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# Conformal Gravity as a Gauge Theory

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It is shown that the same two features that determine the structure of ordinary gravity and supergravity theories also determine the structure of conformal gravity. The relevant local gauge symmetry turns out to be the inhomogeneous Weyl group. The transformation laws of the fields follow from geometry. Some consequences of breaking the scale symmetry are briefly discussed.

The conformal theory of gravity was introduced by Weyl in 1918.[1] During the succeeding six decades, it has been revived and fallen to disfavor several times. The most recent interest in such theories is related to the global supersymmetry invariances of the $S$ matrix.[2] Motivated by this, conformal and superconformal theories have been investigated in a number of articles.[3,4] Despite these interesting attempts, the overall status of such theories is unclear because the properties which have emerged so far are tied down to the specific choice of Lagrangians as well as a number of additional constraints.

On the other hand, it has recently been pointed out[5] that the structure and invariances of ordinary gravity and OSp$(N;4)$ supergravity theories can be understood in terms of two physically motivated requirements: (a) In contrast to Yang-Mills theories of internal symmetries, gravitation is described by a nonlinear realization of the gauge symmetry.[6-8] (b) To ensure that the gauge group in question describes not an internal but a space-time symmetry, the horizontal (general coordinate) and vertical (gauge) transformations in an appropriate fiber bundle must be "interlocked."[9,10] Unlike nonlinear realizations of internal symmetries, it is the condition (b) which provides the constraint for a nonlinear realization of space-time gauge symmetry and determines the model-independent transformation laws of fields for part of the group which is realized nonlinearly.[5,11] The purpose of this Letter is to show that conditions (a) and (b) also give a geometrical model-independent description of conformal gravity. Some of the features which make this formulation distinct from all previous ones are these: (i) The local gauge group will be seen to be the inhomogeneous Weyl group, consisting of local Poincaré and scale transformations. (ii) As in ordinary gravity the "local" algebra of the group differs from its global form. (iii) Just as in Yang-Mills theory the transformation laws of the fields follow directly from geometry. (iv) No *ad hoc* constraints are introduced.

*Choice of gauge group.*—To determine the relevant gauge group one must decide in what way the symmetries of conformal gravity differ from the conventional gravity. The latter theories are based on local Lorentz and general coordinate invariance, and our requirements (a) and (b) can be used to formulate them as local gauge theories, although nonlinear ones, by noting that general coordinate transformations are just local translation.[5] So the origin of the symmetry which is gauged is traced to the structure of space-time. In conformal gravity the origin of the gauge symmetry must also be traced to the structure of space-time. Otherwise one would be hard pressed to justify it. With this in mind we note that in conformal gravity the structure of space-time allows for local scale invariance, so that from the knowledge of *global* conformally invariant theories it would be tempting to take the local gauge group to be SU(2, 2). However, let us compare the variations of the coordinates $X^\mu$ under global translations and conformal boosts (accelerations) on the one hand and the corresponding local variations on the other. Globally, we have

$$X^\mu \to X^\mu + a^\mu, \quad \text{translations;} \tag{1}$$

$$X^\mu \to (X^\mu + C^\mu X^2)(1 + C\cdot X + C^2X^2)^{-1}, \quad \text{accelerations.} \tag{2}$$

So it is clear that the two transformations differ in character. One is $x$ dependent, the other is not. As local transformations $a^\mu$ is replaced by $a^\mu(x)$, and $C^\mu$ by $C^\mu(x)$, where both $a^\mu(x)$ and $C^\mu(x)$ are arbitrary nonsingular functions of coordinates. Therefore, after making these replacements in Eqs. (1) and (2), it becomes clear that they are no longer distinct. In fact, the transformation $X^\mu \to X^\mu + a^\mu(x)$ contains all the general coordinate transformations including local accelerations. Since such transformations have already been represented by local translations, then introducing separate gauge fields to preserve local invariance with respect to local conformal boosts amounts either to gauging the same symmetry twice or to associating them with a hitherto unknown external symmetry of nature. An alternative view, pursued in this article, is to gauge (introduce gauge fields for) that subgroup of SU(2, 2) transformations which can be traced to the structure of space-time. These are scale and Poincaré transformations, i.e., the elements of the inhomogeneous Weyl group (IW). It is to be emphasized that in this approach the local conformal symmetry is *not* reduced. It is the introduction of redundant gauge fields which is rendered unnecessary.

