Effect of Earth's Rotation on the Quantum Mechanical Phase of the Neutron

S. A. Werner and J.-L. Staudenmann
Physics Department and Research Reactor Facility, University of Missouri–Columbia, Columbia, Missouri 65211

and

R. Colella
Physics Department, Purdue University, West Lafayette, Indiana 47907
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Using a neutron interferometer of the type first developed by Bonse and Hart for x rays, we have observed the effect of Earth's rotation on the phase of the neutron wave function. This experiment is the quantum mechanical analog of the optical interferometry observations of Michelson, Gale, and Pearson.

In 1925 Michelson, Gale, and Pearson[1] carried out a remarkable experiment designed to detect the effect of Earth's rotation on the speed of light. Using an interferometer in the form of a rectangle of the size 2010 ft× 1113 ft they were able to detect a retardation of light due to Earth's rotation corresponding to about $\frac{1}{4}$ of a fringe, in agreement with the theory of relativity. An experiment demonstrating that angular rotation could be detected by optical interferometry was carried out earlier by Sagnac.[2] In view of the differences in the coordinate transformation properties of light waves and matter waves, it is not obvious that an analogous quantum mechanical effect should exist for neutrons. We find that it does.

A schematic diagram of our experiment is shown in Fig. 1. We use a perfect-silicon-crystal interferometer of the type first developed for x rays by Bonse and Hart.[3] The first demonstration that such a device could be used for neutrons was achieved by Rauch, Treimer, and Bonse.[4] In this experiment a nominally monoenergetic neutron beam of wavelength $\lambda = 1.262$ Å is reflected vertically by a beryllium crystal. This beam passes through a collimator and subsequently through a 7-mm-diam cadmium aperture onto the interferometer. The beam incident on the interferometer is coherently split in the first Si-crystal slab at point A by Bragg reflection from the (220) lattice planes. The two resulting beams are coherently split again in the second Si slab near points B and C. Two of these beams are directed toward point D in the third Si slab, where they overlap and interfere. The outgoing interfering beams are detected in two ^{3}He proportional detectors, labeled C_1 and C_2 in Fig. 1. If the beam traversing the path ACD is shifted in phase by an angle β relative to the beam traversing the path ABD, it can be shown[5] that the expected intensities observed at detectors C_1 and C_2 are

$$I_1 = \alpha(1 + \cos\beta) \tag{1a}$$

and

$$I_2 = \gamma - \alpha\cos\beta. \tag{1b}$$

The constants α and γ depend on the incident flux. The perfect contrast predicted by these equations is never exactly realized in practice. In this experiment we have observed a phase shift β of the neutron wave function, which we will call β_{Sagnac}, resulting from the rotation of Earth. According to the theory developed below, this phase shift

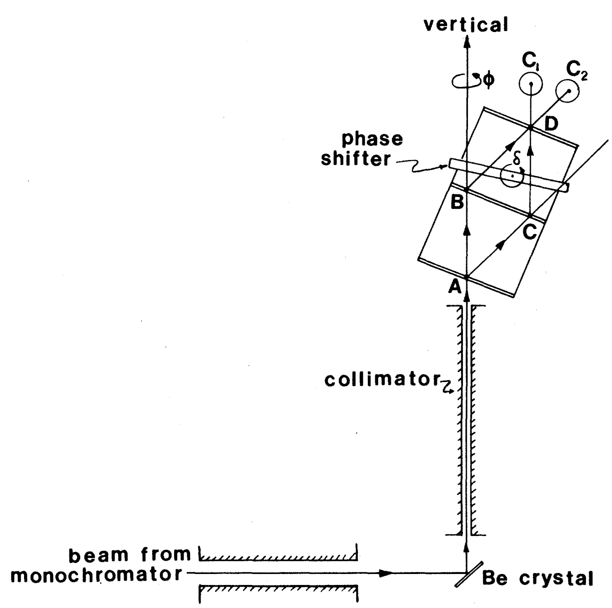

FIG. 1. Schematic diagram of the apparatus. The drawing is not to scale. The collimator is approximately 1 m in length and the interferometer is approximately 8 cm long from point A to point D. The angle δ of the phase-shifting slab is zero when it is parallel to the three interferometer slabs.

