tions to occur only between A and B but not between A and A or B and B, it is possible to restrict the number of loops to a maximum of one per polymer ($N_L \leq N_p$). Thus, it is possible to approach gelation described essentially by a zero-state Potts model. If a number N_3' of trifunctional groups A_2-R-B are added to a solution of A-R-B_2 and A-B's, it is possible to increase the maximum number of allowed loops to $N_p + N_3'$. It may, therefore, be possible to create a distribution of molecules with well defined $\Lambda_p\Lambda_m$ between zero and one by allowing reactions to proceed to a certain point, diluting the system to inhibit intermolecular interactions, allowing the reaction to continue to another point, and then removing some solvent to recreate a more concentrated solution.

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Sea-Level Muon Charge Ratio of Cosmic Rays at High Energies

O. C. Allkofer, K. Carstensen,[a] W. D. Dau, and H. Jokisch
Institut für Kernphysik, University of Kiel, 23 Kiel, Germany
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The charge ratio of cosmic-ray muons with energies up to 1 TeV in the zenith-angle range 68°–82° has been measured. The data clearly show an enhancement of the charge ratio with increasing energy thus indicating a "normal" mass composition of galactic cosmic rays.

The charge ratio of cosmic-ray muons at sea level provides the only information on the mass composition of the primary cosmic-ray flux, in particular on the ratio of neutrons (bound in nuclei) to all nucleons,[1] in the energy range beyond 10^{11} eV. The mass composition is an essential tool to decide between different models for galactic confinement of cosmic rays.[2]

Measurements have been performed in a magnetic spectrometer of Carstensen *et al.*[3] and Carstensen.[4] The mean maximum momentum $\bar{p}_{\max}$ defined by $p\psi = \bar{p}_{\max}\sigma_\psi$ (with ψ the angular deflection, σ_ψ the angular resolution, and p the momentum)[5] amounts to 750 GeV/c. The original charge ratio R_μ at muon productions is modified for low- and high-energy muons: Because the viewing direction is towards east [$75 \pm 7°$ zenith angle and $288 \pm 20°$ azimuth angle], low-energy positive muons run a longer path in the atmosphere than the negative muons by geomagnetic

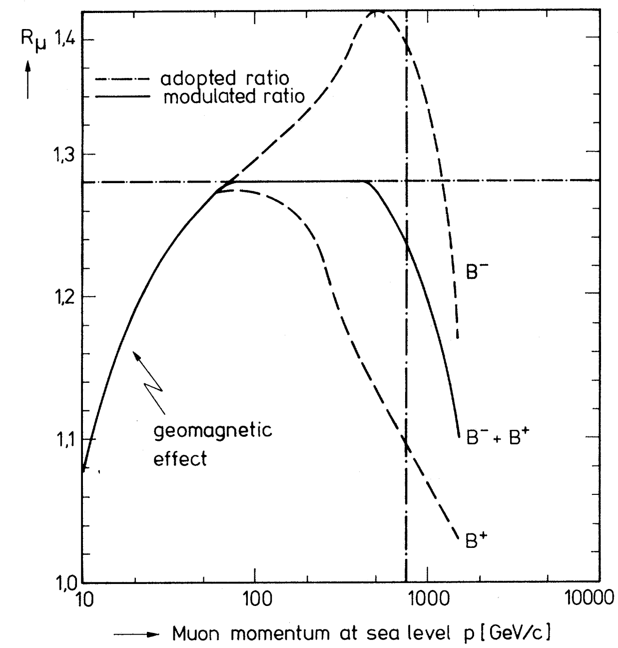

FIG. 1. Geomagnetic effect and mean-maximum-momentum effect on the charge ratio R_μ. An original ratio of $R_\mu = 1.28$ was adopted for the calculations. The curves B^+ and B^- refer to fixed signs of the field B in the spectrometer magnet. They display the effect of an assumed systematic alignment error of $\Delta\alpha = 0.1\sigma_\psi = 0.074$ mrad, i.e., $\frac{1}{10}$ of our angular resolution. $\bar{p}_{max}$ is indicated by the vertical dash-dotted line.

