β has to increase much further for the disruption to occur in the second case than in the first one. This would be seen as longer sawteeth in the second case. Both results are in qualitative agreement with the results of TFR. Nonideal effects, like finite resistivity and finite-Larmor-radius stabilization, may also be important in explaining the experimental results.[17] However, a theory including these effects which is also valid near the magnetic axis remains to be developed.

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Confining a Tokamak Plasma with rf-Driven Currents

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Continuous toroidal electron currents, which sustain the poloidal magnetic field in tokamaks, may be generated by injecting waves with net parallel momentum into the plasma via phased waveguide arrays. Waves with high phase velocity can produce a current capable of confining a reactor plasma so that steady-state tokamak operation with acceptable power dissipation becomes possible.

Plasma confinement in tokamak fusion devices is maintained, in part, by a poloidal magnetic field sustained by a toroidal current. The current is usually driven by an inductively produced dc electric field, so that the tokamak operates only in a pulsed mode. For steady-state tokamak operation, a method of continuously driving the toroidal current is essential. One scheme of producing continuous current relies upon the Landau damping of high-phase-velocity rf waves traveling in only one direction parallel to the magnetic field. These waves may be launched from an endfire waveguide array, which directs the power flow substantially in one of the directions parallel to its axis. The waves have net parallel momentum, which upon being absorbed by electrons traveling with the wave parallel phase velocity, exerts a force that drives an electric current. The current is mainly carried by these resonant high-velocity electrons because, being relatively collisionless, they retain momentum longer than bulk electrons. The effective plasma resistivity is diminished, allowing steady-state tokamak reactor operation with acceptable power dissipation, an achievement previously thought infeasible because of the incorrect modeling of the current as a single electron fluid.[1]

The presence of the rf power results in parallel velocity diffusion which competes with the collisional relaxation of the plasma, so that the evolution of the space-averaged electron velocity distribution is governed by

$$\partial f/\partial t = [(\partial/\partial v_z)D_{\mathrm{QL}}(v_z)]\partial f/\partial v_z + (\partial f/\partial t)_c, \qquad (1)$$

where we have neglected any dc electric fields and where D_{QL} is the quasilinear diffusion coefficient and $(\partial f/\partial t)_c$ is the Fokker-Planck collision operator, which describes both parallel and perpendicular velocity scattering. My main interest, however, is in the dynamics in the parallel direction, where the quasilinear diffusion tends to flatten the distribution and the collisions tend to restore it to a Maxwellian. These dynamics are retained when Eq. (1) is integrated over the perpendicular velocity direction, where f is assumed to be Maxwellian, obtaining for high-velocity electrons in a singly ionized plasma[2]

$$\frac{\partial f}{\partial \tau} = \frac{\partial D(w)}{\partial w}\frac{\partial f}{\partial w} + \frac{\partial}{\partial w}\left(\frac{1}{w^3}\frac{\partial f}{\partial w} + \frac{1}{w^2}f\right), \qquad (2)$$

where I have normalized $w = v_z/v_{\mathrm{th}}$, $\tau = \nu_0 t$, $\nu_0 = \nu w^3$, $\nu = \omega_p^4 \ln\Lambda/2\pi n v_z^3$, and $D(w) = D_{\mathrm{QL}}/\nu_0 v_{\mathrm{th}}^2$. The perpendicular-velocity-space structure, which is neglected, manifests itself mainly in a flattening in the resonant region of velocity space and may be important in other plasma dynamics, e.g., interaction with a dc electric field or rf runaway in a high-Z_{eff} plasma.[3,4] It can be shown, however, that for parameters typical for current generation in reactors, this flattening does not significantly affect the number of resonant electrons nor their absolute velocity.[4] The collisionality of the resonant electrons, which depends on their speed and determines the power dissipated, is thus not affected. Since the perpendicular flattening affects neither the parallel current nor the power dissipated, the one-dimensional analysis is adequate for my purposes. Furthermore, it allows me to determine that the distribution can eventually have many more resonant electrons than initially, a significant but previously overlooked possibility, and to find the time in which this steady state is reached.

