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[3]It would be interesting to pursue similar studies for hadron production of J/ψ since its decay shows even larger inhibitions than does the φ. Unfortunately, the production cross section is sufficiently small that no exclusive channel has been observed.

[4]A. Etkin *et al.*, Phys. Rev. Lett. 40, 422 (1978).

[5]Since a threshold Cherenkov counter was used (to veto pions) our data sample consists of K^+K^- and $p\bar{p}$ pairs. We can estimate the $\bar{p}p$ contamination using information from observation of single $\bar{p}p$ pair production in the reaction $\pi^-p \to p\bar{p}n$ at 15 GeV/c. The ratio of single $\bar{p}p$ to single K^+K^- pairs is about 1:1 [see H. H. Williams, SLAC Report No. 142, 1972 (unpublished)], while at 11 GeV/c the $p\bar{p}$ pair production is less than 20% of the K^+K^- production [B. D. Hyams *et al.*, Nucl. Phys. B22, 189 (1970)]. Our experiment was at higher energy, but an additional pair of particles is produced. We estimate, therefore, that the $\bar{p}p$ contamination is less than 50%, integrated over the whole K^+K^- spectrum.

[6]Our K^+K^- effective-mass resolution is about 14 MeV/c^2 (full width at half-maximum). Since the width of the φ (Γ) is 4.1 MeV/c^2, the true enhancement is about a factor of 3 larger.

[7]Note that every K^+K^- pair accompanied by a φ meson is plotted in the spectrum, hence each $\varphi\varphi$ event is plotted twice. This procedure preserves the correct signal-to-background ratio.

[8]This ratio is typical of resonance-to-background ratios in allowed reactions—for example, C. Baltay *et al.*, Phys. Rev. Lett. 40, 87 (1978), find that the ω production accounts for about 5% of the $\pi^+\pi^-\pi^0$ spectrum in the reaction $\pi^-p \to \Delta^{++}\pi^+\pi^-\pi^0$ at 15 GeV/c.

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Gluon Jets from Quantum Chromodynamics

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Following the idea of Sterman and Weinberg, we calculate the jet angular radius of a gluon jet and its energy dependence in the framework of perturbative quantum chromodynamics. The result is free of infrared and mass singularities and does not depend on the gluon fragmentation function. We find that the gluon jet spreads much more (i.e., less jetlike) than a quark jet; this renders the detection of gluon jets very difficult.

The hope that quantum chromodynamics (QCD) provides a complete description of hadron physics is widely spread. One of the key characteristics of QCD is the existence of gluons. A crucial verification of QCD would thus be the observation of its gluonic structure. Partly motivated by the observation of hadronic jets (which we shall refer to as quark jets, since they are believed to be jets arising from energetic quarks) in e^+e^- annihilation,[1] we expect the gluons to reveal themselves in the form of gluon jets (i.e., hadronic jets arising from energetic gluons). In fact, gluon jets are expected to be produced in many experimental processes such as large-p_T scattering, e^+e^- annihilation, lepton-proton deep inelastic scattering, and Drell-Yan dimuon experiments.

Recently it has been emphasized that, in QCD, the three-gluon decay mode[2] of the upsilon[3] Υ (or some other heavy-quark-antiquark bound states) would provide a very clean place to find and study the gluon jet.[4] From e^+e^- annihilation we learn that a quark jet is observable when the initial quark has an energy $E_q \geqslant 3$ GeV. Since the mass of Υ is around 9 GeV, the average energy per gluon is 3 GeV. Hence, whether a gluon jet from

ϒ decay is observable or not depends crucially on its jetlike nature; i.e., the jet angular radius and its energy dependence as compared to those of the quark jet.

In this paper we shall study these gluon-jet properties using the approach suggested by Sterman and Weinberg[5]; they showed that the quark-jet structure follows from a perturbative calculation in QCD without assuming a transverse-momentum cutoff. Furthermore, their result does not require any knowledge of the fragmentation function.[6] This means that their approach is particularly useful to the study of gluon jets since how a gluon fragments into hadrons is totally unknown (presumably, fragmentation is a consequence of quark confinement, an issue which we shall not discuss here).