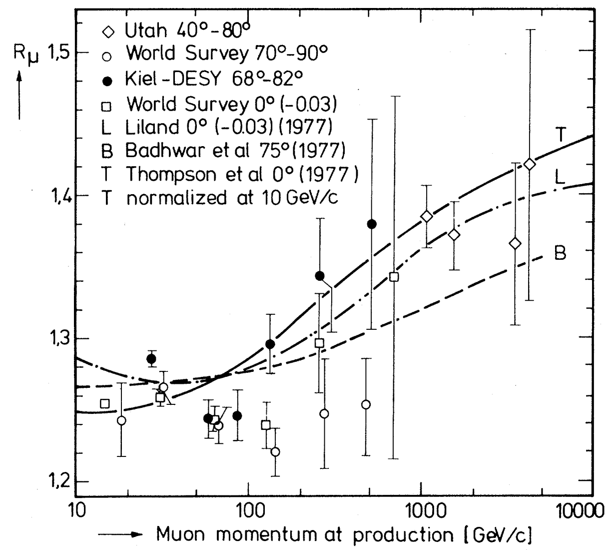

FIG. 2. Present status of charge-ratio measurements and calculations. Utah (Ref. 6) (33 000 events > 1 TeV/c); world survey 70°–90° (Refs. 7–22) (120 000 events > 20 GeV/c); Kiel-DESY, this work (310 000 events > 20 GeV/c); world survey 0° (Refs. 7, 16, 23–32) (shifted down by 0.03, 420 000 events > 20 GeV/c); Liland 0° (Ref. 33) (shifted down by 0.03); Badhwar and Stephens, 75° (Ref. 34); Thompson, Thornley, and Whalley, 0° (Ref. 35) (normalized to 1.25 at 10 GeV/c).

deflection (Fig. 1) and thus will be relatively suppressed. A severe deviation from the correct charge ratio occurs for even a small systematic misalignment of the apparatus (curves B^+ and B^- in Fig. 1) if it is not canceled out by frequent reversals of the magnetic field (curve $B^- + B^+$ in Fig. 1). We took care of this effect by equalizing the run times for both signs of the spectrometer magnetic field. At the highest energies the limited resolution of the spectrometer causes a dilution of the charge ratio (curves $B^- + B^+$ in Fig. 1).

Figure 2 shows our experimental results, together with a world survey of combined data of other experiments, in comparison with the most recent theoretical calculations. All data besides the results of Ashley, Keuffel, and Larson (Ref. 6) were binned according to their momenta at production within following values: 10, 20, 50 (70 only for our data), 100, 200, 400, 1000 GeV/c. No data points above the mean maximum momentum of any experiment were included. Also, no results which were subject to geomagnetic effects were included in the world surveys. Only our two data points in the 20–50-GeV/c and 50–70-GeV/c bins have been corrected for the geomagnetic effect according to Fig. 1 (the corrections were +8.7% and +1.1%, respectively). The measured points and calculated curves for 0° zenith angle have been shifted down by 0.03 [36] for better comparison with these values at large zenith angles. Only the curve of Thompson, Thornley, and Whalley[35] has been normalized to 1.25 at 10 GeV/c. All the data are remarkably consistent in the 50–100 GeV/c range. In the 20–50-GeV/c bin, our value may be too high due to a presumably overestimated correction for the geomagnetic effect. At energies above 100 GeV, the world survey for 70°–90° data do not show any enhancement in contradiction to our data, the world survey at 0°, and the data of Ref. 6.

A charge ratio rising with energy is also expected from theory because of the growing importance of the K decay (the K^+/K^- ratio is larger than the π^+/π^- ratio in nucleon collisions). The calculated curves in Fig. 3 clearly show this effect. If one takes into account that the absolute values of calculated charge ratios have an error of 0.03–0.05 [34] or even more,[35] then there is reasonable agreement between theory and experiment.

An interesting feature which seems to show up more or less in all the data is a minimum around

100 GeV/c. The only theoretical curve which shows a similar effect is given by Liland.[33] He explains it by a decrease of the muon charge ratio (down to a constant value) when scaling is approached in meson production, in combination with a rise due to the kaons which become more important as muon source at higher energies.

