aligned} \not{D}\varphi_n(x) &= \lambda_n \varphi_n(x), \\ \int \varphi_n(x)^\dagger \varphi_m(x) d^4x &= \delta_{n,m}. \end{aligned} \tag{7}$$

The coefficients a_n and $\bar{b}_n$ in (6) are the elements of the Grassmann algebra. We note that $\psi(x)$ and $\bar{\psi}(x)$ are independent quantities in the classical level. (In the chiral form, ψ_L and $\bar{\psi}_L$ are expanded in φ_L and $\varphi_R{}^\dagger$, respectively.) The functional-integral measure is then defined by

$$\begin{aligned} d\mu &\equiv \textstyle\prod_x [\mathfrak{D}A_\mu(x)]\,\mathfrak{D}\bar{\psi}(x)\,\mathfrak{D}\psi(x) \\ &= \textstyle\prod_x [\mathfrak{D}A_\mu(x)] \prod_{n,m} d\bar{b}_m\, da_n \end{aligned} \tag{8}$$