Under the assumption that the temperature of the bulk particles is essentially constant, the steady-state solution of Eq. (2) is given by

$$f = C\exp\left(-\int^w \frac{w}{1+w^3 D(w)}\,dw\right), \qquad (3)$$

where C is a constant determined by conservation of particles. The situation of interest for rf heating schemes and especially for current generation is when $D(w)$ is very large in a finite velocity interval, say $w_1 < w < w_2$, and vanishes elsewhere. The steady-state solution is then Maxwellian outside this interval and flat in the resonant region. For $Dw_1^2 > \Delta > 1/w_1$, where $\Delta = w_2 - w_1$, the plateau in the resonant region, which contains far more electrons than initially, acts as a fast beam with current density

$$J = (6.5\times10^8) n_{14} T_{10}^{1/2} \times f(w_1)(w_2^2 - w_1^2)/2\ \mathrm{A/m^2}, \qquad (4)$$

where n_{14} is normalized to 10^{14} cm^{-3} and T_{10} is normalized to 10 keV. In the steady state, the power dissipated by the waves in the resonant electrons balances the power dissipated in the bulk particles from the collisional slowing down of this fast beam of resonant electrons and so is given by

$$P_D = (5\times10^9) n_{14}^2 T_{10}^{-1/2} \times f(w_1)\ln(w_2/w_1)\ \mathrm{W/m^3}. \qquad (5)$$

In order to appreciate the power cost for generating the current it is helpful to define an effective rf resistivity

$$\eta_{\mathrm{rf}} \equiv \frac{P_D}{J^2} = \frac{10^{-8}}{T_{10}^{3/2}}\frac{\ln(w_2/w_1)}{f(w_1)(w_2^2 - w_1^2)^2/4}\ \Omega\ \mathrm{m} \approx \frac{10^{-8}}{T_{10}^{3/2}}\frac{n/n_p}{w_1^3}\ \Omega\ \mathrm{m}, \qquad (6)$$

where the last approximate equality was written for $\Delta/w \ll 1$, and n_p/n is the fraction of electrons in the plateau. Note that for equal rf and Ohmic current densities, we have $\eta_{\mathrm{rf}}/\eta_\parallel \approx 10/w_1^2 v_{D0}$, where $\eta_\parallel$ is the Spitzer resistivity and v_{D0} is the bulk drift velocity of the Ohmic current. The factor of 10 arises, in part, because the rf current suffers friction with all bulk particles, not just ions. The ratio of the effective resistivities is simply the ratio of the dynamic friction encountered by the different current carriers. This implies that whereas the rf current dissipates more power than the Ohmic current at low current levels (v_{D0} small), as the current level increases, or as the spectrum is shifted to higher phase velocities at constant current, the rf current begins to dissipate less power than the Ohmic current.

The current generated by rf power differs in several important ways, other than simply in the amount of power dissipated, from an inductively generated current. It must be appreciated that when net-momentum waves are damped, the plasma itself acquires a net momentum, and tends to rotate. A similar situation occurs in unidirectional neutral beam injection. A calculation determining the extent of the plasma rotation is beyond the scope of the present study; note, however, that momentum may be lost to particles leaving the plasma or to particles trapped in ripple fields.[5] In any case, one may write momentum-balance equations of the form

$$(\partial/\partial t+\gamma_e)m_e n_0 v_{De} = \nu_{ee}^{P/B} m_e n_p v_p + \nu_{ei}^{B/B} n_0 m_e (v_{Di} - v_{De}), \tag{7}$$

$$(\partial/\partial t+\gamma_i)m_i n_0 v_{Di} = \nu_{ei}^{P/B} m_e n_p v_p + \nu_{ie}^{B/B} n_0 m_i (v_{De} - v_{Di}), \tag{8}$$

where $\nu_{ee}^{P/B}$, for example, indicates the collision frequency for slowing down or momentum transfer of plateau with bulk electrons and γ_e and γ_i model any momentum sinks for electrons and ions, respectively. Equations (7) and (8) describe the transfer of momentum from the fast plateau electrons, at velocity v_p and density n_p, to the bulk ion and electron distributions, drifting respectively with velocities v_{Di} and v_{De}, which, in turn, experience a mutual friction to the extent that these drift velocities differ. Solving Eqs. (7) and (8), not for the overall toroidal rotation, which necessitates an evaluation of γ_e and γ_i, but for the relative drift between the ions and electrons, and assuming $\gamma_e, \gamma_i \ll \nu_{ei}^{B/B}$, I find a steady-state bulk current, which is negligible compared to the plateau current, i.e.,