Instead of studying a possible three-gluon jetlike structure in the ϒ decay directly, we choose to consider a simplified process where a pair of energetic gluons is produced by a color-singlet, gauge-invariant scalar

$$(F_{\mu\nu}{}^a)^2 = (\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g f^{abc} A_\mu{}^b A_\nu{}^c)^2$$

in the center-of-mass frame. For the two-gluon decay mode, this scalar source effectively represents a color-singlet 0^{++} quark-antiquark system in the limit of heavy-quark mass. It is our basic assumption (which, we believe, is very plausible) that the jet angular radius and its energy dependence do not depend on the source. Calculations using the pseudoscalar source $F_{\mu\nu}{}^a \tilde{F}_{\mu\nu}{}^a$ have also been carried out.[7] The result supports our assumption. This will mean that the essential features of gluon-jet structures do not depend on how the energetic gluon is produced and that our results are applicable to gluon jets produced in all scattering processes.

To study gluon jets we calculate, to order g^2, the partial cross section $\sigma(E,\epsilon,\delta)$ for the two-gluon jetlike events, where all but a small fraction ϵ of the total energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within the solid angle $d\Omega$ $(\pi\delta^2 \ll d\Omega \ll 1)$. For high-energy phenomena in perturbative QCD, there appear degenerate states within the resolution of detector devices. These degenerate states lead to infrared divergences and mass singularities,[8] which, however, are expected to be canceled for physically sensible cross sections such as $\sigma(E,\epsilon,\delta)$.

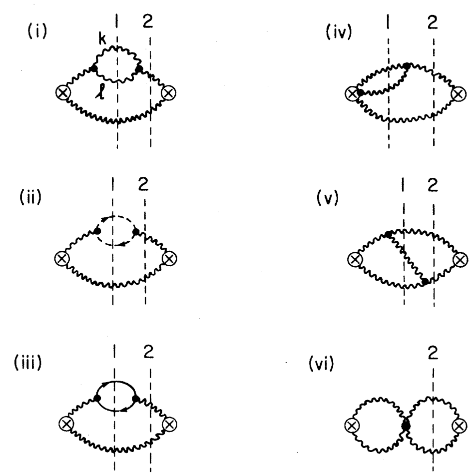

FIG. 1. Leading-order corrections to the two-gluon production cross section. Wavy lines are gluons, dotted lines are Faddeev-Popov ghosts, and solid lines are massless quarks. The dashed lines numbered 1 and 2 denote the unitary cuts. Mirror reflections of the diagrams shown above and diagrams that do not contribute in our calculation are not shown.

Figure 1 shows six types of diagrams that contribute to $\sigma(E,\epsilon,\delta)$ to order g^2. Quark masses are set to zero. Each diagram divided at line 1 [diagrams (i)–(v)] represents a real-particle (i.e., gluon, ghost, or quark) emission process. The hard-bremsstrahlung region (where the energies of emitted particles $k^0 > \epsilon E$ and $l^0 > \epsilon E$ are within the angular cone $\pi\delta^2$) gives rise to mass singularities and determines the jet angular radius. [For diagram (iii), quark pairs are produced within the angular cone $\pi\delta^2$]. The soft-bremsstrahlung region (where $k^0 < \epsilon E$ or $l^0 < \epsilon E$) gives rise to infrared divergences as well as mass singularities. As we shall see below, these infrared divergences and mass singularities are canceled by those arising from the virtual corrections to the external gluon lines and the source[9]; diagrams (i)–(vi) divided at line 2 represent these virtual corrections.

We perform our calculation in the Feynman gauge and use the dimensional-regularization method[10] to control the ultraviolet and infrared divergences and mass singularities. Proper combinatoric factors are taken into account. The real-particle emission processes give the partial cross section

$$\sigma(1) = \sigma_0 \frac{g^2}{4\pi^2} f(D) \left\{ C_2(G) \left[\left(\frac{2}{D^2} - \frac{11}{3D} \right) E^D - \left[4\ln(2\epsilon) + \tfrac{11}{3} \right] \ln\delta - \tfrac{5}{6}\pi^2 + \tfrac{67}{9} \right] + \tfrac{4}{3} T(R) N_f \left[D^{-1} E^D + \ln\delta - \tfrac{23}{12} \right] \right\} \quad (1)$$

as the space dimension $n = 4 + D \to 4$, where $f(D) = [(2\sqrt{\pi})^D \Gamma(\frac{1}{2}D + 1)]^{-1}$, and $\sigma_0 \equiv (d\sigma/d\Omega)_0$ is the lowest-order differential cross section for the two-gluon production from the source. N_f is the number of quark flavors and $C_2(G)$ and $T(R)$ are color factors for the gluon representation G and the quark representation R, respectively[11]: In QCD, $C_2(G) = 3$ and $T(R) = \frac{1}{2}$ for quark triplets. The double-pole term $1/D^2$ and the $\ln(2\epsilon)\ln\delta$ term originate solely from diagram (v) where an infrared divergence and a mass singularity show up simultaneously. Finite terms proportional to ϵ and/or δ have been neglected.

