Are "Beats" of Electron Waves at Optical Frequencies a Potential for Free-Electron Lasers?

Helmut Schwarz[(a)]
Department of Physics, Rensselaer Polytechnic Institute, Troy, New York 12181
(Received 6 November 1978)

If two coherent electron beams originating from the same source interfere after one of the beams has been accelerated or decelerated by a few electron volts, a wave-mechanical beat phenomenon should occur. Continuously tunable optical radiation should be emitted. Feedback of this radiation within an optical cavity may turn the system into a free-electron laser.

A laser is essentially an electron oscillator and feedback amplifier working at optical frequencies. In conventional lasers the oscillating electrons are bound to atoms and molecules and are excited to higher energy levels by illumination from light sources (flash lamps *inter alia*) called "optical pumps." Such higher energy levels are discrete which means that light reemitted from such a system falling back into the ground states, spontaneously or by stimulation, can only have a specific wavelength. Conventional lasers are quantum devices and are limited to single characteristic wavelengths that cannot be changed at will. Even the "continuously tunable" dye lasers have a series of absorption and emission lines so close together that they appear to represent a continuous spectrum—however, within a relatively narrow range.

Free electrons in a vacuum, as for example within an electron beam generated by an electron gun, should radiate at any frequency and any reasonable amplitude if accelerated and decelerated periodically coherently and, therefore, in phase. That this is possible is known from the generation of radio waves and microwaves. The first indication that the production of such waves could be extended into the optical region was the modulation of an electron beam at optical frequencies.[1,2] Even though the original experiment of 1969 has not yet been reproduced, it has stimulated quite a number of theoretical works[3] which led to the use of an electron beam to amplify the 10.6-μm radiation from a CO_2 laser.[4] Recently it was even possible to operate the first free-electron laser[5] at a wavelength of 3.4 μm. In this process a relativistic electron beam at 43 MeV passed through a transverse periodic magnetic field[6] and the resulting radiation was fed back into the system by two resonator mirrors. In case this feedback occurs at the right time and at the right phase, the system will become an oscillator and even an amplifier, if the energy of the "stimulated" oscillation is higher than the energy due to thermal motion of the electrons (noise). Whereas a conventional laser is a quantum device, such a free-electron laser could be treated classically.[7]

We propose a wave-mechanical class of free-electron lasers. If two coherent electron beams originating from the same source interfere with each other after one of the beams has been accelerated or decelerated slightly, let us say by $\Delta E = 0.2$–100 eV, a beat phenomenon should occur, i.e., the electron probability density in the interference region should vary at frequencies $\omega = \Delta E/\hbar = 3\times10^{14} - 1.5\times10^{17}$ rad/s ($2\pi\hbar = h$, Planck's constant); electromagnetic radiation, including the optical spectrum, of wavelength $\lambda = 10$ nm—6 μm will be emitted as a result of the variation of the charge density. The wavelength of such radiation could be tuned continuously at will by changing the energy difference ΔE. This "beat" phenomenon could be achieved by using, for example, the electron prisma as developed by Möllenstedt and Düker[8]; see Fig. 1. Hereby, an electron beam is split up into two branches by a thin wire electrode I at negative potential and led together by a positively charged wire electrode II. A slightly negatively charged electrode III decreases before they join and interfere. If one branch has an energy E and the other a slightly lower energy $E - \Delta E$ the beating frequency created at the interference region will be $\Delta E/\hbar$. The difference in energy and momentum of the two branches could be created, for example, by a constant time change of the magnetic flux φ produced in a small coil placed inside the electron prisma ($\Delta U = d\varphi/dt$).

Let one branch have the energy E and momentum p and the other the energy $E - \Delta E$ and momentum $p - \Delta p$; then their respective wave functions will be

$$\psi_1 = \exp[(\vec{p}\cdot\vec{r} - Et)i/\hbar], \tag{1}$$

and

$$\psi_2 = \exp\{[(\vec{p} - \Delta\vec{p})\cdot\vec{r} - (E - \Delta E)t]\,i/\hbar\}, \tag{2}$$

with r the length of the electron beam. The wave

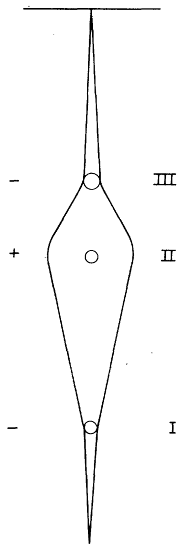

FIG. 1. Electron prisma according to Möllenstedt and Düker (Ref. 8).