*Geometry.*—Following the methods of Ref. 5, we want to construct a nonlinear realization of the gauge group $G = \mathrm{IW}$ which is linear with respect to the subgroup $H = \mathrm{SL}(2, C)\otimes \mathrm{U}(1)$. We denote the generators of $G$ by $\{X_A\} = \{X_{ij}, D, X_i\}$ such that $\{X_{ij}, D\}$ generate $H$ and $\{X_i\}$ the "local" $G/H$. Consider the object $\hat{D}_\mu$ with values in the Lie algebra:

$$\hat{D}_\mu = \partial_\mu + H_\mu{}^{ij}X_{ij} + S_\mu D + K_\mu{}^i X_i \equiv D_\mu + K_\mu. \tag{3}$$

To realize the symmetry nonlinearly, $D_\mu$ and $K_\mu$ must *separately* be covariants under $G$.[5] To ensure that the nonlinear realization so obtained represents not an internal but a space-time symmetry, we must implement our requirement (b). To this end we go to the local horizontal basis $\{\hat{X}_i\}$ such that

$$\hat{X}_i = \hat{K}_i{}^\mu, \quad D_\mu = \hat{K}_\mu{}^i \hat{X}_i, \tag{4}$$

$$\hat{K}_i{}^\mu \hat{K}_\mu{}^j = \delta_i{}^j, \quad \hat{K}_\mu{}^i \hat{K}_i{}^\nu = \delta_\mu{}^\nu. \tag{5}$$

The "interlocking" of horizontal and vertical transformations[5] allows one to set $\hat{K}_\mu{}^i\hat{X}_i$ in (4) equal to $K_\mu$ in (3):

$$\hat{K}_\mu{}^i \hat{X}_i \equiv K_\mu. \tag{6}$$

This identification puts the base manifold into one-to-one correspondence with $G/H$ part of the fiber, thus clarifying the notion "local" $G/H$. As usual the components of curvature two-form are given by

$$[D_\mu, D_\nu] = -R_{\mu\nu}{}^{ij}X_{ij} - S_{\mu\nu}D, \tag{7}$$

where

$$R_{\mu\nu}{}^{ij} = H_{\mu,\nu}{}^{ij} - H_{\nu,\mu}{}^{ij} + f_{klmn}{}^{ij}H_\mu{}^{kl}H_\nu{}^{mn} \tag{8}$$

and

$$S_{\mu\nu} = S_{\mu,\nu} - S_{\nu,\mu}. \tag{9}$$

From Eqs. (4)–(7) it follows that

$$[X_i, X_j] = -K_i{}^\mu K_j{}^\nu(R_{\mu\nu}{}^{mn}X_{mn} + S_{\mu\nu}D + T_{\mu\nu}{}^k X_k), \tag{10}$$

where $T_{\mu\nu}{}^k$ is the torsion tensor:

$$T_{\mu\nu}{}^k = D_\mu K_\nu{}^k - D_\nu K_\mu{}^k. \tag{11}$$

The algebra of $G$ is completed by supplementing (10) with

$$[X_{ij}, X_{kl}] = f_{ijkl}{}^{mn}X_{mn}, \quad [X_{ij}, X_k] = f_{ijk}{}^l X_l, \quad [X_j, D] = X_j, \quad [X_{ij}, D] = 0. \tag{12}$$

Thus the local realization of the algebra differs from its global one by the nonvanishing of the commutator $[X_i, X_j]$.

Just as in Yang-Mills theory,[9] the transformation law of the fields under an element $g \in G$ is given by $D_\mu \to gD_\mu g^{-1}$. Infinitesimally, we have

$$D_\mu \to (1 + \epsilon^A X_A)D_\mu(1 - \epsilon^A X_A). \tag{13}$$

For local translations this gives, using our interlocking requirement,

$$\delta K_\mu{}^k = -D_\mu \epsilon^k + \epsilon^j K_j{}^\lambda T_{\mu\lambda}{}^k, \tag{14}$$

$$\delta H_\mu{}^{mn} = \epsilon^j K_j{}^\lambda R_{\lambda\mu}{}^{mn}, \tag{15}$$

$$\delta S_\mu = \epsilon^j K_j{}^\lambda S_{\lambda\mu}, \tag{16}$$

$$\delta R_{\mu\nu}{}^{ij} = K_k{}^\lambda(-D_\mu \epsilon^k R_{\lambda\nu}{}^{ij} + D_\nu \epsilon^k R_{\sigma\mu}{}^{ij}) + \epsilon^\lambda(D_\nu R_{\lambda\mu}{}^{ij} - D_\mu R_{\lambda\nu}{}^{ij}) - \epsilon^m K_m{}^\sigma K_k{}^\lambda(R_{\lambda\mu}{}^{ij} D_\nu K_\sigma{}^k - R_{\lambda\nu}{}^{ij} D_\mu K_\sigma{}^k). \tag{17}$$

Since scale transformations are realized linearly, the variations of the fields due to them are exactly as in any U(1) theory:

$$H_\mu{}^{ij} \to H_\mu{}^{ij}, \tag{18}$$

$$S_\mu \to S_\mu - \partial_\mu \lambda(x), \tag{19}$$

$$K_\mu{}^i \to e^\lambda K_\mu{}^i, \quad K_i{}^\mu \to e^{-\lambda} K_i{}^\mu,$$

$$K \equiv \det(K_\mu{}^i) \to e^{+4\lambda} K. \tag{20}$$

The reader familiar with the literature of supergravity theories will note that the above geometry- and model-independent transformation laws can be immediately generalized to conformal supergravity theories. This generalization has been made and will be reported elsewhere.[12]