$$\frac{J_B}{J_P}=\frac{q n_0 (v_{De}-v_{Di})}{q n_p v_p}\approx\frac{\nu_{ee}^{P/B}}{\nu_{ei}^{B/B}}\sim\frac{1}{w_1^{3}}\ll 1. \tag{9}$$

An rf-driven current also differs from an inductively driven current in that there is no dc electric field in steady-state operation. However, as the rf current is turned on in a tokamak, it does generate a time-varying magnetic field, which, in turn, induces a toroidal dc electric field that opposes the motion of the plateau electrons. The dc field instantaneously produces a counter current of electrons, primarily in the bulk of the velocity distribution (since most electrons are situated there), so as to oppose any abrupt change in the flux linkage to the plasma. The counter-current decays in an L/R time of the tokamak, where L and R are the plasma inductance and resistance, after which the rf current flows in the absence of the dc field. When even very intense rf power is turned on, there is, initially, very little rf current, since the number of electrons initially in the resonant region is quite small. I can define an rf current turn-on time, $\tau_{\text{t-o}}$, occurring on a collisional time scale, during which bulk electrons are collisionally scattered into the resonant region to form the "raised" plateau that is characteristic of the time-asymptotic distribution. In the event that the rf power is turned on in an inductively driven tokamak, the current is transferred from bulk carriers to plateau carriers in a time $\tau_{\text{t-o}}$, but the total current does not change, although the dc electric field is effectively shut off. In the event that the rf current exceeds the original Ohmic current in less than an L/R time, the rf current will begin to drive the primary transformer coils in reverse. In steady-state tokamak operation, it is desired to switch from the Ohmic current to the rf current, so that after a turn-on time the Ohmic coils should be disconnected. It should be noticed that the absence of a dc electric field in the steady-state operation implies that no runaways are produced.

To calculate the turn-on time, $\tau_{\text{t-o}}$, I assume that during the turn-on, the bulk electron temperature is essentially constant; in other words, that $\tau_{\text{t-o}} \ll \tau_h$, where τ_h is a heating time defined, in normalized units, by $\tau_h \equiv 4\exp(w_1^2/2)/\ln(w_2/w_1)$. It may be seen, upon use of Eq. (5), that in the steady state the bulk electron temperature is roughly doubled in a time τ_h. A rigorous derivation, given elsewhere,[3] corroborates the following rough calculation of the turn-on time. The flux of electrons from the region $w<w_1$ into the resonant region is found by successive approximations of Eq. (2), assuming in the lowest order that $\partial f/\partial\tau=0$, but with slowly varying boundary conditions on f at $w=0$ and $w=w_1$, obtaining

$$S(w_1)\approx[f(0,\tau)\exp(-w_1^2/2)-f(w_1,\tau)]/w_1^2. \tag{10}$$

A comparison of the time-asymptotic state with the initial distribution indicates that $f(0,\tau)$ is slowly varying compared to $f(w_1,\tau)$ and that very few electrons are scattered into the region $w>w_z$. Furthermore, under intense rf excitation, the resonant region is nearly flat, so that for $\Delta w_1>1$, we have, by conservation of electrons, $S(w_1) =\Delta\,\partial f(w_1,\tau)/\partial\tau$. Solving now Eq. (10), I find $\tau_{\text{t-o}} \approx \Delta w_1^2$, so that the rf current in, as we shall see, typical reactors can be turned on in about a millisecond.

