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Precritical Behavior in Pionlike Nuclear Excited States

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(Received 27 December 1978)

The possible occurrence of precritical phenomena in finite nuclei due to the relative proximity of the pion condensation threshold is investigated. We point out that such precritical behavior may occur in inelastic scattering differential cross sections for the excitation of unnatural-parity states at high momentum transfers $[q \simeq (2-3)m_\pi]$. As an example, we discuss inelastic proton scattering to 1^+ states in ^{208}Pb.

The question whether pion condensation appears as a new phase of dense baryonic matter has been of continuous interest in recent years.[1-3] While early estimates[1] have suggested that the critical density for condensation in nuclear matter should be lower than normal nuclear matter density, $\rho_0 = 0.5 m_\pi^3$, more realistic approaches incorporating Δ isobars, short-range repulsive baryon-baryon correlations, and the density dependence of the effective nucleon mass come to the conclusion that pion condensation in symmetric nuclear matter is very unlikely to appear around or below ρ_0.[4] Nevertheless, the question has been raised which nuclear properties could serve as a possible indicator of critical behavior in channels carrying pion quantum numbers, even if a pionic soft mode is not expected to appear in ordinary finite nuclei.[5,6]

The possible occurrence of precritical phenomena has been suggested by Gyulassi and Greiner[7] and by Ericson and Delorme,[8] who use the term "critical opalescence" for the physical consequences of an effective enhancement of the pion field inside the nucleus, as the pion condensate is approached. Let us illustrate the nature of precritical behavior close to a pion condensate in the case of infinite nuclear matter. Consider the coupling of a low-frequency ($\omega \ll m_\pi$) pion, for example by inelastic proton scattering, to a pionlike particle-hole excitation, as illustrated in Fig. 1, through various virtual intermediate nucleon-hole and Δ-isobar-hole states. This many-body renormalization of the pion propagation

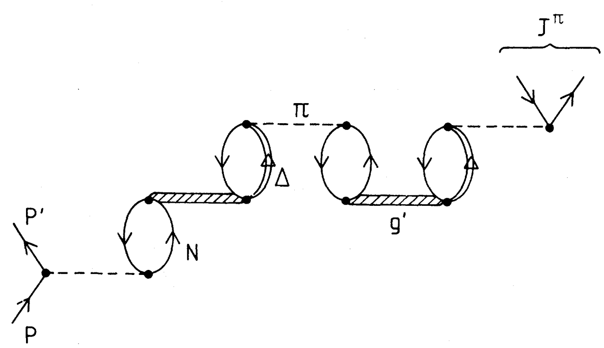

FIG. 1. Inelastic proton scattering into a low-lying "pionlike" excited state through intermediate excitation of high-lying nucleon- and isobar-hole states. Shown are one-pion-exchange pieces (π) and additional contributions from short-range baryon-baryon correlations (g').

modifies the basic one-pion–exchange interaction such that the effective interaction in this channel can be written

$$\tilde{V}_\pi(\vec{q}) = V_\pi(\vec{q})/\epsilon(q)\,, \tag{1}$$

where the driving term contains the free one-pion–exchange interaction plus short-range correlations which are described by a Landau parameter g':

$$V_\pi(\vec{q}) = -[f^2(q^2)/m_\pi^2][(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q})/(q^2+m_\pi^2) - g'\,\vec{\sigma}_1\cdot\vec{\sigma}_2]\vec{\tau}_1\cdot\vec{\tau}_2\,, \tag{2}$$

where $f^2(q^2)/4\pi = 0.08[(\Lambda^2 - m_\pi^2)/(\Lambda^2+q^2)]^2$ is the pion-nucleon vertex factor, and the static limit has been taken in Eq. (2) since we consider excitation energies $\omega \ll m_\pi$ only. The Landau parameter g' is meant to incorporate ρ exchange, additional short-range correlations and other many-body vertex corrections[9].

The polarization denominator in Eq. (1), which we may call the "dimesic function," becomes

$$\epsilon(q) = 1 + [g' - q^2/(q^2+m_\pi^2)][f^2(q^2)/m_\pi^2][U_N(q) + 4U_\Delta(q)]. \tag{3}$$

The Lindhard functions $U_N(q)$ and $U_\Delta(q)$ include all the effects of the virtual intermediate nucleon-hole and isobar-hole excitations, respectively. The form of U_N and U_Δ is given in Ref. 4 and the factor of 4 multiplying U_Δ in Eq. (3) is the standard ratio of $\pi N\Delta$ and πNN coupling strength.[2]

Note that pion condensation appears in this model at the density ρ_c where $\epsilon \to 0$. This situation takes place at large momenta, typically $q \sim (2-3)m_\pi$. Thus, if the interaction of Eq. (1) is used in the description of an inelastic scattering process, one expects that ϵ has the tendency of becoming small around the critical range of momentum transfers q even if the density of the system is still much below the critical density for condensation. This can lead to an enhancement in $\tilde{V}_\pi(q)$, and therefore in the scattering cross section. This characteristic enhancement is what we would like to refer to as "precritical behavior."

