similar results.
[6]R. Lee and M. Lampe, Phys. Rev. Lett. 31, 1390 (1973).
[7]S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 5, p. 216.
[8]S. S. Yu, H. L. Buchanan, F. W. Chambers, and E. P. Lee, Ref. 2, p. 55.
[9]E. P. Lee and R. K. Cooper, Part. Accel. 7, 83 (1976).

Influence of Toroidal Effects on the Stability of the Internal Kink Mode

R. M. O. Galvão, P. H. Sakanaka, and H. Shigueoka
Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas, 13.100 Campinas, São Paulo, Brazil
(Received 17 May 1978)

Using the σ-stability technique, we study the stability of the internal kink mode in toroidal geometry. We show that there are two unstable regions separated by a stable one in a β-q_c stability diagram. In one of these regions toroidal effects are stabilizing and in the other they are destabilizing. Discrepant results of previous analytical theories and experimental results are explained.

The internal kink mode has been the object of intense theoretical research recently[1-6] because of its relevance to the triggering mechanism of internal disruptions in tokamak discharges.[7-11] In a cylindrical plasma column of length $2\pi R_0$ and radius a with an equilibrium magnetic field $(0, B_\theta(r), B_z(r))$, the mode is excited when $q(r) = rB_z(r) < 1$ in some region inside the plasma; the mode poloidal and toroidal wave numbers are $m = 1$ and $|n| = 1$, respectively.

Two different analytical theories have been developed for the internal kink mode in toroidal geometry so far.[4-6] The one developed by Bussac *et al.*[4,5] is based upon the energy principle. According to their results, the toroidal effects are stabilizing if

$$\beta_p{}^* = [2/B_\theta{}^2(r_0)]\int_0^{r_0}(r/r_0)^2(-dp/dr)\,dr$$

is below some critical value; $p(r)$ is the plasma pressure. The theory developed by Pao[6] is based upon a normal-mode analysis. He uses $\epsilon = a/R_0$ and r_0/a as expansion parameters; here r_0 is the radius where $q(r_0) = 1$. According to Pao's theory, the internal kink mode is *not* stabilized by toroidal effects, as predicated by Bussac *et al.* To our knowledge, the discrepancy between the two theories has not been elucidated yet.

Experimental results have shown the presence of the $m = |n| = 1$ instability superimposed on the sawtooth oscillations characteristic of the internal disruptions.[10,11] The TFR group's results indicate also that the $m = |n| = 1$ oscillations vanish when the plasma density is increased.[10] Since this corresponds to an increase in the plasma pressure, this result is somewhat surprising and not explained by previous theories. Using the σ-stability technique,[12] we show that in toroidal geometry there are stable and unstable regions for the $m = 1$ internal mode in the β-q_c parameter space, as found by Freidberg and co-workers for the external kink mode.[13] Here we define $\beta = p(0)/[p(0) + \frac{1}{2}B_T{}^2(0)]$, $q_c = q(0)$, and $B_T(0)$ is the toroidal component of the equilibrium magnetic field $B(r)$ at the magnetic axis. The apparent discrepancy between the two analytical theories and the experimental result described above are explained on the basis of the β-q_c stability diagrams. We also calculate the mode spectrum for different values of β.

Copenhaver[14] has derived a simple σ Euler equation starting from a theory of the magnetohydrodynamic spectrum in toroidal geometry developed by Goedbloed.[15] The relevant equations are flux averaged and expanded in powers of ϵ. Copenhaver's equation, with first-order toroidal terms included, is

$$\frac{d}{dr}\left[f(r)\frac{d}{dr}(r\xi)\right] - g_M(r)(r\xi) = 0, \tag{1}$$

where

$$f(r) = N/rD, \quad N = (\rho\sigma^2 + F^2)[\rho\sigma^2(\gamma p + B^2) + \gamma p F^2],$$

