Vortex Waves in Superfluid ^{4}He

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The motion of ions along vortex lines in the presence of a transverse radio-frequency electric field has been investigated. An anomaly in the velocity—versus—electric-field relation, associated with the "resonant" generation of vortex waves, is observed. These measurements constitute the first observation of vortex waves at frequencies larger than a few hertz.

A quantized vortex is a macroscopic excitation of superfluid helium in which the superfluid executes azimuthal motion about a line with a velocity given by

$$v_s(r) = (\hbar/mr)\,\hat{l}_\theta, \tag{1}$$

where $\hbar$ is Planck's constant divided by 2π, m is the mass of a helium atom, and r is the distance from the line. Vortex waves are traveling helical deformations of the line having a dispersion relationship[1,2]

$$\omega(k) = (\hbar k^2/2m)[\ln(1/ka) + 0.1159], \tag{2}$$

where a is the effective "radius" of the vortex core. These waves are polarized so that an element of vortex line executes circular motion in a sense opposite to the circulation sense of the velocity field. A number of experiments have been done in which vortex waves having a frequency of order 1 Hz have been observed.[3] In this paper we describe an experiment in which the vortex-wave dispersion relation is tested at radio frequencies.

Halley and Cheung[4] and more recently Halley and Ostermeier[5] have suggested that an rf electric field, transverse to a vortex line charged with ions moving along the line, would couple strongly to vortex waves under suitable conditions. It is reasonable to expect strong coupling, i.e., "resonant" generation of vortex waves, when the following two conditions are satisfied:

$$\omega_{\rm rf} = \omega(k) - kv_{\rm ion}, \tag{3}$$

$$v_{\rm ion} = \partial\omega(k)/\partial k. \tag{4}$$

The first condition is simply that the vortex-wave frequency and sense of rotation, in the frame of reference of the moving ion, is the same as the rf frequency and sense of rotation. The second condition ensures that any energy pumped into the vortex wave, which of course moves with the group velocity of the vortex wave, remains in the vicinity of the ion. These two conditions determine a characteristic ion velocity which depends on the rf frequency. If the longitudinal dc electric field, driving the ions along the vortex line, is measured as a function of ion velocity, an anomaly should be observed at the characteristic velocity. The vortex waves produced near "resonant" conditions clearly have phase velocities in the same direction as the ion velocity so that the ion drag should be enhanced.

The only propagating vortex-wave modes in our frequency range are those which are polarized in a sense opposite to the circulation sense of the vortex. Because the vortex-wave group velocity is larger than its phase velocity, the sign of $\omega_{\rm rf}$ is opposite to that of $\omega(k)$ and conditions (3) and (4) can only be satisfied simultaneously for an rf electric field polarization in the same sense as the vortex circulation. The ion-velocity anomaly should therefore be observed in only one rf field polarization for a given sense of rotation of the apparatus.

The experiment involves the use of a rotating ^{3}He refrigerator in which a sample of ^{4}He can be cooled to 0.3 K while rotating at 10 rad/sec. This rotation speed yields a uniform distribution of vortex lines oriented along the rotation axis with a density of 2×10^4 cm^{-2}. A schematic of our experimental cell is shown in Fig. 1 and, except for the rf electrodes, is similar to those used previously for ion mobility studies.[6] In order to properly observe the predicted anomaly, it was necessary to ensure that both the longitudinal dc electric field and the transverse rf electric field were reasonably uniform. This was accomplished by having the drift field defined by four stacks of electrodes, each of which consisted of eight electrodes stepped in dc potential. rf potentials were ac coupled to the electrodes and applied in a circularly polarized mode. Circular polarization not only helped distinguish real from spurious effects (because of the intrinsic polarization of the vortex waves), but also helped achieve rf-field uniformity over the cross section of the drift region.

A plot of ion velocity versus dc electric field,

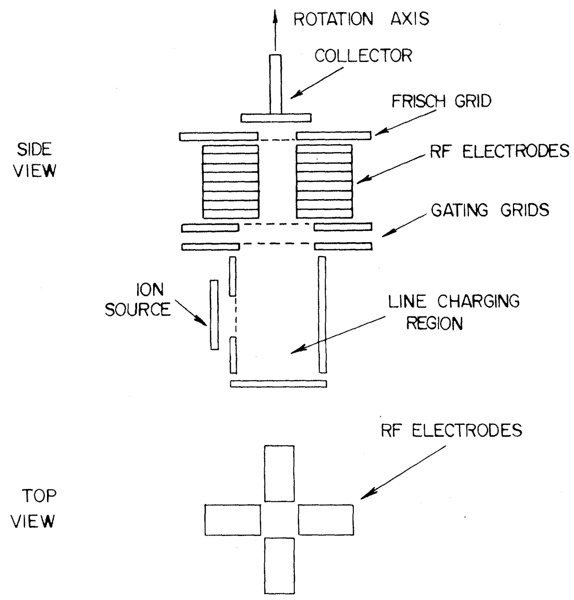

FIG. 1. A schematic drawing of the experimental cell.