function of the electron beam at the interference region is then

$$\psi = a_1\psi_1 + a_2\psi_2. \tag{3}$$

Its current density in the interference zone is given by

$$\vec{j} = i\hbar e(\psi\nabla\psi^* - \psi^*\nabla\psi)/2m_0. \tag{4}$$

Using Eqs. (1)–(3) for the evaluation of Eq. (4) leads to

$$\vec{j} = [(a_1^2 + a_2^2)\vec{p} - a_2^2\Delta\vec{p} + a_1a_2(2\vec{p} - \Delta\vec{p}) \times\cos[(\vec{r}\cdot\Delta\vec{p} - t\Delta E)/\hbar]e/m_0. \tag{5}$$

Maximum amplitude of the beat phenomenon will be obtained for $a_1 = a_2 = a$, i.e., if both electron beam branches carry equal current, so that Eq. (5) will become

$$\vec{j} = (2\vec{p} - \Delta\vec{p})\{1 + \cos[(\vec{r}\cdot\Delta\vec{p} - t\Delta E)/\hbar]\}a^2e/m_0. \tag{6}$$

Considering only the x component of the beam along its main axis and the fact that $\Delta E \ll E$, Eq. (6) can be simplified to

$$j = eA\{1 + \cos[\omega(\alpha_0 x/v - t)]\}. \tag{7}$$

$2\alpha_0$ is a correction factor value of almost unity,

$$\alpha_0 = (1 + \Delta U/4U)/2, \tag{8}$$

where

$$eA = j_0[1 - \alpha_0(\Delta U/U)^{1/2}/2], \tag{9}$$

with

$$j_0 = 2a^2e(2eU/m_0)^{1/2} \tag{10}$$

being the current density of the beam before entering the electron prisma, U the acceleration voltage of the undivided beam, and $\Delta U \ll U$ the difference between the acceleration voltages of the two branches. $v = (2eU/m_0)^{1/2}$ is the velocity of the electrons before entering the prisma.

The electric field created by the fluctuating electron charge density ρ_t can be calculated from the continuity relationship:

$$\nabla\cdot\vec{j} + \partial\rho_t/\partial t = 0. \tag{11}$$

Entering with Eq. (7) into Eq. (11) will yield by simple differentiation and integration

$$\rho_t = (eA\alpha_0/v)\cos[\omega(\alpha_0 x/v - t)], \tag{12}$$

and E_x is obtained from the Maxwell equation

$$\nabla\cdot\vec{E} = \rho_t/\epsilon_0 \tag{13}$$

by further integration[9]:

$$E_x = (eA/\epsilon_0\omega)\sin[\omega(\alpha_0 x/v - t)]. \tag{14}$$

The extension Δx of the dipole is obtained by integrating the force equation

$$m_0\ddot{x} = eE_x, \tag{15}$$

giving

$$x = \Delta x + vt + x_0 \tag{16}$$

(x_0 is the length of the electron beam from the source to the beginning of the interference region), and

$$\Delta x = (e^2A/\epsilon_0 m_0\omega^3)\sin[\omega(\alpha_0 x/v - t)]. \tag{17}$$

In order that radiation really becomes detectable, one has to provide for a length L of the interference region that, according to Eq. (16), would have to be longer than one pulse length v/ω, or in other words the coherence length Δs of the electron beam would have to be larger than the pulse length of the expected radiation,[10] i.e.,

$$\Delta s \simeq \lambda_e^2/\Delta\lambda_e = hv/(e\Delta U_{th}) \geq L \gg v/\omega. \tag{18}$$

(λ_e is the de Broglie wavelength of the electron and ΔU_{th} is the energy spread of the electron beam.) The average energy that such a dipole $e\Delta x$ will radiate during a unit of time will be, for one electron,[11]

$$S = e^2\omega^4\langle\Delta x^2\rangle(6\pi\epsilon_0 c^3). \tag{19}$$

If we assume the density of the electrons to be n_e occupying a volume $V = Lq$ (q is the cross section of the electron beam), the number of dipoles within V would be $n_e Ve\Delta x$.

The average value of Δx^2 over time and space can be derived from Eq. (17). A number $n_e V$ of electrons of speed v will, therefore, radiate on their path through the interaction region of length L a total light energy of

$$S_{tot} = A_s j_0^2 I_0^2 L^3/\omega^2 U^{3/2}, \tag{20}$$

where I_0 is the total electron beam current before entering the electron prisma and $A_s = [e/(2m_0)]^{1/2}/(24\pi c^3\epsilon_0^3) = 2.1\times10^{11}$ kg m^3 V$^{3/2}$ C^{-4} is a constant.