The virtual corrections [diagrams (i)–(vi)] lead to, after renormalization, the partial cross section

$$\sigma(2) = \sigma_0 \frac{g^2}{4\pi^2} f(D) \left\{ C_2(G)\left[-\frac{4}{D^2} E^D + \frac{11}{3D}\mu^D - \frac{67}{18} \right] + T(R) N_f \left[-\frac{4}{3D}\mu^D + \frac{20}{9} \right] \right\}. \tag{2}$$

We have carried out the renormalizations of the gluon propagator and of the source by subtracting only the ultraviolet divergent terms at an arbitrary off-mass-shell point $p^2 = \mu^2$ for gluon momenta[12] and on the mass shell $p^2 = E^2$ for the outgoing source momentum. As expected, cancellations of infrared divergences and mass singularities take place in the sum of $\sigma(1)$ and $\sigma(2)$. Taking the limit $D = 0$, we obtain

$$\sigma(1) + \sigma(2) = \sigma_0 \frac{g^2}{4\pi^2} \{ C_2(G)[-\tfrac{11}{3}\ln(E/\mu) - (4\ln(2\epsilon) + \tfrac{11}{3})\ln\delta + \tfrac{67}{18} - \tfrac{5}{6}\pi^2] + \tfrac{4}{3} T(R) N_f [\ln(E/\mu) + \ln\delta - \tfrac{1}{4}] \}. \tag{3}$$

The coefficient of the $\ln(E/\mu)$ term is simply the anomalous dimension[13] of the source $(F_{\mu\nu}{}^a)^2$. This term reflects the nature of the source and is not related to that of the gluon jet. Let us choose $\mu = E$. Then, to order $\alpha(E)$, the partial cross section $\sigma(E,\epsilon,\delta)$ is given by

$$\sigma(E,\epsilon,\delta) = \sigma_0 \left\{ 1 - \frac{\alpha(E)}{\pi} [4C_2(G)\ln(2\epsilon) + \tfrac{11}{3}C_2(G) - \tfrac{4}{3}N_f T(R)] \ln\delta \right\}, \tag{4}$$

where only the logarithmic terms in Eq. (3) are kept. The E dependence occurs only in the definition of $\alpha(E)$. It is amusing to observe that the coefficient of the single-logarithmic term is precisely the one-loop β function[14] of QCD.

To facilitate comparison to the quark jet we integrate $\sigma(E,\epsilon,\delta)$ over the whole solid angle Ω $(=4\pi)$ and define the fraction f of all jetlike events which have a fraction $(1-\epsilon)$ of the total energy E inside some pair of opposite cones of half-angle δ:

$$f(\text{gluon}) = 1 - \frac{\alpha(E)}{\pi}[4C_2(G)\ln(2\epsilon) + \tfrac{11}{3}C_2(G) - \tfrac{4}{3}N_f T(R)]\ln\delta. \tag{5}$$

Here the double-logarithmic term in f is taken to be independent of the gluon-production source.[15] The same formula for the quark case[5] is

$$f(\text{quark}) = 1 - \frac{\alpha(E)}{\pi}\{4\ln(2\epsilon) + 3\} C_2(R)\ln\delta, \tag{6}$$

where the color factor $C_2(R) = \frac{4}{3}$ for a quark triplet. For the same values of f, E, ϵ, the ratio of the quark-jet angular radius to the gluon-jet angular radius is independent of $\alpha(E)$:

$$\frac{\ln[\delta(\text{quark})]}{\ln[\delta(\text{gluon})]} = \frac{C_2(G)\ln(2\epsilon) + \frac{11}{12}C_2(G) - \frac{4}{3}N_f T(R)}{C_2(R)[\ln(2\epsilon) + \frac{3}{4}]} \simeq \frac{C_2(G)}{C_2(R)} = \frac{9}{4}, \tag{7}$$

where the approximation is qualitatively valid for $\epsilon < 0.2$ and $N_f = 4$. Hence $\delta(\text{gluon}) \simeq [\delta(\text{quark})]^{4/9}$, where δ is measured in radians; this shows that the gluon jet spreads more than the quark jet. If we assume $\alpha(E)$ to be running coupling constant in QCD, then the decrease of the gluon-jet radius with increasing energy E is much slower that that of the quark-jet radius. This general qualitative feature is mainly a consequence of the difference in the color factors associated with the representations of the quark and the gluon in QCD. Typically, a quark jet of angular radius 10° (5°) corresponds to a gluon jet of angular radius 27° (19°). Also the threshold energy for the detection of a gluon jet may be as much as an order of magnitude bigger than that (~ 3 GeV) for the dectection of a quark jet. This renders the observation of gluon jets very difficult.[16] It is intriguing to consider this as the reason why gluon-jet structures have so far escaped detection.