should be given by

$$\beta_{\text{Sagnac}} = (4\pi m_i/h)\vec{\omega}\cdot\vec{A}, \qquad (2)$$

where m_i is the inertial mass of the neutron, h is Planck's constant, $\vec{\omega}$ is the angular rotation velocity of Earth, and $\vec{A}$ is the normal area of the parallelogram enclosed by the beam paths $ABDCA$ in the interferometer. By turning the interferometer through an angle φ about the vertical incident beam direction AB, the dot product $\vec{\omega}\cdot\vec{A}$ will change, giving rise to a change in intensity prescribed by Eqs. (1). The formula (2) was obtained by Page[6] using wave-optical arguments, and by Anandan[7] and Stodolsky[8] within the framework of general relativity.

Because this experiment is done on the surface of rotating Earth, a noninertial frame, the Hamiltonian governing the neutron's motion will involve a third term in addition to the kinetic energy and the gravitational potential energy. According to standard classical mechanics, the appropriate Hamiltonian for the neutron is

$$H = p^2/2m_i + m_g\vec{g}\cdot\vec{r} - \vec{\omega}\cdot\vec{L}, \qquad (3)$$

where $\vec{p}$ is the canonical momentum of the neutron, $\vec{L} = \vec{r}\times\vec{p}$ is the angular momentum of the neutron's motion about the center of Earth ($\vec{r} = 0$), m_g is the gravitational mass of the neutron, and $\vec{g}$ is the acceleration due to gravity. Using Hamilton's equations, one finds that the canonical momentum is

$$\vec{p} = m_i\dot{\vec{r}} + m_i\vec{\omega}\times\vec{r}, \qquad (4)$$

where $\dot{\vec{r}}$ is the neutron velocity in a frame fixed to Earth. We now assume that (3) is also the correct quantum-mechanical Hamiltonian in the frame of rotating Earth, and that we can use (4) and the de Broglie relation

$$\vec{p} = \hbar\vec{k} \qquad (5)$$

to give the neutron wave properties specified by the wave vector $\vec{k}$. Within the WKB approximation, the difference in phase accumulated on the path ACD relative to ABD is then

$$\beta = \oint\vec{k}\cdot d\vec{r} = (1/\hbar)\oint\vec{p}\cdot d\vec{r}. \qquad (6)$$

This line integral along the path $ABCDA$ around the interferometer gives two terms resulting from the two terms in (4):

$$\beta = \beta_{\text{grav}} + \beta_{\text{Sagnac}}. \qquad (7)$$

The first term formed the basis of the analysis of our earlier experiments[9] on gravitationally induced quantum interference, and the second term is given by Eq. (2).

The difficulty in observing the phase shift β_{Sagnac} is that its maximum value is only of order 2% of the maximum value of β_{grav} for thermal neutrons. However, for an incident beam which is precisely vertical (along a plumb line), β_{grav} is independent of the interferometer rotation angle φ. For this orientation, the phase shift due to Earth's rotation can be easily worked out from Eq. (2); it is

$$\beta_{\text{Sagnac}} = (4\pi m_i/h)\omega A\sin\theta_L\sin(\varphi-\varphi_0). \qquad (8)$$

The angle φ_0 is an arbitrary reference angle with respect in north-south, but the difference angle $\varphi-\varphi_0$ is zero when the normal area vector $\vec{A}$ is directed due west. The area of the interferometer used in these experiments is $A = 8.864\ \text{cm}^2$ and the colatitude angle at Columbia, Missouri, is $\theta_L = 51.37°$. Equation (8) thus predicts a phase shift

$$\beta_{\text{Sagnac}} = 91.65\sin(\varphi-\varphi_0)\ \text{deg (theory)}. \qquad (9)$$