If this radiation is fed back through the application of an optical cavity with reflecting mirrors, an optical oscillator and amplifier might be established, or in other words a wave-mechanical free-electron laser could be feasible.

The gain G of such a laser for one path through the interference region is given by the ratio of the total light energy as expressed in Eq. (20) and the energy content of the interaction volume $\epsilon_0 E_0^2 V/2$,

$$G = A_g j_0 I_0 L^2/U^{3/2} \tag{21}$$

where A_g is a constant: $A_g = [e/(2m_0)]^{1/2}/(12\pi c^3\epsilon_0^2) = 3.7$ V$^{3/2}$ (Am)$^{-2}$.

In order to observe the beat phenomenon of electron waves at optical frequencies one will have the greatest difficulty in meeting the requirement for the energy spread ΔU_{th} of the initial beam as expressed in Eq. (18). Certainly the beam has to pass through a monochromator and should have an energy spread not greater than

$$\Delta U_{th} \simeq hv/eL. \tag{18a}$$

The following numerical example which might be hard to realize with presently available electron optical technology shows that a practical free-electron laser based on the beat phenomenon will be quite a weak light source.

Assuming an initial electron beam current of $I_0 = 25$ μA, a beam diameter of 0.5 μm ($q = 2\times10^{-13}$ m^2), $U = 1000$ V, $L = 1$ cm, and $\Delta U = 3$ V would result in optical radiation at a radial frequency of $\omega = 4.55\times10^{15}$ rad/s ($\lambda = 414$ nm), but with a total energy acquired on one path through the interaction region amounting only to $S_{tot} = 3\times10^{-24}$ J which is being emitted during $t = L/v \simeq 0.5$ ns corresponding to a total power of 6×10^{-15} W, which would mean the emission of only 13 000 photons/sec. The observation of the beat phenomenon under the above conditions might not be possible because of the difficulty of achieving requirement [Eq. (18a)] $\Delta U_{th} \simeq 10$ μV, unless one allows for a shorter interaction length L which would diminish even more the number of emitted photons. However, as the emitted energy increases with the square of the current, with the square of the current density, and with the third power of the length of the interference region as well as with the third power of decreasing electron speed, there may be some hope that with improved electron optics the beat phenomenon could be useful for free-electron lasers that do not need relativistic linear accelerators.

The gain of the nonrelativistic quantum-mechanical free-electron laser is comparable with that of relativistic high-power electron-beam lasers. Equation (21) yields for the above numerical example a value of $G = 4\times10^{-5}$ for one path through the interaction region.

Note added.—We have learned at the occasion of a seminar of ours given at the University of Tübingen, West Germany, on 19 January 1979, that G. Möllenstedt and H. Lichte were able to demonstrate experimentally [see Proceedings of the Ninth International Congress on Electron Microscopy, Toronto, 1978 (to be published), Vol. I, pp. 178–179] that two coherent electron waves could indeed produce a Doppler shift and a beat phenomenon at 1 Hz (1.02 sec^{-1}). They have even measured the current as it varies with the beat frequency according to our Eq. (7).

[a]Also with Departamento de Física, Universidade de Brasília, 70.910 Brasília D. F., Brazil.

[1]H. Schwarz and H. Hora, Appl. Phys. Lett. 15, 349 (1969).

[2]H. Schwarz, Trans. N. Y. Acad. Sci. 33, 150 (1971).

[3]A. Salat, J. Phys. C 3, 2509 (1970); E. E. Bergman, Nuovo Cimento 143, 243 (1973); A. D. Varshalovich and M. I. Dyakonov, Zh. Eksp. Teor. Fiz. 60, 90 (1971) [Sov. Phys. JETP 33, 51 (1971)]; L. D. Favro *et al.*, Appl. Phys. Lett. 19, 378 (1971); C. Becchi and G. Morpurgo, Phys. Rev. D 4, 288 (1971), and Appl. Phys. Lett. 21, 123 (1972); P. S. Farago and R. M. Sillitto, Proc. Roy. Soc. Edinburgh, Ser. A 71, 305 (1974); M. A. Piestrup *et al.*, J. Appl. Phys. 46, 132 (1975); C. K. Chen *et al.*, J. Appl. Phys. 49, 41 (1978); F. A. Hopf *et al.*, Phys. Rev. Lett. 37, 1342 (1976), and 39, 1496(E) (1977), and Opt. Commun. 18, 413 (1976); W. J. Cocke, Phys. Rev. A 17, 1713 (1978); W. H. Louisell *et al.*, Phys. Rev. A 18, 655 (1978).

[4]L. R. Elias *et al.*, Phys. Rev. Lett. 36, 717 (1976).