It remains to assess the practicality of rf current generation in a reactor environment. Economic considerations dictate tokamak reactor operation near $\beta_p = R/a$ which, assuming $T_e \approx T_i$ and neglecting the α-particle pressure, requires a density of rf current carriers

$$n_p/n = 4c/w_1\omega_{pe}(aR)^{1/2}. \tag{11}$$

Since $n_p/n \approx \Delta f(w_1)$ is most sensitive to w_1, I immediately locate the spectrum at $w_1 \approx 4$, assuming only that $\Delta, n_{14}, a_1, R_1 \sim 1$, where a_1 and R_1 are the minor and major radii in meters. Assuming operation near $\beta_p = R/a$ with $\Delta/w_1 < 1$, I calculate the crucial parameter

$$\epsilon = P_D/P_f = 4/w_1^2(n_{14}T_{10}a_1R_1)^{1/2}, \tag{12}$$

where P_f is the fusion power density for D-T reactions at temperatures near 10 keV. Low ϵ, implying low circulating rf power, is necessary for the scheme of rf current generation to be of interest. Furthermore, low ϵ implies that in the ignited reactor the rf power does not significantly affect the plasma temperature, which, in any case, may be stabilized by external means. Note that low ϵ is easily achieved in reactors, for example, with UWMAK[6]-type parameters ($n_{14} \approx T_{10} \approx 1$, $a_1 \approx 5$, $R_1 \approx 13$), where my nonrelativistic calculation of P_D is adequate.

It should be appreciated that for a reactor with acceptable wall loading, the rf power may be conveniently brought into the tokamak by means of waveguides, for example, by coupling to lower hybrid waves. The power requirements on the waveguides, assuming a D-T fuel cycle, are bounded by the neutron wall loading H, i.e., $P_{wg} < (\epsilon/\eta_D)(H/0.8)(A/A_{wg})$, where A_{wg}/A is the fraction of available wall area for the rf, and η_D is the fraction of incident rf that is damped. Since H is typically 2–4 $\mathrm{MW/m^2}$, only modest demands on waveguide capabilities are required if a reasonable fraction (~1%) of the wall is used and most of the incident rf power is deposited, i.e., $\eta_D \approx 1$.

Accessibility of the lower hybrid wave to the tokamak center restricts the wave spectrum to a parallel index of refraction $n_\parallel > 1 + 0.6\beta_{4\%}T_{10}^{-1}$, where $\beta_{4\%}$ is the toroidal β, again neglecting α-particle pressure, normalized to 4%. The minimum required $n_\parallel$ corresponding to $w_1 \approx 4$ but $\Delta \ll w_1$ is $n_\parallel(\mathrm{min}) \approx 2(1-\Delta/w_1)T_{10}^{1/2}$, which allows $\beta \approx 4\%$ at $T = 10$ deV and $\Delta \approx 1$. Significant absorption of the incident rf power concommitant with penetration of the plasma center implies, for lower hybrid waves in the vicinity of the lower hybrid frequency, $1 \approx 2k_{xi}a = (\pi/\Delta w_1 D)(n_p/n)(a\omega_{pe}/v_T)$, whereupon, using Eq. (11), we find

$$\Delta D \approx (4\pi/w_1^2)(a/R)^{1/2}(c/v_T)$$

$$\approx 4T_{10}^{-1/2}(3a/R)^{1/2}, \tag{13}$$

implying, as expected, significant plateauing. Although the penetration of the lower hybrid waves to the plasma center is accomplished, essentially, through the decrease in the linear Landau damping due to plateauing at high power levels, $Dw_1^2 \gg \Delta$, nevertheless, for reactor-type plasmas, the same intense rf waves do not suffer parametric decay nor induce significant nonresonant power absorption. Nonresonant power dissipation scales as $P_{\mathrm{nr}} \approx \nu_0 nmv_{\mathrm{osc}}^2$, where v_{osc} is the nonresonant jitter velocity. Using Eqs. (5), (11), and (13), I find $P_{\mathrm{nr}}/P_D \approx n_{14}a_1/4w_1T_{10}^2$, which is small for typical reactor parameters. Also, using Eqs. (11) and (13), I may calculate the quantity $(E_\perp/B)/(T_e/m_i)^{1/2} \approx \beta n_\parallel^3(\nu_0/\omega_{pi})(a/R)$ which, typically being quite small, implies that parametric-decay interactions[7] are unlikely. It may also be shown that strong turbulence or trapping effects are not important here. I conclude that rf current generation, in particular with lower hybrid waves, represents an attractive possibility for steady-state tokamak reactor operation.

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