$$D = \rho^2\sigma^4 + \rho\sigma^2(m^2/r^2 + n^2/\overline{R}^2)(\gamma p + B^2) + (m^2/r^2 + n^2/\overline{R}^2)\gamma p F^2,$$

and

$$g_M(r)=\frac{1}{r}\left[\rho\sigma^2+F^2+2B_\theta\frac{d}{dr}\left(\frac{B_\theta}{r}\right)-\frac{2q^2B_\theta^2}{r^2}\left\{2+\frac{\zeta'\overline{R}}{r}+\Delta\left(\frac{rI'}{I}-1\right)\right\}-4\frac{n^2B_\theta^2}{r^2\overline{R}^2}\frac{(\rho\sigma^2B_\theta^2+\gamma pF^2)(1+qm\Delta/n)^2}{D}\right.$$
$$\left.+r\frac{d}{dr}\left\{\frac{2nB_\theta}{r^2\overline{R}}\left(\frac{mB_T}{r}-\frac{nB_\theta}{\overline{R}}\right)\frac{[\rho\sigma^2(\gamma p+B^2)+pF^2]}{D}\left(1+\frac{qm\Delta}{n}\right)\right\}\right],$$

where primes indicate derivatives with respect to r. The variable ξ in (1) is the radial component of the perturbed displacement of an element of fluid, ρ is the fluid density, σ is the prefixed parameter of the σ-stability technique,[12] γ is the ratio of specific heats,

$$I=\overline{R}(r)B_T(r),\quad F=mB_\theta/r+nB_T/\overline{R},$$
$$\Delta=2+(R_0/r)\int_0^r(\zeta'/r)dr,$$

and $\zeta(r)$ is the displacement of the center of a magnetic surface $p(r)=\text{const}$ with respect to the geometric center of the plasma column at $\overline{R}(a)=R_0$, $z=0$, i.e., $\overline{R}(r)=R_0+\zeta(r)$. Copenhaver's equation is strictly valid only for weak toroidal mode coupling. This restricts the applicability of the equation to the neighborhood of the magnetic axis because the boundary conditions for the *coupled* $m=0, 1$, and 2 modes are $\xi_0\to0$, $\xi_1\to1$, $\xi_2\to0$ as $r\to0$, *strongly* reducing mode coupling in the region.

The equilibrium equations are

$$\frac{dp}{dr}+\frac{1}{2(R_0+\zeta)^2}\frac{dI^2}{dr}+\frac{1}{2}\frac{dB_\theta^2}{dr}+\frac{B_\theta^2}{r}=0 \qquad (2)$$

and

$$\frac{d\zeta}{dr}=-\frac{1}{R_0rB_\theta^2}\int_0^r\left(B_\theta^2-2s\frac{dp}{ds}\right)s\,ds. \qquad (3)$$

We solve Eqs. (1)–(3) numerically as described in Ref. 12. We find that the spectrum and the stability regions of the internal kink mode depend critically on the profiles of the equilibrium quantities. In an actual experiment, the value of q at the wall can be held approximately constant during a major part of the discharge. However, the value q_c of the q at the magnetic axis goes below unity due to localized Ohmic heating. This decreases the plasma resistivity increasing the current density locally, thus decreasing q_c. In order to simulate this effect, we choose the current profile as $j=j_0(1-r^2/a^2)^\nu$ and change q_c by varying j_0 and ν continuously in such a way that the value of $q_w=q(a)\simeq2B_T(a)(\nu+1)/R_0j_0$ remains approximately constant. This approach has also the advantage of keeping the resonance surface near $r=0$, as q_c decreases below unity, consistent with the usage of Eq. (1). The pressure profile is chosen as $p=p_0\exp(\alpha_2r^2+\alpha_4r^4)$, where p_0, α_2, and α_4 are constants. In the numerical procedure Eq. (1) is expanded in a power series up to $O(r^6)$ at the resonance surfaces and at the magnetic axis. The latter region is of particular relevance to the experimental situation because, during the slow rise of a sawtooth oscillation, the resonance surface may occur first near $r=0$ and then move away to larger r.[16] Also it is not clear whether the toroidal theories of Bussac *et al.*[4,5] and of Pao[6] are strictly valid in the neighborhood of $r=0$. Actually not even the cylindrical analysis of Rosenbluth, Dagazian, and Rutherford[3] is valid in this region. The point is that near $r=0$ there is a region of constant pitch of the size of the mode width itself.[9] In this region the $m=1$ mode may be driven mainly by the plasma pressure (as opposed to being driven mainly by the plasma current).

