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Is the Up-Quark Mass Zero?

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We consider, within the framework of current algebra, the possibility that the up-quark mass vanishes (as an alternative to the axion). We argue that the contrary current-algebra value, $m_u/m_d = 1/1.8$, is unreliable. A critical analysis leads to the conclusion that $m_u = 0$ is not unreasonable and furthermore leads to a surprisingly good prediction for the δ-meson mass.

Weinberg[1] and Wilczek[2] have recently discussed the issue of *CP* invariance in the strong interactions in the light of *CP*-nonconserving effects of instantons in quantum chromodynamics (QCD). They propose as the two most satisfactory alternatives to avoid large *CP* nonconservation in the presence of the weak and electromagnetic interactions, that (a) the interactions possess an additional U(1) symmetry as suggested by Peccei and Quinn,[3] the spontaneous breaking of which results in a new, weakly interacting, light boson called the axion; or (b) the mass, m_u, of the up quark in the QCD Lagrangian is zero, although this contradicts the standard current-algebra estimate[4] $m_d/m_u = 1.8$. Other less attractive possibilities are also discussed by Wilczek,[2] among them the setting of the renormalized value of the vacuum phase angle θ arbitrarily equal to zero.

If a particle with the properties predicted for the axion is found experimentally, it will be an exciting confirmation of the current theoretical ideas about the strong and the weak and electromagnetic interactions. However, the currently available experimental evidence suggests that the axion may not be found.[1,5]

Suppose that the axion is not found. Is the alternative $m_u = 0$ a reasonable possibility? Or are we forced to resort to some other device for maintaining strong *CP* invariance? The aim of this paper is to examine the hypothesis $m_u = 0$ in the light of what is known from current algebra in order to illuminate these questions. We assume that the strong-interaction Lagrangian is

$$\mathcal{L} = \mathcal{L}_0 - m_u \bar{u}u - m_d \bar{d}d - m_s \bar{s}s + (\text{counter terms}), \qquad (1)$$

where $\mathcal{L}_0$ is the QCD Lagrangian. The quantities m_a are the renormalized quark masses. (We have in mind dimensional renormalization by minimal subtraction.) A meaning that is independent of renormalization scheme can be attached to the m_a by considering the quark propagators $S_a(P)$ in some convenient gauge. They have the form

$$S_a(P) = A_a(P^2)[\not{P}^2 - M_a(P^2)]^{-1}. \qquad (2)$$

The quark mass functions $M_a(P^2)$ obey a renormalization-group equation,

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - 2\alpha\gamma_A\frac{\partial}{\partial\alpha} + \gamma_m m_b\frac{\partial}{\partial m_b}\right)M_a = 0,$$

from which one deduces the large behavior of M_a:

$$M_a(e^{2t}P_0^2) \sim m_a \exp\left[\int_0^t d\tau\, \gamma_m(g(\tau))\right] + O(g(t)^2),$$

where $g(t)$ is the running coupling constant. Thus the ratio of any two of the quark mass functions, $M_a(P^2)/M_b(P^2)$, equals the corresponding ratio m_a/m_b whenever P^2 is large enough so that the running coupling constant can be neglected.

Let us now recall the standard arguments that give the quark mass ratios in current algebra.[4,6] The $SU(3)_L \otimes SU(3)_R$ chiral symmetry of $\mathcal{L}_0$ is broken in the vacuum and is also broken explicitly by the quark mass terms so that, for instance, $\partial_\mu \bar{u}\gamma_5\gamma^\mu s = -i(m_u + m_s)\bar{u}\gamma_5 s$. The matrix element of this equation between the vacuum and the K^+ state is

$$F(K^+)\mu(K^+)^2 = (m_u + m_s)\sqrt{Z}(K^+). \qquad (3a)$$

Here $\mu(K^+)$ is the K^+ mass in the strong-interaction theory (without electromagnetic radiative corrections),

