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Can Pions Be Strongly Bound in Nuclei?

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On the basis of new data on pionic atoms, it is argued that the criticality condition for the existence of strongly bound π-nuclear states is likely to be met in most nuclei. Numerical solutions for ^{27}Al, ^{48}Ca, and ^{209}Bi are reported, with the conclusion that the widths of such π states are generally prohibitively large for their observability except, possibly, in heavy nuclei.

The possible existence of pion-nuclear strongly bound states has recently been suggested by Ericson and Myhrer[1] (EM) who noted that the p-wave attractive term $\nabla\alpha(r)\cdot\nabla$, with $\mathrm{Re}\,\alpha(r)>0$, in the pion-nuclear optical potential may overcome the pion kinetic energy, thus leading to an infinity of strongly bound states. While a criticality condition must be met for such states to exist mathematically, it is by no means clear that their strong-interaction widths are not prohibitively large for any conclusive observation. Also, it was concluded by EM, on the basis of a particular parametrization[2] of the optical potential, that the criticality condition is not met in most ordinary nuclei.

In this Letter we point out that the new precise data[3] on $2p$ levels in pionic atoms (Al to Zn) lead to a different parametrization [procedure (b) of Ref. 3] of the pion-nuclear zero-energy optical potential, for which the criticality condition is satisfied in many nuclei. This parametrization also reproduces[4] all existing data for $1s$ states. We report here numerical results for pionic bound states in the representative nuclei ^{27}Al, ^{48}Ca, and ^{209}Bi and mention the far-reaching consequences of nuclear absorption on these states.

The pion-nuclear low-energy potential is given by

$$2\mu V(r) = q(r) + \nabla\alpha(r)\cdot\nabla, \tag{1}$$

where $q(r)$ is the local (s-wave) part which for π^- reads

$$q(r) = 4\pi\{(1+\mu/m)[b_0(\rho_n+\rho_p)+b_1(\rho_n-\rho_p)] + (1+\mu/2m)B_0 4\rho_p\rho_n\}, \tag{2}$$

μ and m being the pion-nucleus reduced mass and the nucleon mass, respectively. B_0 is a complex number, simulating both dispersive and absorptive effects ignored by the first-order term. $\rho_{p,n}$ are the nuclear densities. The nonlocal (p-wave) part is parametrized here as follows:

$$\alpha(r) = \frac{\alpha_0(r)}{1+\xi\frac{1}{3}\alpha_0(r)} + 4\pi\left(1+\frac{\mu}{2m}\right)^{-1} C_0 4\rho_p\rho_n, \quad 0\leq\xi\leq 1, \tag{3a}$$

$$\alpha_0(r) = 4\pi(1+\mu/m)^{-1}[c_0(\rho_n+\rho_p)+c_1(\rho_n-\rho_p)], \tag{3b}$$

where C_0, again, is a complex number. The strength ξ of the Lorentz-Lorenz (LL) renormalization is at present, not uniquely determined by pionic-atom data.

In the absence of compelling theoretical motivation, the two-nucleon term of Eq. (3a) is stripped of its customary LL renormalization. This results in a value for C_0 which differs from the value report-

ed in Ref. 3, but the quality of the atomic fit remains as high.[4] The set of parameter values (for $\xi = 0$) adopted in the present work is given (in pion mass units) by

$$b_0 = -0.017, \quad b_1 = -0.13,$$
$$\mathrm{Im}B_0 = 0.0475, \quad \mathrm{Re}B_0/\mathrm{Im}B_0 = -1; \tag{4a}$$
$$c_0 = 0.21, \quad c_1 = 0.17,$$
$$\mathrm{Im}C_0 = 0.0425, \quad \mathrm{Re}C_0/\mathrm{Im}C_0 = -0.6. \tag{4b}$$

The negative sign of $\mathrm{Re}C_0$, relative to that of c_0, gives rise to a repulsive modification of the first-order p-wave attraction, much the same as otherwise caused by the ($\xi = 1$) LL modification for which case the value of $\mathrm{Re}C_0/\mathrm{Im}C_0$ obtained in the fit[4] is 2.1, all other parameters remaining the same.

