tion and phase and whether the low-frequency excitations observed recently by Shapiro *et al.*[11] in Nb and earlier by Chowdhury[12] in Pd-Ag alloys have, in fact, the same origin.

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Possibility of Vortex-Antivortex Pair Dissociation in Two-Dimensional Superconductors

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The possible existence of a Kosterlitz-Thouless vortex-antivortex dissociation transition in thin superconducting films is discussed. It is found that in practice the situation should be closely analogous to that predicted for superfluid-helium films. A simple relationship is found between the Kosterlitz-Thouless transition temperature and the sheet resistance. This relationship is compared with the observed broadening of the resistive transition of superconducting films with high sheet resistance.

Kosterlitz and Thouless[1] have predicted that in two-dimensional neutral superfluids a thermodynamic instability should occur in which vortex-antivortex pairs, bound at low temperatures, spontaneously dissociate into free vortices at a characteristic transition temperature T_{KT}. Similar ideas have also been discussed by Berezinskii.[2] At this transition Nelson and Kosterlitz[3] have established a universal relation between the Kosterlitz-Thouless transition temperature T_{KT} and the superfluid sheet density. Experiments on superfluid-helium films apparently confirming this prediction have been reported by several groups.[4,5] Some objections to the interpretation of these experiments have been raised, however.[6] Because of the importance of these theories to the questions of the nature of phase transitions and long-range order in two-dimensional systems generally, additional systems suitable for testing the proposed theories are of considerable interest. In this Letter we argue that if these theories are correct, a situation closely analogous to that expected in helium films should arise in very thin superconducting films, and we discuss the conditions under which such effects should be observable. A particularly simple relationship is found between T_{KT} and the sheet resistance of the film.

The existence of the predicted vortex-antivortex dissociation instability in helium films is intimately connected with the fact that in such films the interaction energy between vortex pairs depends logarithmically on the separation between them. As first shown by Pearl,[7] vortex pairs in thin superconducting (i.e., charged superfluid) films have a logarithmic interaction energy out to a characteristic distance $\lambda_\perp = \lambda^2/d$ beyond which the interaction energy falls off as $1/r$. Here λ is the bulk penetration depth of the material of the film and d is the film thickness. Physically $\lambda_\perp$ plays the role of the magnetic penetration depth for fields perpendicular to the film, and as $\lambda_\perp$ increases the diamagnetism of the superconductor becomes increasingly less important. Correspondingly, as $\lambda_\perp$ increases, the vortices in a superconducting film become progressively like those in helium (i.e., progressively governed by the superfluid velocity field along with diminishing effect of the vector potential).

Kosterlitz and Thouless[1] have pointed out that the finite range of the logarithmic interaction in a superconducting film destroys the precise analogy to helium films. We certainly do not dispute this point of principle. However, we wish to point out that (as we show explicitly below) in the superconducting systems of interest $\lambda_\perp$ is so large (on the order of a few millimeters) that in practice the situation in superconducting and helium films cannot be too different. Some differences certainly will exist. For example, because of the finite energy of a free vortex in a superconducting film, in equilibrium a few free vortices will exist in the film below T_{KT} in addition to the bound vortex-antivortex pairs envisaged by Kosterlitz and Thouless. On the other hand, if dissociation of these bound pairs occurs in a helium film at T_{KT}, it certainly will have occurred by T_{KT} in a superconducting film where dissociation is inherently easier. Moreover, above T_{KT} where a "plasma" of interacting free vortices exists, the situation in superconducting and helium films should be quite comparable provided that the density of vortices is such that the mean separation between them is less than $\lambda_\perp$. Thus, although it is hard to be quantitative, in practice we expect only some broadening of the dissociation transition in the superconducting case because of the finite range of the logarithmic interaction. Of course, even with helium the finite size of the sample in any real experiment makes the energy of a free vortex finite and leads to similar broadening.

