Multipion Production in Relativistic Heavy-Ion Collisions

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We show that recent measurements of π^- multiplicity distributions produced in relativistic heavy-ion collisions are in remarkable agreement with the predictions of the collective tube model.

Recently two independent experiments have reported[1,2] measurements of π^- multiplicity distributions produced in relativistic heavy-ion collisions. It was noted[2,3] that the observed distributions are significantly different from those expected from an aggregate of individual nucleon-nucleon collisions.[3,4] In this Letter we show that the collective tube model[5] (CTM) which has been applied with considerable success[6] to predict inelastic particle-nucleus interactions at high energies also reproduces remarkably well the observed π^- multiplicity distributions in relativistic nucleus-nucleus collisions.

Consider a high-energy nucleus-nucleus collisions at impact parameter B (the transverse distance between the nuclear centers) in the center-of-mass system. Let us focus on the collision inside a cylinder of cross section δ along the incident direction at transverse coordinate b. In this cylinder i_1 right-moving projectile nucleons collide with i_2 left-moving target nucleons. $i_j(b) = \sigma T_j(b)$ where $T_j(b) \equiv \int_{-\infty}^{\infty} \rho_j(b, z)dz$ is the nuclear density function normalized such that $\int \rho_j d^3r = A_j$. In the c.m. system the colliding tubes are Lorentz contracted into narrow disks. The CTM thus assumes that tube-tube collisions resemble elementary particle-particle collisions. It further assumes that all the simultaneous $N(B)$ tube-tube collisions in the intersection area of the colliding nuclei are independent. [$N(B)$ is equal to the intersection area divided by σ the pp total inelastic cross section.[7]] For calculating multiplicity distributions the CTM makes use of the observation that the average multiplicity and the multiplicity distribution of negatively charged particles produced in high-energy particle collisions depend only on Q, the available energy for particle production in the c.m. system; and not on the specific nature[8,9] (quantum numbers) of the colliding particles [Q is defined by $Q \equiv \sqrt{s} - (m_1 + m_2)$ where $\sqrt{s}$ is the total c.m. energy and m_1, m_2 are the masses of the colliding particles]. Thus the multiplicity distribution of π^- produced in a high-energy collision of a tube of i_1 nucleons with a tube of i_2 nucleons is taken from pp collisions at $Q = Q(i_1, i_2, \bar{E})$, where

$$Q(i_1, i_2, \bar{E}) \equiv [m^2(i_1^2 + i_2^2) + 2mi_1i_2\bar{E}]^{1/2} - m(i_1 + i_2). \qquad (1)$$

m is the nucleon mass and $\bar{E}$ is the incident energy per nucleon. The probability P_{n_-} of producing n_- negative pions in an inelastic nucleus-nucleus collision is obtained by averaging $P(n_-|B)$, the probability to produce n_- negative pions in a collision with impact parameter B, over all impact parameters, i.e.,

$$P_{n_-} = \int d\sigma(B)P(n_-|B)/\int d\sigma(B). \qquad (2)$$

$d\sigma(B)$ is the contribution to the total inelastic cross section from impact parameter B. It is given by[10]

$$d\sigma(B) = \{1 - \exp[-\sigma \int T_2(b)T_1(b - B)d^2b]\}d^2B. \qquad (3)$$

Since we assume that all the tube-tube collisions are independent we obtain that

$$P(n_-|B) = \sum_{\{n_j\}} \delta\left(\sum n_j - n_-\right) \prod_{k=1}^{N(B)} P_{n_k}(k), \qquad (4)$$

where $P_{n_k}(k)$ is the probability for producing n_k π^- in the kth tube-tube collision in the intersection area. In the incident energy range 1 GeV/c $\leq P_{\rm lab} \leq$ 70 GeV/c the multiplicity distributions in pp collisions can be well represented by Poisson distributions.[11] With the substitution of Poisson distributions for $P_{n_k}(k)$, Eq. (4) reduces to another Poisson distribution:

$$P(n_-|B) = e^{-\langle n_-(B)\rangle} \frac{\langle n_-(B)\rangle^{n_-}}{(n_-)!}, \qquad (5)$$

where

$$\langle n_-(B)\rangle = \sum_k^{N(B)} \langle n_k(k)\rangle = \int \frac{d^2b}{\sigma} \langle n_-(Q(b, B))\rangle. \qquad (6)$$

$Q(b, B)$ can be calculated from expression (1) with $i_1 = \sigma T_1(b)$ and $i_2 = \sigma T_2(b - B)$. Equations (2), (3), (5), (6), and (1) with $\langle n_-(Q)\rangle$ taken from high-energy pp collisions, uniquely determine the CTM predictions for the multiplicity distribution of π^- produced in high-energy nucleus-nucleus collisions.