The parameter set (4) gives, as a rule, values of $\mathrm{Re}\alpha(0)$ greater than the critical value of 1. These, for example, are (in parentheses the values for $\xi = 1$ are given) 1.09 (1.09) for ^{27}Al, 1.03 (1.04) for ^{40}Ca, 1.16 (1.13) for ^{44}Ca, 1.17 (1.14) for ^{48}Ca, and 1.05 (0.99) for ^{209}Bi. Two- and three-parameter Fermi distributions which fit electron scattering data[5] were employed for protons, with identical parameters for neutron distributions in ^{27}Al and $^{40-44}$Ca. The neutron rms radius r_n was assumed to be larger by 0.11 fm than the proton rms radius r_p for ^{48}Ca and ^{209}Bi. We note that with $r_n = r_p$ it is possible, also for $\xi = 1$, to obtain $\mathrm{Re}\alpha(0) > 1$ (1.03) in ^{209}Bi.

We have also verified that, with $\mathrm{Im}C_0 \neq 0$ as in (4b), a gradual change of parameters so that $\mathrm{Re}\alpha(0)$ passes through 1, leads to a very smooth and predictable trend in the values of atomic binding energies and widths (B, Γ). Finally, the Schrödinger equation, not the Klein-Gordon, was used in the nuclear case, since our primary interest is in π-nuclear states near $E = 0$. The generalization is straightforward.

Since the optical-potential parameters (4) were determined primarily on the basis of atomic $2p$-level shifts and widths, it was natural to look for strongly bound states in the appropriate mass region. For ^{27}Al and ^{48}Ca we failed to obtain bound states with widths of less than the order of GeV. In fact, for $l = 0$ in ^{48}Ca, the state which is closest to threshold is found to have (B, Γ) = (9, 2400) MeV.

EM suggested an averaging procedure around r_0, the point at which $\mathrm{Re}\alpha(r)$ crosses the value 1, in order to avoid the far-reaching consequences of the singularity of the wave equation with potential (1). In its simplest form they suggested linearization of the wave function in some small interval $\pm\Delta r$ around r_0. Naturally, any regularization has to be constrained by the π^--atomic data. An iterative procedure was therefore developed by us for the atomic case, where the parameter C_0 is refitted as a consequence of applying linearization. The refitted C_0, in turn, shifts the value of r_0, and so the Δr interval is now moved to cope with this change and a newly refitted value of C_0 emerges, etc., until convergence is reached. Without such an adjustment, the atomic results deviate, e.g., for ^{44}Ca and $\Delta r = 0.2$ fm, by $(\Delta B, \Delta\Gamma) \sim (0.15, 0.25)$ keV (considerably larger than the experimental uncertainties[6]). Similar deviations were found for non-iterative linearization about arbitrary points within the nucleus, which signifies that the singularity of the wave equation at the point r_0 has no special role in the π^--*atomic* case.

The least bound π^--*nuclear* states in ^{48}Ca are listed in Table I for the lowest l values. The net repulsion, about 18 MeV for $l = 0$ states, of the local interactions, the repulsive $q(r)$, and the attractive Coulomb potential prevents binding of states with radial number $n_i < 6$ for the exact $l = 0, 1$ solutions. In the absence of these interactions, $l = 0, 1$ bound states with n_i as low as 2 do appear, with widths comparable to those of the corresponding solutions (also listed in the table) obtained within the iterative linearization procedure. For the latter solutions, the spacings between adjacent levels is considerably larger

TABLE I. Least-bound states for π^- in ^{48}Ca. B and Γ are the binding energy and width, respectively. The sequential number n_i is determined by the number of nodes of the real part of the radial wave function within the critical region $r \lesssim r_0$, and l is the angular momentum.

n_i	l	B (MeV)	Γ (MeV)
		Exact solution	
6	0	9.0	2400
	1	11.8	2730
		Linearization	
0	2	7.2	178
1	0	26.9	219
	1	57.5	329
	2	92.2	456
2	0	109	498
	1	160	672
	2	215	864