Applications to finite systems are complicated by the fact that momentum conservation is lost in the summation over intermediate particle-hole excitations which lead to the dimesic function ϵ. In order to calculate the pion self-energy, or polarization function, in a finite nuclear system (omitting short-range correlations for the moment), we expand in partial waves:

$$\begin{aligned}\Pi(\vec{q},\vec{q}\,') &= -\textstyle\sum_{JM}\sum_{\mathrm{p,h};J}(q)Y_{JM}(\hat{q})[1/(E_\mathrm{p}-E_\mathrm{h})]Q_{\mathrm{p,h};J}{}^*(q')Y_{JM}{}^*(\hat{q}')\\ &\equiv \textstyle\sum_J \Pi_J(q,q')[(2J+1)/4\pi]P_J(\hat{q}\cdot\hat{q}')\,,\end{aligned} \tag{4}$$

where $\vec{q}\,'$, and $\vec{q}$ are the incoming and outgoing pion momenta and

$$Q_{\mathrm{p,h};J}(q) = if(q^2)\alpha(q/m_\pi)\textstyle\sum_L \langle L1;J|j_\mathrm{p}j_\mathrm{h};J\rangle(J0|0|L0)F_{\mathrm{p,h};L}(q)\,, \tag{5}$$

where $\alpha = 2$ for nucleons and $\frac{8}{3}$ for isobars, respectively. Furthermore, $\langle L1;J|j_pj_h;J\rangle$ denotes the LS-jj transformation coefficients and $F_{\mathrm{p,h};L}(q)$ is

$$F_{\mathrm{p,h};L}(q) = (4\pi)^{1/2}(-i)^L(-1)^{l_\mathrm{h}}\left(\frac{(2l_\mathrm{p}+1)(2l_\mathrm{h}+1)}{2L+1}\right)^{1/2}(l_\mathrm{p}0l_\mathrm{h}0|L0)\int_0^\infty r^2\,dr\,j_L(qr)R_\mathrm{p}(r)R_\mathrm{h}{}^*(r)\,. \tag{6}$$

In the process of iterating the pion self-energy, we encounter in second order integrals of the type

$$\Pi_J{}^{(2)}(q,q') = \int_0^\infty \frac{k^2\,dk}{(2\pi)^3}\,\Pi_J(q,k)D_0(k)\Pi_J(k,q')\,, \tag{7}$$

where $D_0(k) = -(k^2+m_\pi^2)^{-1}$, the free-pion propagator. We have to study the degree of nonlocality in the partial-wave self-energies $\Pi_J(q,k)$. First, let us investigate the diagonal part $\Pi_J(q,q)$ in some detail. A typical example is shown in Fig. 2(a). The various curves in Fig. 2(a) show the degree of convergence with increasing particle space. While convergence is obtained already at $N=10$ (N is the maximum oscillator shell considered) for $q=m_\pi$, thirty major shells are required to collect the full strength for $q\sim(2-3)m_\pi$. This means that a full-sized random-phase-approximation diagonalization taking into account the one-pion-exchange tensor force would require extremely large particle-hole spaces in order to describe correctly the high-momentum-transfer behavior in pionlike excitation nels.

The momentum-space nonlocality in $\Pi_J(q,k)$ is studied in Fig. 2(b). We observe that $\Pi_J(q,k)$ peaks

