Chaotic Response to Periodic Perturbation of a Convecting Fluid

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We have observed a chaotic response to a small periodic thermal perturbation in time-dependent Rayleigh-Bénard convection. The induced noise shows a threshold in the driving amplitude, and occurs when the driving frequency nearly equals the frequency difference between two spontaneous oscillatory modes. The behavior for time-independent boundary conditions is qualitatively similar to that of a fourteen-mode model recently studied by Curry.

A number of recent investigations have been concerned with the onset of turbulence in classical fluid systems. Two cases which have been studied extensively are Rayleigh-Bénard convection[1,2] and Taylor vortex flow.[3] In previous experiments on time-varying flows the boundary conditions have been time independent.[4] Here we describe a study of time-dependent convection in a fluid layer whose lower boundary could be perturbed by a small periodic temperature modulation. We find that this perturbation induces nonperiodic motion when its frequency nearly equals the difference between two spontaneous natural frequencies and its amplitude exceeds a threshold. The generation of chaotic convective motion by a periodic perturbation has not been previously predicted or observed. We have also observed several new phenomena in the absence of modulation which are similar to the behavior of a generalized Lorenz model recently studied by Curry.[5]

Our experiments utilized a rectangular fluid cell of interior dimensions 16×28 mm^2 by 8 mm high containing water at about 70°C where the Prandtl number[1] is 2.5. The vertical temperature difference ΔT across the layer was controlled to a long-term stability of about 0.1% (10^{-3} K) by means of resistive heaters and ac feedback techniques. In addition, ΔT could be modulated by as much as 0.4% (5 mK) with negligible noise in the bandwidth of interest (0.01–0.3 Hz).

In order to study the dynamical state of the fluid, the velocity component parallel to the 28-mm cell edge was measured locally (using laser Doppler techniques) as a function of the relative Rayleigh number[1] $R/R_c = \Delta T/\Delta T_c$, where $\Delta T_c = 37.3$ mK is the temperature difference at which convection begins in an infinite layer of the same thickness. When the velocity field was time independent ($R/R_c < 20$), we used a computer-controlled scanning technique to determine that the flow pattern consists of two convective rolls whose axes are parallel to the 16-mm cell edge. This pattern persists throughout the time-dependent regimes in an average sense. When the velocity was time dependent, we sampled it sequentially up to 8192 times at intervals of 1–2 s, and used standard discrete fast–Fourier-transform techniques to obtain power spectra of the local fluid velocity.

Before describing the effect of modulating the boundary temperature, we summarize the behavior of this system with stationary boundary conditions. For $1 \leq R/R_c < 20$ the fluid is convecting but the velocity field is everywhere time independent. There are three distinct time-dependent regimes which overlap in Rayleigh number, as shown in Fig. 1. For $20 < R/R_c < 39.8$ the velocity field can be strictly periodic, as indicated by a power spectrum containing a single frequency

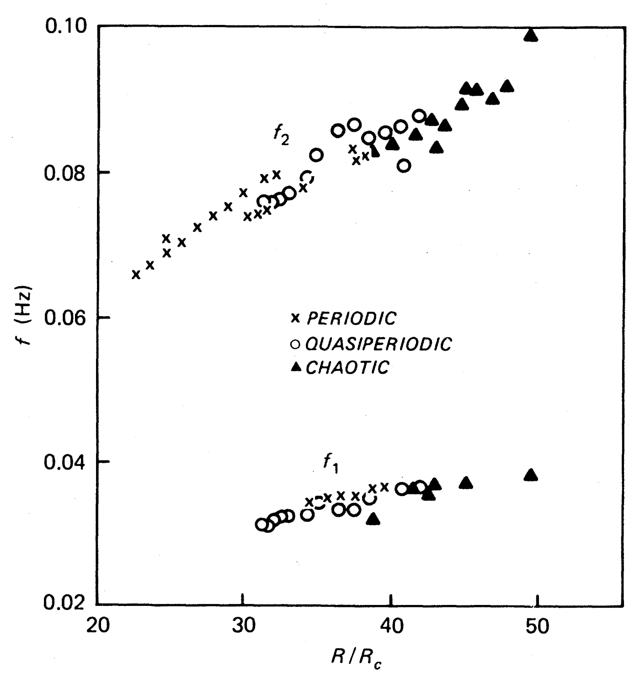

FIG. 1. Frequencies of peaks in the velocity power spectrum as a function of Rayleigh numbers. The symbols are explained in the text.