Before comparing the CTM predictions with experiments let us derive a useful approximation for the average π^- multiplicity. For complex nuclei $d\sigma(B)$ can be well approximated by a step function:

$$d\sigma(B)=\begin{cases} d^2B, & B \leqslant B_{\max}, \\ 0, & B \geqslant B_{\max}, \end{cases} \qquad (7)$$

where $B_{\max}$ is given by $\sigma_{A_1A_2}=\int d\sigma(B)=\pi B_{\max}^2$. With the aid of Eq. (7), Eq. (2) reads

$$P_{n_-}=\int_0^{B_{\max}} d^2B\, P(n_-|B)/\sigma_{A_1A_2}. \qquad (8)$$

We also note that $\langle n_-(Q)\rangle$ is a slowly varying function of Q[8,9] and consequently Eq. (6) can be well approximated by $\langle n_-(B)\rangle = N(B)\langle n_-(\overline{Q})\rangle$ where $\overline{Q} \equiv Q(A_1^{1/3}, A_2^{1/3}, \overline{E})$. It is then easy to see that the average π^- multiplicity is given by

$$\langle n_-\rangle_{A_1A_2} \equiv \sum n_- P_{n_-} \cong \frac{\sigma_{pA_1}\sigma_{pA_2}}{\sigma_{pp}\sigma_{A_1A_2}}\langle n_-(\overline{Q})\rangle_{pp}. \qquad (9)$$

According to Ref. 11 nuclear cross sections can be well represented by

$$\sigma_{A_1A_2}=\sigma_{pp}(A_1^{1/3}+A_2^{1/3}-d)^2,$$

where

$$d=\begin{cases} 1.028-0.028\min(A_1,A_2), & \min(A_1,A_2)<40; \\ 0, & \text{otherwise}. \end{cases}$$

Consequently Eq. (9) reduces to

$$\langle n_-\rangle_{A_1A_2} \cong \frac{A_1^{2/3}A_2^{2/3}}{(A_1^{1/3}+A_2^{1/3}-d)^2}\langle n_-(\overline{Q})\rangle_{pp}. \qquad (10)$$

In order to calculate the CTM predictions for the π^- multiplicity distributions as given by Eqs. (2)–(6) we need the nuclear density distributions, the average π^- multiplicity, and the total inelastic cross section in pp collisions. In our calculations we have used nuclear density functions that were deduced from electron-nucleus scattering.[12] $\langle n_-\rangle_{pp}$ and σ_{pp} were taken from the compilations in Refs. 8 and 9. For complex targets results were properly averaged with the respective probabilities for inelastic interaction of the projectile with the various target nuclei.

Figure 1 compares the CTM predictions and experimental π^- multiplicity distributions[2] produced by Ar^{40} with 1.8-GeV kinetic energy per nucleon and by C^{12} with 2.1-GeV kinetic energy

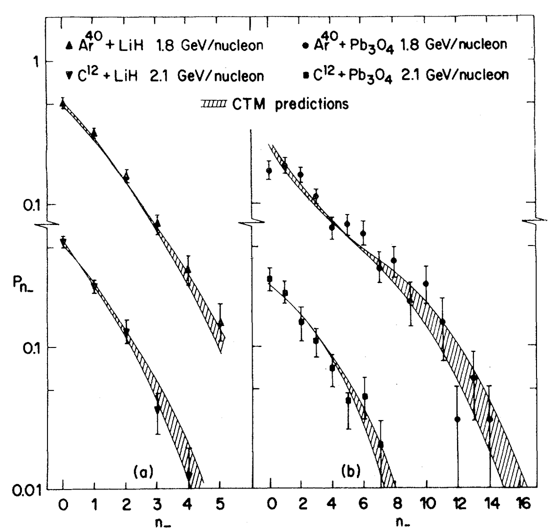

FIG. 1. Comparison between experimental results (Ref. 2) and the CTM predictions for multiplicity distributions of π^- produced in heavy-ion collisions. The shaded areas reflect the uncertainty in the input experimental data on $\langle n_-(Q)\rangle$ in pp collisions.