The parameters which mainly influence the charge ratio R_μ of muons are as follows: the charge composition of the incident primary cosmic rays, i.e., η, the ratio of neutrons to all nucleons; the charge-exchange probability, W, of the interacting nucleon; and the charge ratios R_π of pions and R_K of kaons at production by a proton-air collision (for a more detailed discussion see Thompson and Whalley[37] and Badhwar, Stephens, and Golden[38]). The values used by the authors cited in Fig. 2 are given below.

	η	R_π	R_k	W	Calculated R_μ at 100 GeV/c
Badhwar *et al.*[a]	0.105	1.38	1.8	0.26	1.276
Liland[b]	0.125	1.40	2.85	0.44	1.309
Thompson *et al.*[c]	0.130	1.63	3.14	0.26	1.430

[a]Refs. 3, 8, and 34.
[b]Ref. 33.
[c]Ref. 37.

Because of the large R_π—and R_k—numbers of Thompson *et al.*, their R is very high and therefore normalized in Fig. 3. According to Elbert *et al.*[39] the charge ratio R_μ can roughly be expressed by the equation $R_\mu \approx [\eta + (1-\eta)R_\pi]/[\eta R_\pi + (1-\eta)]$. Taking Badhwar's mean values for R_π and η from the table we get $R_\mu = 1.289$, and for a variation $\Delta\eta$ we obtain a variation in R_μ of $\Delta R_\mu = -0.837\Delta\eta$. So, using the curve B in Fig. 2 as reference line, we can convert the measured R_μ values of Fig. 2 to η values in Fig. 3. The muon energy at production E_μ is scaled up by a factor of 7 to get the median primary nucleon energy $E_N{}^1$. The error bars in Fig. 3 only display the statistical errors. The systematic errors can be estimated by taking Lilands curve in Fig. 2 as reference for the R_μ-η conversion: All muon data points move upwards as shown by the dotted lines.

Two remarks should be made concerning Fig. 3: (i) Although the interaction properties of hadrons are relatively well known at 100 GeV, the muon data do not fit well to the direct measurements at this energy. (ii) Nevertheless, the data show that a rising η, i.e., an overabundance of heavy primaries, is not needed to explain the muon charge ratio. This conclusion is in contrast to the results of Erlykin, Ng, and Wolfendale,[1] Adair,[40] and Hoffman.[41] Adair deduces the high value $\eta = 0.25$ from his calculations to fit the measured R_μ. Using a small $\eta = 0.11$ he obtains a rather high R_μ. The direct determination of R_μ from proton collisions on copper nuclei at Fermilab (Adair *et al.*[42]) also resulted in a high μ^+/μ^- ratio. But, as was recently shown by Ramana Murthy,[43] R_μ decreases again by going from the high-density copper target to the actual air target (five orders of magnitude smaller density), so that in fact the Fermilab results support a low η.

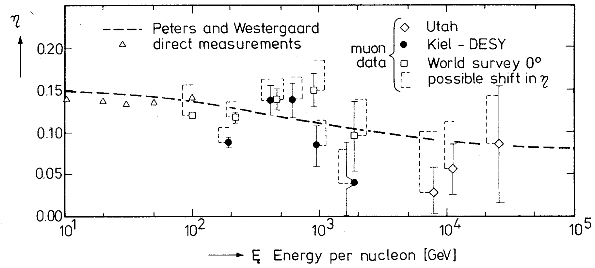

FIG. 3. The ratio of neutrons to all neutrons, η, of the galactic cosmic rays. The muon data have been obtained by comparing the measured muon charge ratio R_μ to that calculated by Badwar and Stephens (Ref. 34). A comparison with the calculations of Liland (Ref. 33) would yield a shift indicated by the dotted line. The direct measured data are those from a summary by Erlykin, Ng, and Wolfendale (Ref. 1). The Peters-Westergaard curve is taken from the publication of Shapiro and Silberberg (Ref. 2).

Our data are consistent with a constant composition but they seem to show even a reduction of η at higher energies. This reduction is expected if one assumes confinement of cosmic rays not only in the galactic disk but also in the halo (Peters-Westergaard model, Fig. 3).[2]

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(a)Now at Abteilung Medizinische Statistik und Dokumentation, University of Kiel, 23 Kiel, Germany.

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