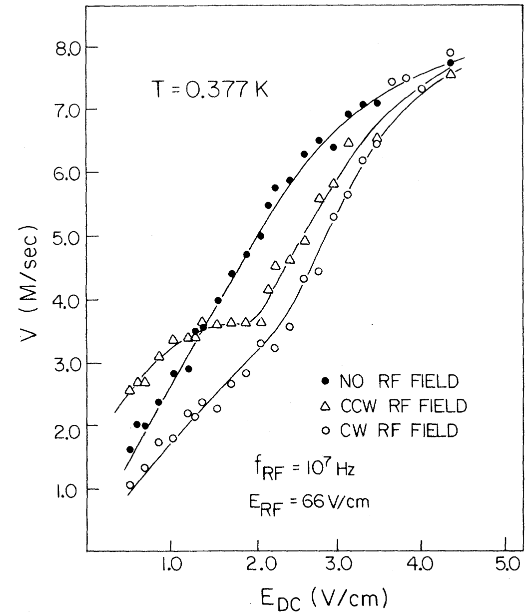

FIG. 2. A plot of ion velocity versus dc electric field for no rf field and for the rf field polarized in the clockwise and counterclockwise senses. The solid lines are freely drawn through the data points in order to guide the eye.

for the cases of no rf field and for the rf field polarized in the clockwise and counterclockwise senses, is shown in Fig. 2 (in this case the ion cell was rotated in the counterclockwise sense). Note that the ion velocity is anisotropic with respect to rf field polarization. Reversing the apparatus-rotation sense, and hence the vortex-circulation sense, reproduces the data with the roles of the two rf-field polarizations reversed. An anomalous kink and plateau in the velocity versus dc electric field curve, for counterclockwise rf-field polarization, is observed near the characteristic velocity determined by Eqs. (3) and (4). A simultaneous solution of Eqs. (3) and (4) yields $v_{\text{ion}} = 3.0$ M/sec for the rf frequency used. The small discrepancy between this value and the plateau velocity observed can be explained in terms of the ac field inhomogeneity—the ions move much faster near the top and bottom of the drift region where the ac-field amplitude is small. At the characteristic velocity, the wavelength of the resonantly generated vortex waves (2000 Å) is two orders of magnitude larger than the ion radius and three orders of magnitude larger than the vortex-core radius. The kink is not perfectly sharp, of course, because of finite vortex-wave damping as well as residual field inhomogeneities. This observation confirms the most important prediction of Halley and Ostermeier[5] and constitutes a measurement of the vortex-wave dispersion relation at rf frequencies.

Halley and Ostermeier[5] go beyond a calculation of the characteristic ion velocity and, using the formalism of Ohmi and Usui[7] to describe the dynamics of the coupled ion–vortex-line system, they calculate the complete velocity–versus–electric-field relation. Our results differ in some important ways from those of the calculation.[8] For counterclockwise (i.e., "resonant") rf polarization and for ion velocities smaller than the characteristic velocity, we observe that the effect of the rf field is to increase the ion velocity significantly. One can show that the only vortex waves that ought to be generated under these circumstances propagate in the direction of the ion velocity, thus *increasing* the ion drag. Furthermore, the magnitude of the effect is much larger (by two orders of magnitude) than predicted. For clockwise ("nonresonant") polarization, the effect of the rf field is observed to be comparable in magnitude, but also has the opposite sign from the calculated effect. Vortex waves propagating both along the direction of the ion velocity and opposite to it ought to be generated under these circumstances. Assuming, quite reasonably, that the damping coefficient for vortex waves is not very dependent on wave number, the calculation shows that the net effect is to *decrease* the ion drag rather than increase it as observed.

These discrepancies should not be too surprising given that the Ohmi-Usui formalism fails by orders of magnitude to account for the observed ion velocity in the absence of an rf field.[6] The nature of the failure, which disappears in the presence of a small concentration of ^{3}He impurities, is not at all clear but is probably related to a lack of sufficient damping experienced by the vortex line or the ion–vortex-line system. Ohmi and Usui calculate the reflection coefficient for vortex waves assuming that the ion is located on the axis of an otherwise perfectly straight vortex line. It is quite possible that the reflection coefficient would be much smaller for an ion not at rest and located off the vortex-line axis. A transverse rf field might enhance the damping if properly polarized and destabilize the system in the opposite polarization. In any event, the existence of a plateau at the characteristic velocity should not be affected by these considerations.

In the course of studies of the mobility of ions along vortex lines in the absence of an rf field, Ostermeier and Glaberson[9] reported an interesting electric-field dependence at large electric fields. In weak fields, the ion velocity increased linearly with field, characteristic of ordinary mobility behavior. As the electric field was increased further, the velocity saturated and became completely field independent over a range of more than two orders of magntiude in field. They suggested that the limiting velocity was associated with strong coupling between the ion and an appropriate vortex wave. Conditions identical to Eqs. (3) and (4) were proposed, with the rf frequency replaced by the natural frequency of the bound ion in its hydrodynamic potential well. The velocity plateau observed in the present experiment lends support to this interpretation of the limiting velocity. It is worth pointing out that the determination of the ion natural frequency based on this interpretation is the only measurement to date which probes the details of the structure of a vortex core.

The quality of our data is not as high as we had hoped and was principally limited by rf heating in the apparatus which, in turn, severaly limited the rate at which data could be acquired. We have, however, been able to observe vortex-wave generation and have demonstrated the feasibility of the technique for mapping out of the vortex-wave dispersion relation over a wide frequency range.

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[2]A general treatment of vortex oscillations is presented by A. L. Fetter, in *Lectures in Theoretical Physics*, edited by K. T. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969), Vol. XI-B, p. 321. Vortex oscillations having wavelengths comparable to the core radius or associated with deformations of the shape of the core itself are not relevant to the work described in this paper.

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[8]In Ref. 5, the authors erroneously ignored the intrinsic polarization of vortex waves and therefore included a larger class of vortex waves in their calculation than they should have. In this paragraph we compare our results with their revised calculation. Note that the figures in Ref. 5 are drawn for a situation in which the rf field is linearly polarized and therefore represent a combination of the effects of the two circularly polarized fields. The effect of the revision is not qualitatively significant for linearly polarized fields, but is important in determining the expressions appropriate for circularly polarized fields (J. W. Halley and R. M. Ostermeier, private communication).

[9]R. M. Ostermeier and W. I. Glaberson, Phys. Rev. Lett. 35, 241 (1975); see also Ref. 6.