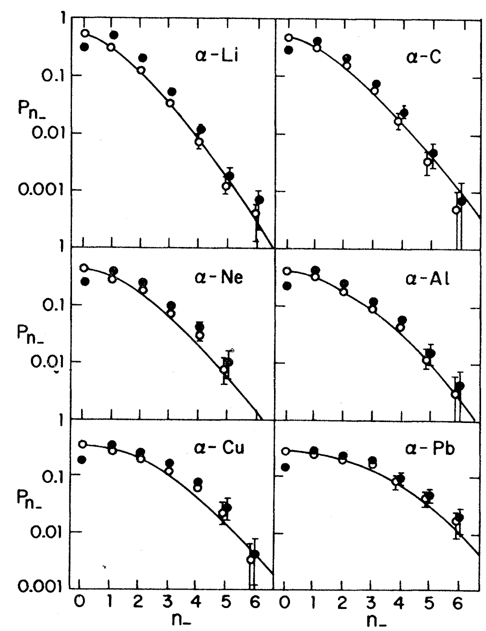

FIG. 2. Comparison between experimental results (Ref. 1) and the CTM predictions for multiplicity distributions of π^- produced in α-nucleus collisions at 17.8 GeV/c. The corrected results (open circles) were obtained from the original experimental results (full circles) by normalizing P_0 to the CTM predictions.

per nucleon incident on LiH and Pb_3O_4. Figure 1 demonstrates very good agreement between the CTM predictions and experiment. Note that slightly low experimental values of P_0 can result from the experimental triggering mode which according to Ref. 2 causes loss of events with small pion multiplicities. Note also that the calculated distributions cannot be represented by a single Poisson distribution.

Figure 2 compares the CTM predictions and experimental[1] π^- multiplicity distributions in α-nucleus collisions at 17.8 GeV/c. The discrepancy at $n_- = 0$ probably results from loss of events with small pion multiplicities and events where only π's are produced. Such losses are strongly suggested by the multiplicity distributions of Ref. 2. They lead to an experimental underestimate of P_0 and thus to a slight overestimate of P_{n_-} for $n_- \geq 1$ since $\sum P_{n_-} = 1$. To demonstrate the effect of such losses, in Fig. 2 we have also drawn the P_n's obtained by changing P_0 to the values predicted by the CTM. The corrected distributions are in excellent agreement with the CTM predictions as can be seen from Fig. 2. They are not much different from Poisson distributions although systematic deviations develop at large multiplicities. A similarity between the input distribution and the calculated one is obtained in the CTM only for very light projectiles colliding with heavy nuclei. This has been observed in both high-energy π-nucleus and p-nucleus collisions.[5,6] The agreement between the CTM predictions and experiment is not destroyed by using other nuclear density functions, other parametrizations of $\langle n_-\rangle_{pp}$, or slightly different values of σ_{pp}. This is demonstrated in Table I where we compare the CTM predictions and experimental measurements[1,2] of the average π^- multiplicities in fourteen projectile-target combinations. We list there results of three sets of calculations: (a) Predictions based on the approximate formula (10). (b) Predictions based on Eqs. (2)–(6) where

TABLE I. Comparison between average π^- multiplicities (Refs. 1 and 2) and the CTM predictions as given by (a) Eq. (10); (b) Eqs. (2)–(6) for nuclear spheres of constant density and radii $R_A = 1.27A^{1/3}$ fm; (c) Eqs. (2)–(6) for nulcear density functions deduced from electron scattering (Ref. 12). The errors in the theoretical predictions reflect mainly the experimental uncertainty in $\langle n_-\rangle_{pp}$.[13]