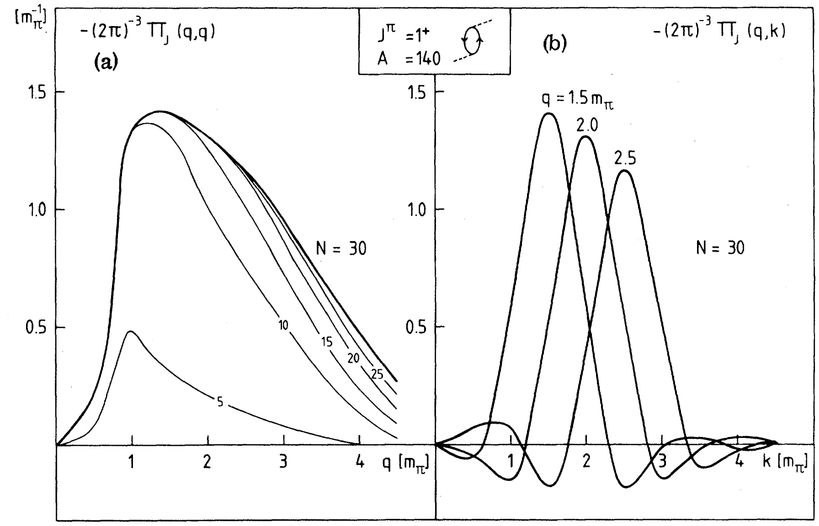

FIG. 2. The partial-wave pion self-energy $\Pi_J(q,k)$ of Eq. (4), for the example of $J^\pi = 1^+$ and a hypothetical closed-shell system with $A = 140$. Particle-hole excitations are calculated in a harmonic-oscillator basis with oscillator constant $\hbar\omega = 41A^{-1/3}$ MeV. (a) Diagonal part $\Pi_J(q,q)$. Numbers indicate the maximum principal quantum number which truncates the particle space. (b) Nondiagonal values $\Pi_J(q,k)$, showing the momentum-space nonlocality. Approximation by the form Eq. (9) gives $R_J = 5.94$ fm.

strongly at $q = k$ as a result of cancellations for intermediate excitations with $q \neq k$. We choose the representation

$$\Pi_J(q,k) = -g_J(q) d_J(q-k) g_J(k), \tag{8}$$

where $d_J(q-k)$ is a distribution and $d_J(0) = 1$. According to Fig. 2(b), this distribution can be very well approximated for not too small nuclei ($A \geq 16$) by a Bessel function: $d_J(q) \simeq j_0(qR_J)$, where the parameter R_J, by its inverse, measures the size of the momentum-space nonlocality and turns out to be close to the nuclear radius R, as expected. In this particular case, Eq. (7) can be approximated for $q, q' > m_\pi$ to a very good degree of accuracy by

$$\Pi_J^{(2)} = (8\pi^2 R_J)^{-1} g_J(q) d_J(q-q') g_J(q') \times [\bar{q}^2/(\bar{q}^2 + m_\pi^2)]\, \Pi_J(\bar{q},\bar{q}), \tag{9}$$

where $\bar{q}$ is the mean value of q and q'.[10] Successive iteration of Π_J to all orders, using Eq. (9), yields

$$\Pi_J^{(\infty)}(q,q') = \Pi_J(q,q')/\epsilon_J(\bar{q}), \tag{10}$$

where

$$\epsilon_J(\bar{q}) = 1 + [\bar{q}^2/(\bar{q}^2 + m_\pi^2)]\, \gamma_J\, \Pi_J(\bar{q},\bar{q}). \tag{11}$$

Although the form of Eq. (11) is similar to that for nuclear matter (in the absence of short-range correlations), we note that the dimesic function ϵ_J, Eq. (11), is now J dependent. The inclusion of short-range correlations (in terms of g') into Eq. (11) is straightforward.

We shall now present an application of the formalism developed above to the case of inelastic proton scattering into $J^\pi = 1^+$ states in ^{208}Pb,[11] in order to show how precritical behavior might develop. Here the distribution $d_J(q)$ in Eq. (8) is localized in a relatively small range ($\Delta q \simeq \pi/R$), and the approximations Eqs. (9) and (10) concerning the momentum-space nonlocality are supposed to be well founded. The scattering amplitude is then calculated according to the diagram of Fig. 1, using plane waves for the ingoing and outgoing proton.

Calculations of the 1^+ states are performed as follows. The "$0\hbar\omega$" excitations ($\pi h_{9/2} h_{11/2}^{-1}$ and $\nu i_{11/2} i_{13/2}^{-1}$) are taken into account explicitly, whereas all other intermediate particle-hole states, including also the isobar excitations, are treated in terms of the pion self-energy [see Eq. (4)]. Thus, most of the polarization strength is described by the dimesic function $\epsilon_{J=1^+}$. As a result, we observe a pronounced enhancement in the effective interaction at $q \sim (2-3) m_\pi$. On the other hand, the $0\hbar\omega$ particle-hole matrix elements and therefore the excitation energies of the two 1^+ states show relatively small influence from the polarization effect. [$E_x(1_\downarrow^+) = 6.7$ MeV,

$E_x(1_\uparrow^+) = 7.8$ MeV for $g' = 0.6$ and $E_x(1_\downarrow^+) = 5.7$ MeV, $E_x(1_\uparrow^+) = 7.0$ MeV for $g' = 0.42$; the variation is mainly due to the change in g'. Since ^{208}Pb has $N \neq Z$, we also take into account a spin-spin interaction of the form $g\vec{\sigma}_1 \cdot \vec{\sigma}_2$ with $g = g'$.] This is clearly due to the fact that the momentum distributions of the $0\hbar\omega$ wave functions are peaked at $q \sim m_\pi$ and decrease so rapidly that the pronounced enhancement at large momenta is hardly felt by the small-space matrix elements.