TABLE II. Least-bound states for π^- in ^{209}Bi.

n_i	l	B (MeV)	Γ (MeV)
		Exact solution	
0[a]	0	1.73	25.3
	1	1.78	34.4
	2	2.04	45.6
1	0	1.01	50.7
	1	2.10	67.1
	2	3.22	85.4
2	0	4.41	89.8
	1	6.19	113
	2	7.87	138
3	0	8.99	142
	1	11.2	172
	2	13.4	204
		Linearization	
6	0	1.5	764
7	0	3.2	1002

[a]Results for $n_i = 0$ are obtained using $b_1 = -0.11 m_\pi^{-1}$. See also caption to Table I.

than for exact solutions, so that the local repulsion destroys fewer states, with the *de facto* conclusion that the widths of states closest to threshold are significantly reduced by the use of linearization. These widths are still too large to acquire a meaning to the bound states.

The results for nuclear π^- states in ^{209}Bi are given in Table II. Although the local repulsive $q(r)$ and the local attractive Coulomb potential to a large extent cancel each other, their net effect in ^{209}Bi is a repulsion of about 2 MeV which causes one bound state, for each of the low partial waves listed for exact solutions, to disappear. These somewhat narrower states would have become stable already for $b_1 = -0.11 m_\pi^{-1}$. However, the latter value of b_1 is found to take the atomic $(B, \Gamma)_{4f}$ values about 3 standard deviations away from the measured values.[4,7] It is interesting to note that the ratio Γ/B quickly becomes constant, with increasing binding, independently of l; the value of this constant, however, is considerably larger than that argued by EM.

Somewhat dramatic effects occur when the *ad hoc* linearization procedure is applied to π^- ^{209}Bi. The binding energies and widths of the states closest to threshold are hardly changed for a naive $\Delta r = \pm 0.2$ linearization around $r_0 = 5.55$ fm. However, the iterative linearization procedure described above brings r_0 down to 3.92 fm, corresponding to the refitted value of $\mathrm{Re}C_0/\mathrm{Im}C_0 = -1.12$. For such a relatively large change in r_0, the ratio Γ/B reacts drastically. The states closest to threshold have now widths of the order 1000 MeV, the least bound one found to have $(B, \Gamma) = (1.5, 764)$ MeV.

Although $\mathrm{Re}\alpha(0) = 1.05$ for ^{209}Bi, modest changes in the densities or in pionic parameters may significantly change the above picture. For example, increasing r_n by another 0.11 fm relative to r_p results in $\mathrm{Re}\alpha(0) = 0.99$, marking an end to the present considerations. It is quite plausible that with a single-particle density distribution for ^{209}Bi the value $\mathrm{Re}\alpha(r) = 1$ may even be crossed more than once. The effects expected on the widths of π^--nuclear states are large.

On the basis of the above analysis we conclude that the chances of unambiguously observing π^--nuclear states are poor. Since the energy dependence of the potential (1) is here ignored, only the least-bound states are potentially credible (a linear dependence, for example, is expected to destroy all states bound by more than about 20 MeV). However, the widths of such states were found to vary most sensitively with r_0: The lower r_0 is, the larger Γ turns out to be. To obtain widths of the order of $\Gamma \lesssim 50$ MeV, heavy elements are necessary; but for such elements —as the ^{209}Bi example shows—the central nuclear densities are smaller compared to those obtained for medium-size nuclei. Consequently, $\mathrm{Re}\alpha(0)$ only marginally exceeds 1 and r_0 is susceptible to wide variations.

While attempts to reduce the widths of π^--nuclear states by order of magnitude through *ad hoc* linearization succeed for medium-size nuclei, the resulting widths are still prohibitively large; this was also the case for ^{32}Na, discussed in Ref. 1. However, such attempts may generally fail for heavier elements as our ^{209}Bi example shows. Moreover, it can be proved that for *real* $\alpha(r)$, where the call for regularization is mandatory, the regularized[8] wave function cannot follow a linear trend at the vicinity of r_0; the effects of regularization—leading ultimately to suppression of the π^- binding phenomenon—must be treated within a self-consistent framework.

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