Although we expect a marked increase in sphericity (or spherocity) when the energy in e^+e^- annihilation approaches that of a heavy-quark-antiquark bound state, its distinct three-gluon angular distributions would be difficult to measure. A better place to test the gluonic structure in QCD is probably the photon angular and energy distribution in the two-gluon–one-photon decay mode[17] of Υ. This test is even better if there exists another heavy charge-$\frac{2}{3}$ quark.

We were informed that this problem is also under investigation by M. B. Einhorn and B. Weeks. We thank our colleagues at Fermilab, in particular C. K. Lee, for discussions.

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[5]G. Sterman and S. Weinberg, Phys. Rev. Lett. 39, 1436 (1977). We refer the readers to this work for a general discussion on the validity and applicability of their method. Here we follow their notations.

[6]H. Georgi and M. Machacek, Phys. Rev. Lett. 39, 1237 (1977); E. Farhi, Phys. Rev. Lett. 39, 1587 (1977).

[7]This pseudoscalar source will be suitable for the description of the two-gluon decay from a possible 0^{-+} $q\bar{q}$ bound state produced in the radiative decay of Υ. However instanton effects may be important in this case. Neglecting this complication, the scalar and pseudoscalar sources give, to order g^2, the same gluon-jet angular radius and its energy dependence [i.e., Eq. (1)]. Here we choose to discuss the scalar source since the renormalization of this source has been extensively studied in the literature. See, e.g., H. Kluberg-Stern and J. B. Zuber, Phys. Rev. D 12, 467 (1975); C. K. Lee, Phys. Rev. D 14, 1078 (1976).

[8]The problem of infrared and mass singularities has been discussed exhaustively in the literature. We refer the readers to Ref. 5 for an extensive list. In addition, see also J. M. Cornwall and G. Tiktopoulos, Phys. Rev. D 13, 3370 (1976); J. Frenkel and J. C. Taylor, Nucl. Phys. B116, 185 (1976); T. Kinoshita and A. Ukawa, Phys. Rev. D 16, 332 (1977); A. Sugamoto, Phys. Rev. D 16, 1065 (1977); G. Sterman, Phys. Rev. D 17, 616 (1978). For specific applications motivated by Ref. 5, see, e.g., C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. D 17, 2298 (1978); S.-Y. Pi, R. L. Jaffe, and F. E. Low, Phys. Rev. Lett. 41, 142 (1978); D. Amati, R. Petronzio, and G. Veneziano, CERN-TH-2470, 1978 (unpublished).

[9]The scattering of a gluon by an external source $(F_{\mu\nu}{}^a)^2$ has been studied by A. G. Alvarez, Nucl. Phys. B120, 355 (1977). He demonstrated the finiteness of the leading-order correction and obtained a formula which is similar to ours. Here we investigate the production of two gluons in the center-of-mass frame with the inclusion of quarks.

[10]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); C. G. Bollini and J. J. Giambiagi, Nuovo Cimento 12B, 120 (1972). See also, W. J. Marciano and A. Sirlin, Nucl. Phys. B88, 86 (1975); R. Gastmans and R. Meuldermans, Nucl. Phys. B63, 277 (1973).

[11]Here we follow the group notation of D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).

[12]By means of this off-mass-shell renormalization procedure for gluon momenta the ultraviolet divergences are separated from the infrared and mass singularities. In Eq. (2) we have included the contribution arising from the ultraviolet-finite renormalization factor $[Z_3(\mu^2)/Z_3(0)]^{1/2}$ for other external gluon line; this factor contains an infrared divergent piece although it is ultraviolet finite.

[13]The gluon part of the anomalous dimension in Eq. (3) agrees with that obtained in Ref. 7. In our present case, where the external gluons are massless and the outgoing source momentum is timelike, there is no mixing of the anomalous dimension with those of other operators at the one-loop level, as shown in C. K. Lee, Ref. 7.