The result for $d\sigma/d\Omega$ in plane-wave Born approximation for the lower 1^+ state is presented in Fig. 3 as a function of momentum transfer q. The various curves show the dependence on g', which is strong, as expected at $q \sim (2-3)m_\pi$,

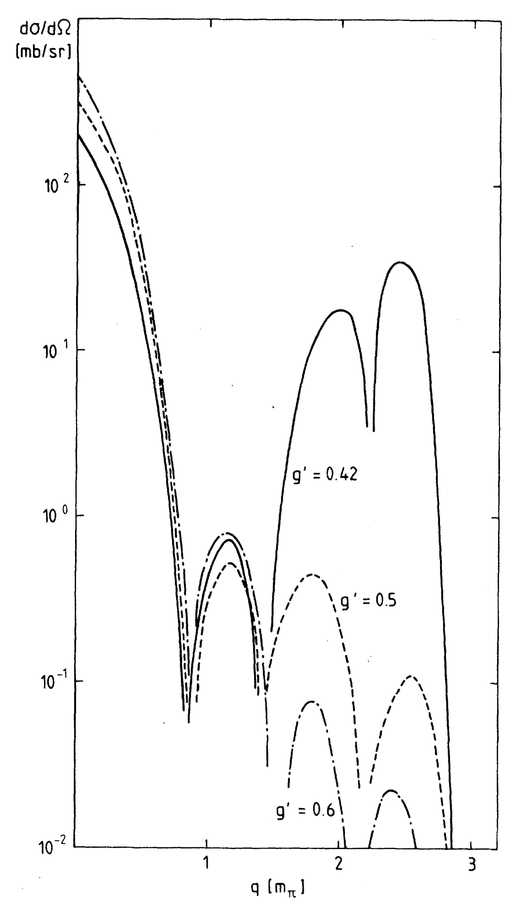

FIG. 3. Proton inelastic-scattering cross section to the lowest 1^+ state in ^{208}Pb. The calculations are performed within the plane-wave Born approximation for a proton energy $E_p = 5m_\pi$. The various curves show the dependence on the Landau parameter g' which describes the short-range interaction. The long-range polarization effects are included in terms of the dimesic function $\epsilon_{J=1^+}$ (see text) in the effective particle-hole interaction.

while only small changes appear for $q < 1.5m_\pi$. We note that the precritical behavior in this 1^+ excitation becomes critical (in the sense that $\epsilon_{J=1^+} \to 0$) for $g' = 0.40$ which is somewhat smaller than the critical g' in nuclear matter ($g' = 0.45$ according to Ref. 4). Note that for $g' = 0.5$ the precritical enhancement in $d\sigma/d\Omega$ around $q \sim (2-3)m_\pi$ would still be one order of magnitude as compared to a calculation with $\epsilon_{J=1^+} = 1$. Distortion effects of the ingoing and outgoing proton waves have been estimated by using wave functions $\chi^{(\pm)} \propto e^{i\vec{k}(1\pm i\gamma)\vec{r}}$ normalized to obtain the proper damping in the nuclear interior. The resulting $d\sigma/d\Omega$ is then reduced by an overall absorption factor, but the relative enhancement due to precritical behavior at $q \sim (2-3)m_\pi$ is not destroyed as long as $\gamma \lesssim 0.1$ (a reasonable upper limit at energies as high as $E_p \sim 5m_\pi$).

In conclusion, we have outlined the possibility of a precritical enhancement in differential cross sections for inelastic scattering to pionlike excitations in heavy nuclei. A strong enhancement at momentum transfers $q \sim (2-3)m_\pi$ would indicate the proximity of the pion condensation threshold, whereas the energies of such states would still remain relatively unaffected. Such precritical behavior, if existent, would be the result of strong coherence in a large number of very high-lying particle-hole states, and would be invisible in standard random-phase-approximation calculations using small model spaces.

This work was supported in part by Deutsche Forschungsgemeinschaft under Contract No. WE-655/4.

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[10]For $q, q' < m_\pi$, Eq. (9) becomes inaccurate, but this

is also the region where the polarization effects are unimportant.

[11]Note, however, the experimental situation of these 1^+ states is yet unclear, though in a pure shell model one expects two states around 7-MeV excitations. See, for recent experimental information, W. Knüpfer, R. Frey, A. Friebel, W. Mettner, W. Meuer, A. Richter, E. Spamer, and O. Titze, Phys. Lett. 77B, 367 (1978); S. Raman, M. Mizumoto, and R. L. Macklin, Phys. Rev. Lett. 39, 598 (1977), and references therein.