To establish when such a transition should be observable we recall the relation between T_{KT} and the superfluid sheet density

$$k_B T_{KT} = \tfrac{1}{2}\pi \hbar^2 n_s^{2D}/m^*, \quad (1)$$

shown by Nelson and Kosterlitz,[3] to be universal, independent of any external effects that may influence n_s^{2D} (e.g., interaction with a substrate). In (1) n_s^{2D} is the superfluid particle sheet density and m^* the particle mass. (Note $m^* = 2m$ for the superconducting case.) Equation (1) can be expressed in a more useful form for a superconductor by first expressing the sheet density in terms of the bulk superfluid pair density n_s of the film and then expressing the bulk density in terms of the bulk penetration depth λ, namely

$$n_s^{2D} = n_s d = \frac{1}{2}\frac{mc^2}{4\pi e^2}\frac{d}{\lambda^2}. \quad (2)$$

Substitution of (2) into (1) yields

$$k_B T_{KT} = \frac{\Phi_0^2}{32\pi^2}\frac{d}{\lambda^2} = \frac{\Phi_0^2}{32\pi^2}\frac{1}{\lambda_\perp}, \quad (3)$$

where $\Phi_0 = hc/2e$ is the superconducting flux quantum.

From (3) we see immediately that at T_{KT}

$$\lambda_\perp(T_{KT}) = \frac{\Phi_0^2}{32\pi^2}\frac{1}{k_B T_{KT}} = \frac{0.98}{T_{KT}(\mathrm{K})}\ \mathrm{cm}, \quad (4)$$

which demonstrates directly that $\lambda_\perp$ is very large at the instability temperature.

To evaluate $\lambda_\perp$ and T_{KT} explicitly in terms of readily accessible material parameters, we note that in the dirty limit appropriate to very thin films[8]

$$\lambda_\perp \equiv \frac{\lambda^2}{d} = \frac{\lambda_L^2(0)}{d}\left(\frac{\xi_0}{l}\right)\left\{\frac{\Delta(T)}{\Delta(0)}\tanh\left[\frac{\beta\Delta(T)}{2}\right]\right\}^{-1}, \quad (5)$$

which can be written more usefully as

$$\lambda_\perp = 1.78\,\frac{\Phi_0^2}{4\pi^5}\frac{e^2}{\hbar}\frac{R_\square}{k_B T_{c0}} f^{-1}\left(\frac{T}{T_{c0}}\right). \quad (6)$$

Here λ_L is the London penetration depth, Δ the superconducting energy gap, ξ_0 the BCS coherence length, and l the electron mean free path. In addition $R_\square = \rho/d$ is the sheet resistance of the film, T_{c0} the BCS transition temperature of the film, and $f(T/T_{c0})$ the temperature-dependent factor contained within the brackets in (5).

Substitution of (6) into (3) leads to the implicit relation

$$\frac{T_{KT}}{T_{c0}} f^{-1}\left(\frac{T_{KT}}{T_{c0}}\right) = 0.561\,\frac{\pi^3}{8}\left(\frac{\hbar}{e^2}\right)\frac{1}{R_\square} \quad (7)$$

$$= 2.18\,\frac{R_c}{R_\square}, \quad (8)$$

where $R_c = \hbar/e^2$ corresponds to a sheet resistance of 4.12 kΩ/□.

Equation (7) shows that T_{KT}/T_{c0} depends only on fundamental constants and the sheet resistance of the film. Near T_c, (7) reduces to the simple form

$$\frac{T_{\rm KT}}{T_{c0}}=\left(1+0.173\frac{R}{R_c}\right)^{-1},\qquad \left(\frac{T_{c0}-T_{\rm KT}}{T_{c0}}\right)\ll 1. \tag{9}$$

A graph of the full solution of (7) is shown in Fig. 1.

From the figure we see that any substantial reduction of $T_{\rm KT}$ below T_{c0} requires quite large sheet resistances. Since real metallic films with large sheet resistances typically are granular, the validity of the standard dirty-limit result for λ [Eq. (5)] used to obtain (7) comes into question. Fortunately there is both theoretical and experimental evidence[9] that (5) remains valid even for granular systems. A more serious complication in practice may be flux pinning in such films.

The presence of a vortex-antivortex plasma in a superconducting film above $T_{\rm KT}$ should manifest itself strongly in the electromagnetic properties. In particular, it should play a role in the onset of resistance as the temperature is increased. The intrinsic onset of resistance in superconductors