*Invariant Lagrangians and breaking of scale symmetry.*—Since the transformation laws we have derived are deeply rooted in geometry, they do not single out a particular Lagrangian. To construct a representative number of these, observe that local Poincaré invariance can be achieved by the index saturation of available tensors. To maintain scale invariance, we note from Eqs. (17) and (18) that $R_{\mu\nu}{}^{ij}$ and $S_{\mu\nu}$ are scale invariants. Aside from topological invariants, the actions must involve $K = \det(K_\mu{}^i)$ to have invariant volume elements. In view of the last two statements, one can construct several conformally invariant actions which are quadratic in curvatures and quartic in $K_i{}^\mu$. Limiting the discussion to this class, one gets, depending on various ways of contracting the indices,

$$\begin{aligned} &K(K_i{}^\mu K_j{}^\nu R_{\mu\nu}{}^{ij})^2 \equiv KR^2, \\ &K g^{\mu\rho} g^{\nu\lambda} S_{\mu\nu} S_{\rho\lambda}, \\ &K g^{\nu\lambda} K_i{}^\mu K_k{}^\rho \eta_{jl} R_{\mu\nu}{}^{ij} R_{\rho\lambda}{}^{kl} \equiv K R_\mu{}^i R_i{}^\mu, \\ &K \eta_{ik} \eta_{jl} g^{\mu\rho} g^{\nu\lambda} R_{\mu\nu}{}^{ij} R_{\rho\lambda}{}^{kl} \equiv R_{\mu\nu}{}^{ij} R_{ij}{}^{\mu\nu}, \end{aligned} \tag{21}$$

where

$$g^{\mu\nu} = K_i{}^\mu K_j{}^\nu \eta^{ij}. \tag{22}$$

For the description of gravity alone consider the last of these which was favored by Weyl himself[13]:

$$I = -b^2 \int d^4x K R_{\mu\nu}{}^{ij} R_{ij}{}^{\mu\nu}, \quad \hbar = c = 1, \tag{23}$$

where $b^2$ is a dimensionless coupling constant. As far as this action is concerned, suppose that the only consequence of the spontaneous breakdown of scale symmetry is an overall shift of the Riemann scalar curvature by an amount $R_0$. Then using the Gauss-Bonet theorem one finds, up to cosmological terms,

$$I \to I' = \int d^4x K(aR - b^2 R_{\mu\nu}{}^{ij} R_{ij}{}^{\mu\nu}), \tag{24}$$

where $a$ depends on $b^2$ and $R_0$. This action is of the same form as that proposed in Ref. 10 from a somewhat different point of view.[14] Recently, a generalized Birkhoff's theorem for this action has been proved,[15] indicating that with respect to tests carried out within solar system it has the same consequences as Einstein's theory. Although this crude way of breaking the scale symmetry does not directly explain the size of the cosmological constant, it is significant that it leads to an acceptable theory of gravity. Conversely, since the action (24) can be traced to the scale-invariant theory (23) with a *dimensionless* coupling constant, its quantum version may be unitary and renormalizable. We hope to return to a more detailed exploration of these ideas elsewhere.

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# Rates and Properties of Trimuon Events Observed in High-Energy Neutrino Interactions

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We report on 39 trimuon events observed in high-energy neutrino interactions at Fermilab. The observed trimuon rate relative to single-muon production is $(1.1\pm0.5)\times10^{-4}$ for $E_\nu > 100$ GeV. The properties of the trimuons are consistent with those for electromagnetic and direct production of muon pairs, although the production of charm-anticharm pairs may account for approximately 20% of the observed rate. New heavy leptons or heavy quarks do not contribute significantly.

The production of multimuon events by neutrinos and antineutrinos has been a subject of great interest since the discovery of dimuon events in 1974.[1] Dimuon events are now known to arise from the production and semileptonic decay of charm mesons. Explanations for the more recent observation of trimuon[2] and tetralepton events,[3] however, are not yet so clear. It was recognized early that the number of possible sources of trimuon events is large.[2b] Ordinary sources are dimuon events in which pions or kaons decay before interacting, and the production in charged-current interactions of virtual photons that yield muon pairs. More interesting possibilities are the production of charmed-particle pairs or completely new processes such as heavy-lepton or new quark production. In this paper we present results on thirty trimuon events observed recently together with nine events previously reported.[2b,2c,4]

The experimental apparatus has been described previously.[2b,2c] It consists of three targets for neutrino interactions followed by a large-acceptance muon spectrometer comprised of iron toroidal magnets. The three targets are 250 tons of iron, 50 tons of liquid scintillator calorimeter, and 120 tons of iron-plate–scintillator calorimeter. Wide-gap optical spark chambers are located in the target calorimeters and in the muon spectrometer.

The data discussed here were obtained in three runs in the quadrupole triplet and bare-target sign-selected neutrino beams at Fermilab with 400-GeV incident protons.[5] A total of $4.9\times10^{18}$

 