We show the β-q_c stability diagrams for $R_0/a=4$ and $R_0/a=8$ in Figs. 1(a) and 1(b), respectively. In both figures $m=|n|=1$, $\alpha_2=-4/a^2$, $\alpha_4=-6/a^4q_w\simeq4$, and $\sigma=10^{-4}$, where σ is normalized to the Alfvén time scale $\tau_A^{-1}=B_T(0)/\rho^{1/2}a$. The curves for $\zeta_0=\zeta(0)=\text{const}$ are also shown. We see that in toroidal geometry there are two unstable regions for the internal $m=1$ mode separated by a stable one. In the lower unstable region the mode profile resembles closely the usual one for the current-driven internal kink in cylindrical geometry. In the upper unstable region the mode profile has a different nature, resembling more the pressure-driven modes of cylindrical geometry. The two boundaries between the unstable and stable regions move up as the value of the aspect ratio R_0/a decreases. So, if one considers an operation point at fixed q_c and β in the upper unstable region and decreases R_0/a, one sees that the boundary to the stable region moves up towards the point and even passes it. On the other hand, if the operation point is in the lower unstable region of the β-q_c stability diagram, one sees that the boundary to the stable region moves up away from the point as R_0/a decreases. In this sense the toroidal effects are stabilizing in the upper unstable region and de-