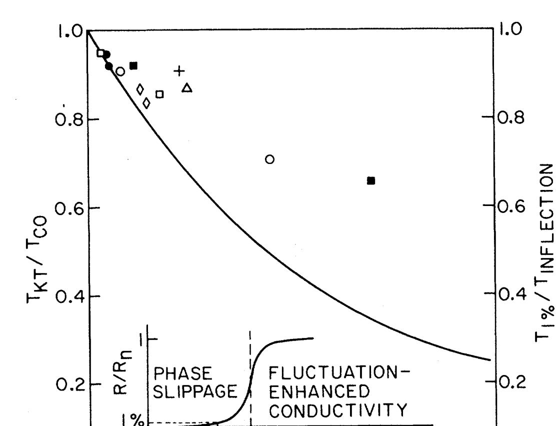

FIG. 1. Solid line, $T_{\rm KT}/T_{c0}$ as a function of $R_\square$, according to Eq. (7). The inset shows a typical resistive transition with the definition of $T_{1\%}$ (where $R/R_n=0.01$) and $T_{\rm infl}$. The data points are values of $T_{(1\%)}/T_{\rm infl}$ as derived from resistance curves in the literature. Triangles, from Ref. 12; diamonds, from Ref. 13; open circles, from Ref. 14; closed circles, from Ref. 15; plus, from Ref. 16; open squares, from Ref. 17; and closed squares from Ref. 18.

has been studied in detail only for one-dimensional filamentary superconductors where it is known to arise from discrete, thermally activated phase-slip events that increase in frequency as $T\to T_{c0}$. Here both the theory and experiment are highly developed.[10] Although to our knowledge the onset of resistance has not been explicitly studied for the two-dimensional case, it seems noncontroversial that phase slippage (in the form of vortex motion across the sample) is the relevant process here as well.[11] The interesting question is whether or not above $T_{\rm KT}$ phase slippage is caused by continuous flow in a vortex-antivortex plasma, in contrast with the discrete thermally activated events which are the direct analog of phase-slips in one-dimensional filaments.

Short of a complete study of the resistive transition, the above suggests that some insight into the possible reality of a vortex-antivortex dissociation transition in superconducting films might be gained by comparing the observed broadening in the resistive transition of high– sheet-resistance films with our calculated $T_{\rm KT}/T_{c0}$. The resistive transition of such films has been widely studied with regard to the phenomenon of fluctuation-enhanced conductivity above T_{c0}.

An attempt to carry out such a comparison is shown in Fig. 1. Here we have collected data from a wide variety of sources for which the resistive transition of high-sheet resistance films could be found.[12-18] What is plotted is the ratio of the temperature $T_{(1\%)}$ (at which the measured resistance equals 1% of the full normal resistance) to the temperature $T_{\rm infl}$ (at which the R-vs-T curve shows an inflection) as a function of $R_\square$. (See the inset of Fig. 1, which schematically shows a typical transition curve and illustrates $T_{(1\%)}$ and $T_{\rm infl}$.)

The motivation for this procedure is as follows. The choice of $T_{(1\%)}$ was dictated by the lowest normalized resistance that we could meaningful extract from all of the references used. Since we are interested only in the broadening caused by phase slippage processes and not by the fluctuation-enhanced conductivity, we need some procedure to subtract out the broadening caused by the latter. If we knew T_{c0}, this would provide the natural dividing point. Unfortunately T_{c0} can only be determined by curve fitting to some theory. Thus we adopt the somewhat arbitrary but physically reasonable and well-defined procedure outlined above.

As seen in Fig. 1 the data so determined show a definite progression as a function of $R_\square$. This strongly suggests that whatever its origins, the observed broadening is intrinsic and a natural function of $R_{\square}$. Moreover, in comparing this empirically determined width with the calculated T_{KT}/T_c curve, note that $T_{1\%}$ has to be higher than T_{KT} and that T_{infl} most probably underestimates T_{c0}.[19] Any compensation for these systematic differences would tend to bring the experimental points into better agreement but with the theoretical curve. The agreement is thus only qualitative but certainly does not rule out a Kosterlitz-Thouless transition in superconducting films.

Another factor that complicates this preliminary test of our prediction is that for each of the three relevant broadening processes—fluctuations toward the superconducting state above T_{c0}, discrete thermally activated vortex flow across the sample, and vortex-antivortex dissociation—the widths are all dependent on $R_{\square}$.[20] At this time it is impossible to give a quantitative prediction of the broadening due to thermally activated vortex flow for the comparison in Fig. 1. We conclude, therefore, that the observed broadening appears to be intrinsic and reasonably consistent with a vortex-antivortex dissociation transition but that it does not unequivocally prove the existence of such a transition in these superconducting films. We feel the case is enticing enough, however, to warrant serious investigation. Progress on the theoretical front appears already forthcoming.[21,22]

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