In principle, if we turn the interferometer about the vertical incident beam direction AB in Fig. 1, the counting rates in detectors C_1 and C_2 will vary according to Eqs. (1). We have found, however, that there is a natural variation of the beam intensities as a function of φ due to the energy-angle correlations in the incident beam resulting from the monochromation process using single crystals. We have therefore measured the relative phase shift for each rotation angle φ directly. In any interferometer there is always an undetermined phase shift β_0 as a result of the fact that the two legs ABD and ACD are not precisely equal. Thus the observed phase shift in this experiment is expected to be of the form

$$\beta = \beta_{\text{Sagnac}} - \beta_0 = 91.65\sin(\varphi-\varphi_0) - \beta_0. \qquad (10)$$

To measure the phase shift directly, we insert a slab-shaped phase shifter into the interferometer as shown in Fig. 1. This slab is another Si single crystal of thickness $T = 0.2931 \pm 0.0001$ cm. Rotating this slab through an angle δ about an axis normal to the parallelogram $ABDCA$ results in a phase shift arising from the mean neutron-nuclear potential given by

$$\beta_{\text{shifter}} = -2\lambda NbT\,\frac{\sin\delta\sin\theta_B}{\cos^2\theta_B - \sin^2\delta}, \qquad (11)$$

where λ is the neutron wavelength (in the laboratory frame), N is the atom density, b is the scattering length of Si, and θ_B is the Bragg angle. As we rotate this phase shifter through various an-

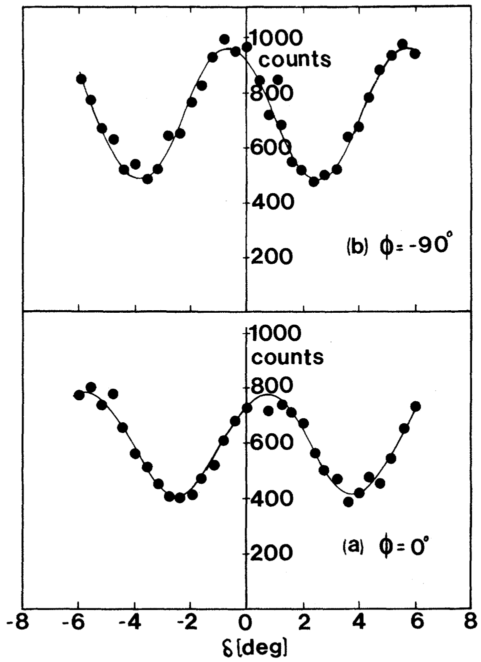

FIG. 2. Typical oscillating counting rates observed in detector C_1 at two orientation settings φ of the interferometer. The counting time for each datum point was approximately 600 sec.

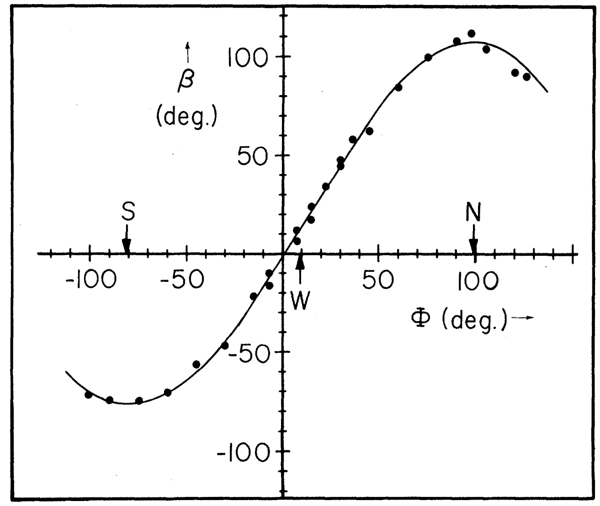

FIG. 3. A plot of the phase shift β due to Earth's rotation as a function of the orientation φ of the normal area $\vec{A}$ of the interferometer about a vertical axis. The symbols N, W, and S indicate north, west, and south.

gles δ, the counting rates in detectors C_1 and C_2 are observed to oscillate. Repeating this process at another setting φ of the interferometer results in another oscillating pattern, of the same period, but shifted in phase with respect to the first pattern. We show in Fig. 2(a) typical data taken at a setting $\varphi = 0°$, and in Fig. 2(b) data taken at $\varphi = -90°$. The phase shift between these two patterns is due to the rotation of Earth. The results of an extensive series of measurements at various interferometer settings φ is shown in Fig. 3. Each datum point in this figure was obtained by least-squares fitting of a sine wave of unknown phase to data of the type shown in Fig. 2. Because of long-term drifts of the interferometer phase, measurements at a reference angle were repeated after each new setting φ. The solid curve in Fig. 3 represents a least-squares fit to the data. We find that

$$\beta_{\text{Sagnac}} = 91.25 \sin(\varphi - 8.8) \text{ deg (experiment)}. \qquad (12)$$