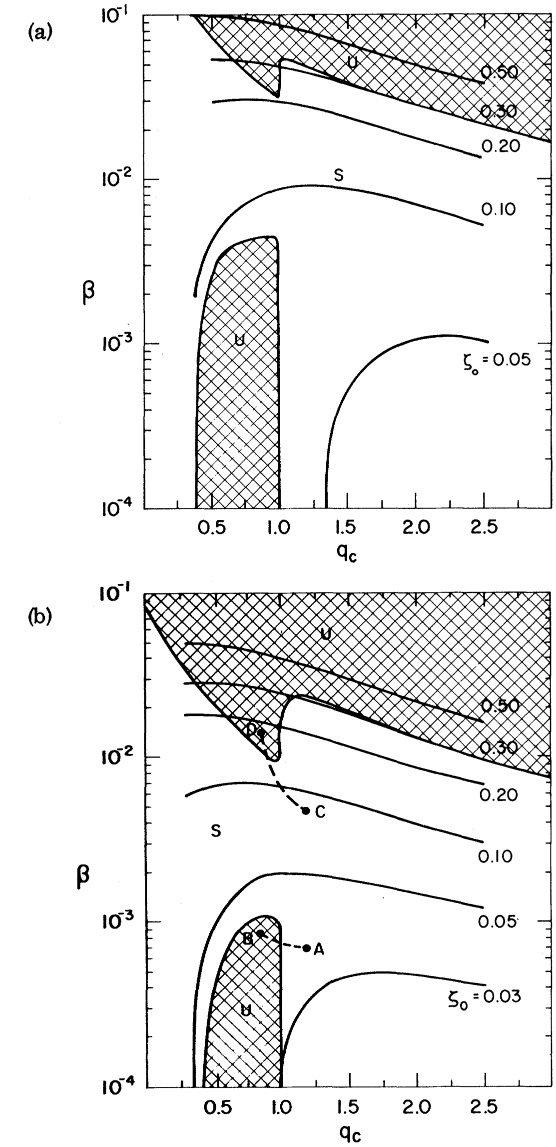

FIG. 1. β-q_c stability diagrams for the $m=1$, $|n|=1$ internal kink mode. The shaded areas indicate unstable regions. (a) The aspect ratio is $R_0/a=4$ while in (b) $R_0/a=8$. In (b) the curves AB and CD indicate *qualitatively* the different paths that operation points (specified by the values of β and q_c) may follow during an internal disruption. The value of ν varies from 0.6 (for $q_c \simeq 2.5$) to 7 (for $q_c \simeq 0.5$).

stabilizing in the lower one. We believe that near the magnetic axis the results of Bussac *et al.* are relevant only to the upper unstable region while Pao's are relevant only to the lower unstable one.

In Fig. 2 we show the normalized growth rate ω_i as a function of q_c for the parameters of Fig. 1(a). We see that, while in the lower unstable

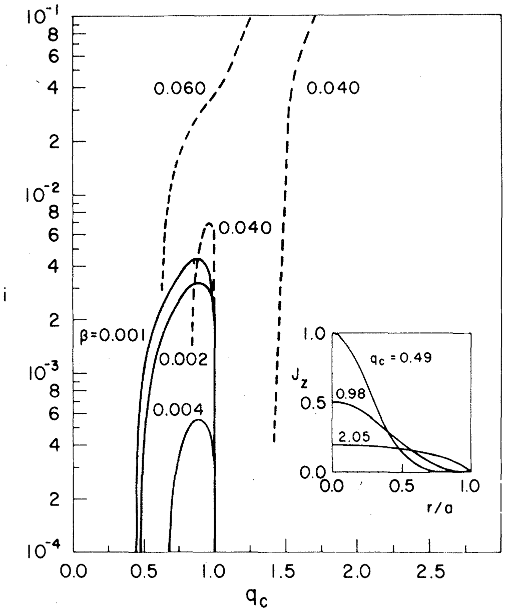

FIG. 2. Internal kink-mode spectrum for the equilibrium parameters of Fig. 1(a). The mode growth rate ω_i is normalized to the inverse of the Alfvén time, $\tau_A^{-1} = B_0/\rho^{1/2}a$. The full and broken lines correspond to the spectrum in the lower and upper unstable regions of the β-q_c stability diagram, respectively. The current-density profile dependence on q_c is shown in the inset.

region the mode spectrum resembles the one for the external kink mode,[1] the mode spectrum in the upper unstable region is of a quite different nature. In this region the mode growth rate increases almost explosively with β, for a fixed value of q_c.

Concerning the experimental data,[10] we recall that for TFR, $R_0/a \sim 5$ and $\beta \sim 10^{-3}$; this device operates in the neighborhood of the boundary between the lower unstable and stable regions. When the equilibrium parameters in the beginning of a sawtooth oscillation are such that the lower unstable region is reached as q_c decreases below unity [as indicated qualitatively by the path AB in Fig. 1(b)], the $m=|n|=1$ mode is observed. When the density (and therefore β) is high enough, the lower unstable region may become inaccessible as q_c decreases below unity. Thus, as the heating of the central region of the plasma column proceeds, q_c decreases and β increases further and the operation point eventually reaches the upper unstable region [path CD in Fig. 1(b)]. Here the mode growth rate is much larger than in the lower unstable region leading to a rapid internal disruption before a mode oscillation is detected. Also, depending on the starting point,

β has to increase much further for the disruption to occur in the second case than in the first one. This would be seen as longer sawteeth in the second case. Both results are in qualitative agreement with the results of TFR. Nonideal effects, like finite resistivity and finite-Larmor-radius stabilization, may also be important in explaining the experimental results.[17] However, a theory including these effects which is also valid near the magnetic axis remains to be developed.