and its harmonics. For $31.3<R/R_c<41.9$ a quasiperiodic state with two incommensurate frequencies occurs. Finally, a chaotic state with broadband spectral noise occurs for $R/R_c>38.2$. In the domains of overlap between these regimes, the state is determined by initial conditions in a manner which is difficult to control. The crosses denote the frequencies of peaks in spectra containing only a single fundamental component, whereas the circles come from spectra containing two incommensurate frequencies. Even when broadband noise is present, the peaks are usually discernible, and these points are plotted as solid triangles. The accuracy of the frequency measurements is better than 0.1%, and so the variability apparent in Fig. 1 must be intrinsic to the fluid system.

Although not shown in Fig. 1, a large subharmonic of frequency $\frac{1}{2}f_2$ appears discontinuously in the spectrum when $R/R_c \gtrsim 29$. This transition is quite obvious in the time domain as well (Fig. 2), and exhibits hysteresis of approximately 1 in R/R_c. We have plotted the frequency f_2 in Fig. 1 even when the subharmonic was dominant.

We now describe the effect of a small periodic modulation (<5 mK) of the lower boundary temperature on the time-dependent flows. A chaotic response is induced by the perturbation if the mean value of R/R_c is above 37 *and* the frequency of the perturbation lies in a narrow band near $f_2 - f_1$. Three spectra illustrating this effect are shown in Fig. 3. The velocity power spectrum at $R/R_c = 37.9$ in the absence of any modulation is given in Fig. 3(a). The system is in a periodic state with spectral peaks at frequencies $\frac{1}{2}pf_2$ where p is any positive integer. The frequency f_1 is not spontaneously present for the initial conditions of this experiment. The effect of a small modulation of the boundary temperature is to induce additional peaks in the spectrum at f_m and sums and differences of f_m with the naturally present frequencies. The peaks in Fig. 3(b) can be written (to an accuracy of 0.02%) as $f_{p,q} = |\frac{1}{2}pf_2 + qf_m|$ where p and q are integers. For example, the lowest-frequency peak in Fig. 3(b) corresponds to $p=1$ and $q=-1$. The low-order peaks are the largest ones, as one would expect for a nonlinear mixing process, and those for which $|q|=1$ dominate when the amplitude of modulation is small.

When the amplitude of modulation A (half of the peak-to-peak value) exceeds a threshold which is a function of (mean) Rayleigh number, broadband noise appears in the spectrum with finite intensity as shown in Fig. 3(c). The transition to the noisy state occurs at $A=3.2$ mK for $R/R_c=37.9$ and $A=3.9$ mK for $R/R_c=37.3$ with hysteresis of about

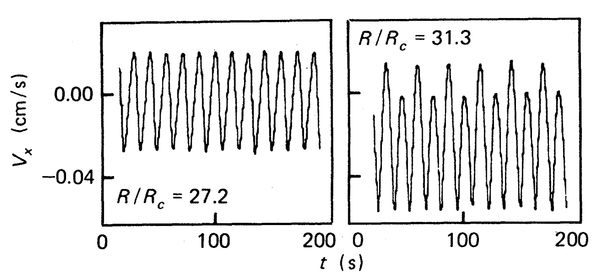

FIG. 2. Time dependence of the x component of the local fluid velocity, showing the presence of the subharmonic $\frac{1}{2}f_2$ when $R/R_c \gtrsim 29$.