Reaction	E GeV/Nucleon	CTM PREDICTIONS			Experiment	
		(a)	(b)	(c)	Corrected	Original
Ar + LiH	2.74	0.76 ± .15	0.73 ± .15	0.68 ± .14	0.82 ± .05	0.97 ± .05
Ar + NaF	2.74	1.50 ± .30	1.68 ± .34	1.59 ± .32		1.91 ± .12
Ar + BaI_2	2.74	3.20 ± .64	3.63 ± .72	3.43 ± .68		3.27 ± .18
Ar + Pb_3O_4	2.74	3.05 ± .61	3.39 ± .68	3.17 ± .62	3.06 ± .15	3.27 ± .15
C^{12}+ LiH	3.04	0.62 ± .12	0.61 ± .13	0.56 ± .11	0.63 ± .07	0.68 ± .07
C^{12}+ NaF	3.04	1.09 ± .22	1.17 ± .23	1.10 ± .22		1.03 ± .08
C^{12}+ BaI_2	3.04	2.04 ± .40	2.06 ± .41	1.96 ± .39		1.91 ± .17
C^{12}+ Pb_3O_4	3.04	1.97 ± .40	1.98 ± .40	1.87 ± .38	1.85 ± .17	1.79 ± .16
α + Li	4.55	0.77 ± .15	0.73 ± .15	0.65 ± .13	0.65 ± .05	1.03 ± .06
α + C	4.55	0.92 ± .18	0.88 ± .18	0.83 ± .17	0.87 ± .03	1.17 ± .04
α + Ne	4.55	1.06 ± .21	1.02 ± .21	0.94 ± .19	0.99 ± .04	1.35 ± .05
α + Al	4.55	1.15 ± .23	1.11 ± .23	1.02 ± .20	1.10 ± .04	1.49 ± .04
α + Cu	4.55	1.41 ± .28	1.36 ± .27	1.26 ± .25	1.39 ± .05	1.75 ± .06
α + Pb	4.55	1.73 ± .34	1.70 ± .34	1.60 ± .32	1.68 ± .07	1.98 ± .08

nuclei are represented by nuclear spheres of constant density and radii $R_A = 1.27A^{1/3}$ fm. (c) Predictions based on Eqs. (2)–(6) and nuclear density functions deduced from electron scattering as in Ref. 12. As can be seen from Table I all three sets of predictions are quite similar and are in very good agreement with experiment!

The remarkable agreement between the CTM predictions and the π^- multiplicity distributions, demonstrated here for fourteen different projectile-target combinations, was obtained without free parameters. We therefore tend to believe that the success of the CTM, even at such relatively low energies, is not accidental. In the present energy range there is an accidental similarity between the CTM predictions and the predictions of statistical thermodynamic models,[14] since multiplicity distributions in pp collisions can be well approximated by Poisson distributions. At higher energies, the multiplicity distributions in pp collisions obey Koba-Nielsen-Olesen (KNO) scaling[15] and cannot be fitted by Poisson distributions.[16] For light projectiles and heavy targets, the CTM predictions will approximately obey KNO scaling. However, for heavy projectiles and heavy targets, the width of the distributions will increase, and large deviations from KNO scaling are expected.

Finally we note that a probabilistic description of nuclei is required in order to treat more accurately peripheral interactions.[6, 17]

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$$d\sigma(B) = d^2B\left(1-\exp\left\{\int\frac{d^2b}{\sigma}\ln\left[\left(1-\frac{i_1}{A_1}\right)^{A_1}+\left(1-\frac{i_2}{A_2}\right)^{A_2} - \left(1-\frac{i_1}{A_1}\right)^{A_1}\left(1-\frac{i_2}{A_2}\right)^{A_2}\right]\right\}\right)$$

which can be approximated by (10) in both regions $i_j \equiv \sigma T_j(b) \ll A_j$ and $i_j \gg 1$, $j = 1, 2$.

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[13]We have used the parametrization $\langle n_-\rangle = 0.25 + 0.19 \ln Q$ for $0.28\ \mathrm{GeV} \leq Q \leq 1$ GeV and $\langle n_-\rangle = 0.25 + 0.17 \ln Q + 0.25 \ln^2 Q$ for $Q \geq 1$ GeV.

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[16]For instance, the relation $(\langle n_-^2\rangle - \langle n_-\rangle^2)^{1/2} = (\langle n_-\rangle)^{1/2}$ is well satisfied in pp collisions in the energy range $1\ \mathrm{GeV}/c \leq p_{\mathrm{lab}} \leq 70\ \mathrm{GeV}/c$. It is badly violated for $p_{\mathrm{lab}} > 100\ \mathrm{GeV}/c$.

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