We are thankful to Dr. C. Copenhaver for providing a copy of his thesis and for warning us about the weak-coupling assumption that he has used in the derivation of Eq. (1). We are thankful also to Dr. K. Weimer, Dr. J. Goedbloed, and Professor B. Coppi for enlightening discussions. This work has been partly supported by Financiadora de Estudos e Projetos, Foundacão de Amparo a Pesquisa do Estado de São Paulo, and Conselho Nacional de Pesquisas (Brazil).

[1]V. D. Shafranov, Zh. Tekh. Fiz. 40, 241 (1970) [Sov. Phys. Tech. Phys. 15, 175 (1970)].

[2]D. C. Robinson, Plasma Phys. 13, 439 (1971).

[3]M. N. Rosenbluth, R. Y. Dagazian, and P. H. Rutherford, Phys. Fluids 16, 1894 (1973).

[4]M. N. Bussac, R. Pellat, D. Edery, and J. L. Soulé, Phys. Rev. Lett. 35, 1638 (1975).

[5]M. N. Bussac, D. Edery, R. Pellat, and J. L. Soulé, in *Proceedings of the Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976* (International Atomic Energy Agency, Vienna, Austria, 1977), Vol. I, p. 607.

[6]Y. P. Pao, Phys. Fluids 19, 1796 (1976).

[7]B. B. Kadomtsev, in Ref. 5, Vol. I, p. 555.

[8]A. Sykes and J. A. Wesson, Phys. Rev. Lett. 37, 140 (1976).

[9]R. B. White, D. A. Monticello, M. N. Rosenbluth, and B. V. Wadell, in Ref. 5, Vol. I, p. 569.

[10]Equipe TFR, in Ref. 5, Vol. I, p. 279.

[11]N. R. Sauthoff, S. von Goeler, and W. Stodiek, Princeton Plasma Physics Laboratory Report No. PPPL-1379, 1978 (unpublished).

[12]J. P. Goedbloed and P. H. Sakanaka, Phys. Fluids 17, 908 (1974); P. H. Sakanaka and J. P. Goedbloed, *ibid.*, p. 919.

[13]J. P. Freidberg and J. P. Goedbloed, in *Pulsed High Beta Plasmas*, edited by D. E. Evans (Pergamon, New York, 1976).

[14]C. Copenhaver, Ph.D. thesis, University of Tennessee, 1976 (to be published).

[15]J. P. Goedbloed, Phys. Fluids 18, 1258 (1976).

[16]J. A. Wesson, to be published.

[17]G. L. Jahans *et al.*, Nucl. Fusion 18, 609 (1978).

Confining a Tokamak Plasma with rf-Driven Currents

Nathaniel J. Fisch[(a)]
Plasma Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 March 1978)

Continuous toroidal electron currents, which sustain the poloidal magnetic field in tokamaks, may be generated by injecting waves with net parallel momentum into the plasma via phased waveguide arrays. Waves with high phase velocity can produce a current capable of confining a reactor plasma so that steady-state tokamak operation with acceptable power dissipation becomes possible.

Plasma confinement in tokamak fusion devices is maintained, in part, by a poloidal magnetic field sustained by a toroidal current. The current is usually driven by an inductively produced dc electric field, so that the tokamak operates only in a pulsed mode. For steady-state tokamak operation, a method of continuously driving the toroidal current is essential. One scheme of producing continuous current relies upon the Landau damping of high-phase-velocity rf waves traveling in only one direction parallel to the magnetic field. These waves may be launched from an endfire waveguide array, which directs the power flow substantially in one of the directions parallel to its axis. The waves have net parallel momentum, which upon being absorbed by electrons traveling with the wave parallel phase velocity, exerts a force that drives an electric current. The current is mainly carried by these resonant high-velocity electrons because, being relatively collisionless, they retain momentum longer than bulk electrons. The effective plasma resistivity is diminished, allowing steady-state tokamak reactor operation with acceptable power dissipation, an achievement previously thought infeasible because of the incorrect modeling of the current as