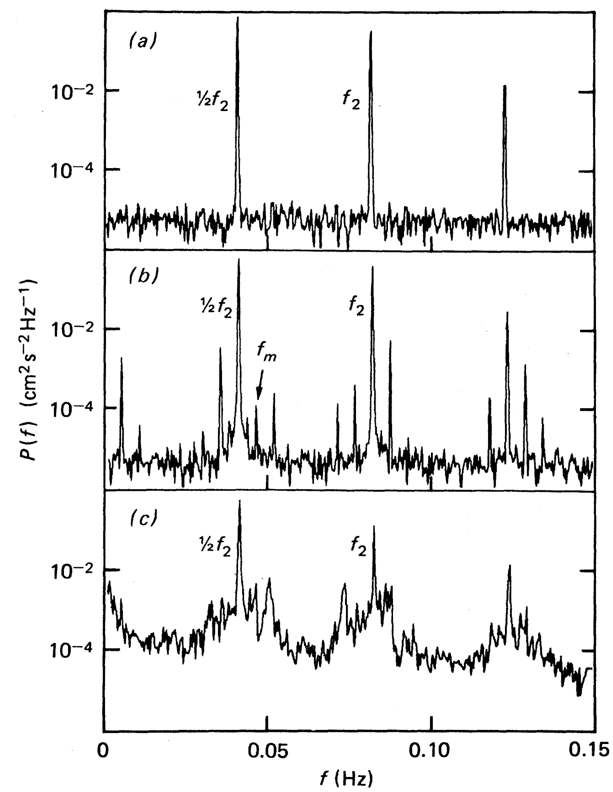

FIG. 3. Velocity power spectra showing induced noise for $R/R_c=37.9$ as the amplitude A of the periodic thermal perturbation is increased: (a) The boundary conditions are time independent, and only the natural frequency $\frac{1}{2}f_2$ and its harmonics are present. (b) Here $A=3.14$ mK, and additional sharp peaks are induced (see text). (c) Here $A=4.86$ mK, and broadband noise is present in the spectrum. The frequency of modulation is 0.0465 Hz. The logarithmic vertical scale should be noted.

0.3 mK. At no time during the experiment does R/R_c exceed the value 38.2 at which noise could occur in the absence of modulation. This would require a modulation amplitude of at least 11 mK at $R/R_c = 37.9$ and 34 mK at $R/R_c = 37.3$.

We quantified the intensity of the induced noise by integrating the spectra with the discrete peaks subtracted out. The variation of noise power with modulating frequency f_m is shown in Fig. 4 for fixed R and A. The transition to the noisy state is apparently discontinuous and exhibits detectable hystersis when f_m is varied, just as is the case when A is varied. The band of frequencies f_m which can induce noise is centered at 0.0465 Hz, and f_2 was measured to be 0.0821 Hz in these experiments. We note that $f_2 - f_m$ is equal to f_1, to within the accuracy with which f_1 can be obtained from Fig. 1 (using the crosses near $R/R_c = 37.9$). Apparently, the system is most responsive to a perturbation that can mix with the mode at frequency f_2 to generate the mode at f_1. There is also a response to our largest available perturbation ($A = 5$ mK) at $f_m = 0.0357$ Hz$\cong f_1$ yielding spectra similar to Fig. 3(b).

All of the above phenomena persist down to $R/R_c = 37$, but the intensity of the induced noise decreases smoothly with decreasing R, A and f_m held constant. For $R/R_c < 37$, sharp components are induced, but broadband noise is not. It is likely that the noise could still be induced for $R/R_c < 37$ by increasing A.

Our observations may be relevant to several recent developments in the theory of dynamical systems. They suggest the applicability to fluid dynamics of a theorem due to Newhouse, Ruelle, and Takens,[6] which implies that under certain conditions a quasiperiodic flow with three or more independent frequencies may be unstable with respect to chaotic flows and hence unobservable. We find experimentally that whether a chaotic flow is induced by varying the Rayleigh number, or the amplitude or frequency of the perturbation, only two distinct frequencies are observable prior to the chaotic transition. However, no detailed theory incorporating thermal modulation is available.

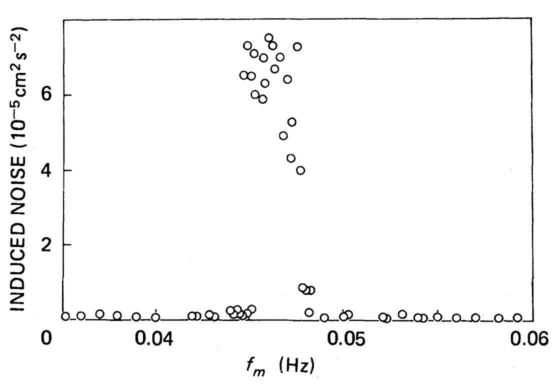

FIG. 4. Induced noise power (obtained by integrating the spectra with sharp peaks removed) as a function of the frequency of the perturbation, for fixed amplitude $A = 4.8$ mK and $R/R_c = 37.9$.