[14]That this coefficient is simply the β function [first evaluated by H. D. Politzer, Phys. Rev. Lett. 26, 1346 (1973), and D. J. Gross and F. Wilczek, Phys. Rev. Lett. 26, 1343 (1973)] is observed by A. G. Alvarez, Ref. 9. The inclusion of the quark contribution in our case makes this connection more transparent.

[15]In Eq. (3), terms independent of logarithms are somewhat ambiguous in the sense that they are related partly to the renormalization of the source and partly to gluon-jet properties. We believe that the former part is removed when one considers the ratio $f = 4\pi\sigma(E, \epsilon, \delta)/\sigma_{\text{tot}}(E)$ defined in Eq. (5), where $\sigma_{\text{tot}}(E)$ is the total cross section computed to order $\alpha(E)$. In Eq. (5) we simply use the lowest-order expression for $\sigma_{\text{tot}}(E)$, since here we are interested only in the leading contribution proportional to $\ln\delta$. As we shall see, our main result depends only on the $\ln(2\epsilon)\ln\delta$ term. Hence our conclusion is applicable to all scattering processes if the coefficient of this double-logarithmic term is independent of the source. This is a weak form of the assumption that we mentioned earlier.

[16]We have implicitly assumed that the transverse momentum of the gluon fragmentation is small compared to that of the bremsstrahlung effect. Otherwise, gluon jets would be even harder to detect. It has even been speculated that gluon jets may not exist. See, e.g., G. L. Kane and Y.-P. Yau, University of Michigan Report No. UM-HE-77-44, 1977 (to be published).

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Average Lifetimes of Collective Transitions in the Spin-(30–50) Region

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A method is developed to determine average lifetimes of continuum gamma rays following heavy-ion compound-nucleus reactions. The resulting enhancement factors for $E2$ transitions depopulating states in the spin-(30–50) regions are of the same order of magnitude as for ground-state rotational bands of deformed nuclei. This shows for the first time that these high-spin states decay through strongly collective bands.

The de-excitation γ-ray spectra of nuclei produced at very high angular momenta in heavy-ion compound-nucleus reactions are composed of three parts.[1] The intensity of the high-energy part of the spectra ($E_\gamma \gtrsim 1.5$ MeV for deformed nuclei in the heavy rare-earth region) falls exponentially with E_γ. In this region the angular correlation is roughly isotropic, suggesting about equal amounts of dipole and quadrupole transitions.[2,3] Furthermore, the multiplicity associated with each transition is roughly independent of E_γ. This part of the spectrum is very likely produced by statistical transitions. The low-energy region of the spectrum ($\lesssim 0.7$ MeV) is dominated by discrete lines, the transitions between the well-separated and strongly populated low-energy states. In this paper, we will focus our attention on the intermediate range of γ-ray energies where the spectrum shows a broad structure (the "bump") with the following characteristics: (i) The multiplicity of transitions at the high-energy edge of the bump is higher than the average multiplicity associated with either the statistical or discrete parts of the spectrum—indicating that these transitions come from the highest spin states populated. (ii) Angular distribution measurements show predominantly a stretched quadrupole component in these decays. These characteristics can be explained by a model which assumes that the quadrupole bump is composed of collective transitions within rotational bands parallel to the yrast line. The effective moments of inertia derived from the multiplicities and excitation energies of these transitions are close to the values for a rigid rotor. However, until now no measurements have shown that these transitions are indeed very highly collective. In the present experiment an attempt is made to measure average lifetimes in the bump region by a Doppler-shift method.

The experimental setup is similar to the one described in Ref. 3, with six 7.6×7.6-cm^2 NaI detectors serving as a multiplicity filter. Two 7.6×7.6-cm^2 NaI detectors were located at 0° and 90° with respect to the beam direction at a distance of 60 cm from the target to allow for neutron discrimination. Average lifetimes of the continuum γ rays were deduced by comparing the Doppler shift for the spectra measured in the 0° detector with a thin self-supporting target and with a target on a thick backing.

The targets and compound nuclei studied using beams of 700-MeV ^{136}Xe from the Lawrence Berkeley Laboratory SuperHILAC are listed in Table I. In each case spectra were measured for an ~1-mg/cm^2 self-supporting target and for a target of similar thickness on a 25-mg/cm^2 Au backing. In order to keep the conditions as equal as possible for the two spectra, a 25-mg/cm^2 Au foil was placed ~2 mm downstream from the thin foil targets. In addition, for the ^{27}Al + ^{136}Xe system, targets with Pb backings were used. In this case the stopping time is almost 2 times longer than in the Au stopper; this provides a check on