[5]D. A. G. Deacon *et al.*, Phys. Rev. Lett. 38, 892 (1977).

[6]H. Motz, J. Appl. Phys. 22, 527 (1951); H. Motz *et al.*, J. Appl. Phys. 24, 826 (1953).

[7]A. Gover, Appl. Phys. (Germany) 16, 121 (1978); T. Kwan *et al.*, Phys. Fluids 20, 581 (1977); also last five papers in Ref. 3.

[8]G. Möllenstedt and H. Düker, Z. Phys. 145, 377 (1956); W. Bayh, Z. Phys. 169, 492 (1962).

[9]SI system of units is used throughout this paper; $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space.

[10]H. Schwarz, in *Proceedings of the Second International Conference on Slight Scattering in Solids, Paris, 1971*, edited by M. Balkanski (Flammarion, Paris, 1971), pp. 123–127.

[11]See, for example, L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1962), p. 199.

Dynamics of Tokamak Discharges Dominated by the Trapped-Electron Instability

H. H. Klein, M. Cotsaftis,[a] N. A. Krall,[b] and J. B. McBride[b]
Science Applications, Inc., La Jolla, California 92037
(Received 15 May 1978)

A space-time analysis of a typical high-current TFR discharge has been performed in terms of the trapped-electron instability whose effects are likely to occur in this parameter range. The evolution of both the electron and ion temperatures is consistent with the transport induced by this instability, but not with other transport models described. The structure of the discharge is calculated and some properties are illlustrated.

There exists a large amount of data from tokamak experiments with significant variation of the main parameters (density, magnetic field, current) and with resolution in both time and space. Various attempts[1] to fit the experimental observations uniformly from physical laws have to date been relatively unsatisfactory. This has led to attempts to reconstruct empirically both the transport coefficients responsible for the observed behavior and the corresponding scaling laws.[2] These approaches suffer from a lack of knowledge of the appropriate physical laws and from the fact that they use only the late-time, steady-state data. This makes the extrapolation to large machines uncertain. So an attempt to fit the space-time data, based on a coherent examination of the existing physical phenomena is made here.

Because both time *and* space variations are considered, the possible set of transport coefficients which can be used to fit the data is strongly restricted. This requires an accurate identification of the physical phenomena generating these coefficients, and occurring in various parameter regions. In fact, two different types of behavior can be exhibited from the data: (a) The more numerous, which corresponds to low energy input, is characterized by a rise of the central electron temperature, $T_e(0,t)$, followed by a leveling off at a stationary value for a constant plasma current [Fig.1(a)]. (b) The less common, which is observed above a certain threshold of energy input, shows an overshoot in which $T_e(0,t)$ rises as in type (a) and then slowly decays afterwards [Fig. 1(b)]. Such behavior is exhibited in the TFR experiment[3] which shows a continu-

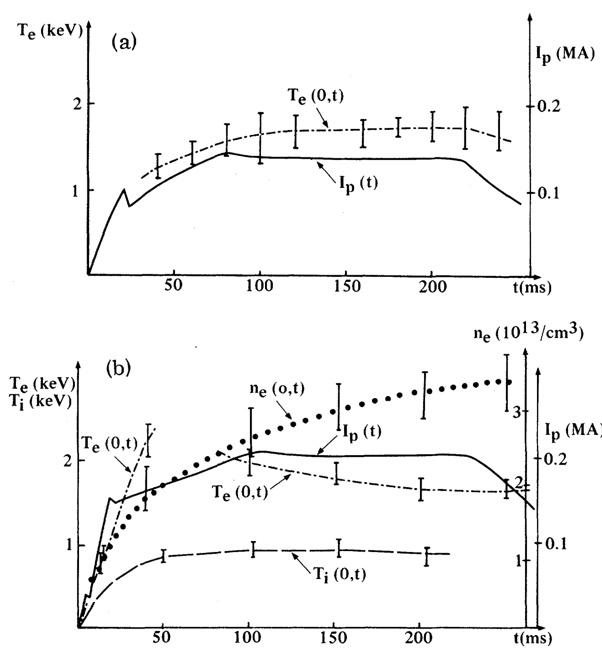

FIG. 1. (a) Experimental central electron temperature $T_e(t,0)$ (error bars) and total current I_p vs time in the type-(a) discharge. $B_T = 40$ kG; $a = 20$ cm. (b) Same parameters in the type-(b) discharge plus experimental central electron density $n_e(t,0)$ vs time (error bars) and as modeled in the calculations (dotted curve). $B_T = 40$ kG; $a = 20$ cm.