The behavior of this system in the absence of modulation is similar to that of a numerical model investigated recently by Curry.[5] The model considers the interaction of a set of fourteen Fourier modes in the spirit originally proposed by Lorenz.[7] We expect a relatively small number of modes to be appropriate to our geometry because the horizontal dimensions are not much larger than the layer thickness.[8] Curry's model shows the following sequence of phenomena: a Hopf bifurcation to a closed orbit in phase space (periodic state); a bifurcation in which stability is transferred to a periodic orbit with half the frequency; bifurcation to an attracting torus (quasiperiodic state); coexistence of both periodic and quasiperiodic attractors for different initial conditions but the same R; and a strange attractor (chaotic state) of the type first discovered by Lorenz. We observe apparently similar phenomena experimentally, and in the same order. Although no quantitative comparison with this model has been made, the similarity supports the hypothesis that the onset of turbulence can sometimes be described by a nonlinear model with a relatively small number of coupled modes. Furthermore the frequency selectivity of the chaotic response to a small periodic perturbation provides new evidence that only a small number of modes are involved in the transition to turbulence in this system.

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Investigation of Strong Turbulence in a Low-β Plasma

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Experimental investigations of strong turbulence in low-β plasmas are reported. The measurements were carried out in a rotating plasma column produced by surface ionization. The turbulence is excited by instabilities associated with the radial gradients of the plasma column. Measured variations of spectral structure and spectral coefficients agree with recent theoretical predictions.

Instabilities associated with gradients in bounded plasmas (usually denoted "universal instabilities") often develop into a strongly turbulent state. We report the results of an experimental investigation of strongly turbulent, gradient-driven, low-frequency instabilities in a low-β plasma. In addition to an investigation of the spectral structure of the turbulence, we also present what we believe are the first measurements of the variation of the coefficient to this spectrum. The experimental results are compared with recent theoretical investigations based on a fluid description using a cascade decomposition.[1] Here we only quote the results concerning power spectra, $G(k)$, of turbulent potential fluctuations that are directly accessible to measurement in contrast to velocity fluctuations. Only two subranges, denoted "production" and "coupling" subranges, were considered: production

$$G(k) = \text{const}\times (T_e/eC_s)^2\Gamma_n^2k^{-3}; \tag{1}$$

and coupling

$$G(k) = \text{const}\times (T_e/eC_s)^2(\omega_0^4/\lambda_c\omega_c)k^{-5}, \tag{2}$$

where $\Gamma_n^2 = (C_s\,\partial\ln\overline{n}/\partial r)^2$, $C_s^2 = \kappa(T_i + T_e)/M$, $\lambda_c = C_s^2/\omega_c$, and $\omega_0^3 = \omega_c J$ with $J = (eC_s/T_e)^2\int_0^\infty k^2G(k)\times dk$. An overbar denotes time average. These results are obtained assuming strongly $\vec{B}$-aligned fluctuations, but allowing for a Boltzmann distribution for the electrons. Locally homogeneous turbulence is assumed in the plane perpendicular to $\vec{B}$. T_e and T_i are assumed constant so that, e.g., buoyancy effects are ignored. For this reason for instance a gravitational or a dc electric force will not appear *explicitly* in (1) and (2), but only through a functional dependence of $\overline{n}$ and Γ_n. The strict derivation of Eq. (1) is found in the unpublished part of Ref. 1. We may note, however, that for small k in the universal part of the spectrum, only two quantities are available for constructing a dimensionally correct spectral subrange: k itself and $(\partial/\partial r)(T_e/e)\ln\overline{n}$ having dimension potential/length. Since $G(k)$ has the dimension (potential)^2 length, the only dimensionally correct combination is the one given by Eq. (1).

In order to be able to rely on Taylor's hypothesis when comparing measured frequency spectra with calculations referring to wave-number spectra, we carried out the experiment in a rapidly rotating magnetized plasma column.[2] Thus, a steady-state plasma was produced by surface ionization of cesium atoms on a hot (~2000 K) spiral made of 2-mm-diam tantalum wire, heated by a dc current of ~160 A supplied by an accumulator battery, and therefore having no disturbing ripple. Because of its spiral structure, the hot filament imposed an almost parablolic potential variation across the plasma column, giving rise to an imposed electric field, E_0, increasing linearly with radius. Together with a homogeneous confining magnetic field, this electric field gave rise to a nearly shear-free, "solid-