The labeling of north, west, and south on Fig. 3 was achieved through an astronomical sighting of the star Polaris. This line of sight was carried inside the reactor hall (which is below ground level) by precision surveying techniques and mechanically transferred onto the interferometer. Comparing Eqs. (9) and (12), we see that the magnitude of the phase shift of the neutron due to Earth's rotation observed in this experiment is in excellent agreement with theory.

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Four-Quark Model for the Mesons

D. D. Brayshaw
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 11 July 1978)

A four-quark mechanism is proposed which generates virtually the entire known spectrum of (noncharm) meson states to remarkable accuracy. The ground-state 0^- and 1^- nonets are taken as input; exclusive of octet-singlet mixing there are no free parameters.

Previous attempts to derive the meson spectrum on the basis of the quark model have focused almost entirely on the $q\bar{q}$ system. Such approaches invariably involve the introduction of a potential (and/or a bag), with associated free parameters. Unfortunately, our experience indicates that the potential must be very complex (and the number of parameters large) in order to generate a detailed description (e.g., at the level of this paper). Recently, several authors have studied four-quark ($q\bar{q}q\bar{q}$) systems with regard to a few select conventional states (0^{++}), and 0^{--} exotics.[1] However, this approach is again within the potential/bag framework, and is not (at present) quantitative. In contrast, taking the ground-state 0^- and 1^- nonets as input, the four-quark mechanism proposed in this Letter involves *no* free parameters, and generates every well-determined meson state to a remarkable accuracy. This implies that the mechanism responsible for confining the $q\bar{q}$ s waves, whatever its nature, does not directly produce low-mass states with $l \geqslant 1$.

Specifically, I propose that the excitations commonly associated with $l \geqslant 1$ arise as the consequence of a simultaneity condition involving the pairwise masses of all four $q\bar{q}$ combinations. In order to understand this condition, we first consider Fig. 1(a), which depicts three mesons m_1, m_2, and m_3 resonating in pairs to produce particles A (m_1m_2) and B (m_1m_3) simultaneously. This can occur only if the invariant three-body mass takes on a particular value M_0 determined by the masses of A, B, and the three mesons. Almost a generation back, Peierls noted the sharp energy dependence of this effect, and proposed that it could be responsible for generating the $N^*(1512)$.[2] Others extended Peierls's treatment to produce excellent predictions for the masses of the A_1, Q, and E mesons. Physically, there is nothing strange about this effective "force"; for example, in the singly ionized hydrogen molecule, molecular binding is produced by exactly such an (exchange) mechanism.

Mathematically, the realization of this effect involves some subtleties. I recently noted that past technical objections to the sheet structure[3] can be eliminated by restating the condition as follows. Consider particles m_1 and m_3 to form again resonance B, but take particles m_2 and m_3 to be relatively at rest. The related singularity is now on the *correct* sheet, and is guaranteed to be strong in the limit that B has zero width.[4] Moreover, if particles m_2 and m_3 are identical, the Peierls kinematic condition is simultaneously satisfied in the same limit. The restriction to

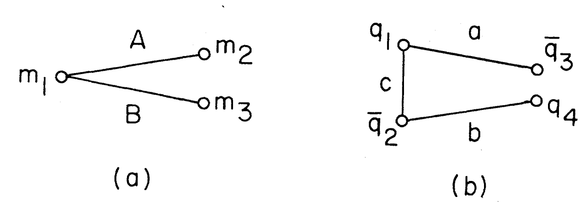

FIG. 1. (a) Three-meson system with pairs forming resonances A and B. (b) Four-quark system with pairs forming mesons a, b, and c. The pair $\bar{q}_3$ and q_4 are relatively at rest.