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 u|K^+(q)\rangle \equiv iq^\mu 2^{1/2}F(K^+),$$

and

$$\langle 0|\bar{s}i\gamma_5 u|K^+\rangle \equiv 2^{1/2}\sqrt{Z(K^+)}.$$

Similarly, one finds

$$F(K^0)\mu(K^0)^2 = (m_d + m_s)\sqrt{Z(K^0)}, \qquad (3b)$$

$$F(\pi^+)\mu(\pi^+)^2 = (m_d + m_u)\sqrt{Z(\pi^+)}. \qquad (3c)$$

From these equations one obtains the equations for the ratio m_u/m_d:

$$\frac{F(K^0)}{\sqrt{Z(K^0)}}\mu(K^0)^2 - \frac{F(K^+)}{\sqrt{Z(K^+)}}\mu(K^+)^2 = \frac{F(\pi^+)}{\sqrt{Z(\pi^+)}}\mu(\pi^+)^2\frac{m_d - m_u}{m_d + m_u}. \qquad (4)$$

Evidently the ratio m_u/m_d is sensitive to the splitting between $\mu(K^0)$ and $\mu(K^+)$. The splitting in the observed masses is $m(K^0) - m(K^+) = 4.0$ MeV, but this includes a radiative contribution. The radiative contribution is usually estimated with use of the result[7] from current algebra in the chiral-symmetry limit:

$$m(\pi^0)^2 \approx \mu(\pi^0) \approx \mu(\pi^+)^2,$$

$$m(K^0)^2 \approx \mu(K^0)^2, \qquad (5a)$$

$$m(K^+)^2 \approx \mu(K^+)^2 + m(\pi^+)^2 - \mu(\pi^+)^2.$$

We further assume the first-order perturbation-theory result

$$\mu(\pi^+)^2 \approx \mu(\pi^0)^2. \qquad (5b)$$

This gives $\mu(K^0) - \mu(K^+) \approx 5.3$ MeV. This estimate can be criticized,[8] but we believe that it is reasonably accurate because a simple quark-model estimate of the radiative correction gives roughly the same result.[9] Thus we will use Eq. (5) throughout the Letter.

If one now sets $F(K^0)/\sqrt{Z(K^0)} = F(K^+)/\sqrt{Z(K^+)} = F(\pi^+)/\sqrt{Z(\pi^+)}$ he obtains the standard result[4] $m_u/m_d = 1/1.8$. This result, however, is extremely sensitive to the assumption that $F(K^0)/\sqrt{Z(K^0)} = F(K^+)/\sqrt{Z(K^+)}$.

We now examine the implications of the hypothesis that $m_u = 0$.[10] Let us allow SU(2) symmetry in the $F/\sqrt{Z}$ value between K^0 and K^+ to be broken by a small amount δ:

$$\frac{F(K^0)}{\sqrt{Z(K^0)}} \equiv \frac{F(K^+)}{\sqrt{Z(K^+)}}(1+\delta). \qquad (6)$$

We then ask how big δ must be in order that m_u vanish. We use the experimental value[11] $F(K^+)/F(\pi^+) = 1.28$ and make the additional assumption (to which the result is *not* sensitive) that $\sqrt{[Z(K^+)/Z(\pi^+)]} \approx 1.0$. This gives

$$\delta = 0.036. \qquad (7)$$

With the same assumptions, Eq. (2) gives for m_s/m_d the value

$$m_s/m_d \cong 17. \qquad (8)$$

Two objections to the $m_u = 0$ hypothesis come to mind. First, if one uses first-order perturbation theory with the Hamiltonian derived from (1), i.e., $H = H_0 + \epsilon_3 u_3 + \epsilon_8 u_8$, where H_0 is SU(3) symmetric and the u_i belong to the octet representation, the ratio of SU(2) breaking to SU(3) breaking is determined by the value $\epsilon_3/\epsilon_8 = \sqrt{3}\,(m_d - m_u)/(2m_s - m_d - m_u)$. The value $\epsilon_3/\epsilon_8 = 0.02$ obtained in the standard model with $m_d/m_u = 1.8$ and $m_s/m_d = 20$ is in fair agreement with experimental mass splitting,[12] while $\epsilon_3/\epsilon_8 = 0.05$ obtained with $m_u = 0$, $m_s/m_d = 17$ is too large. Second, naive first-order perturbation theory predicts $[F(K^0) - F(K^+)]/[F(K^0) + F(K^+) - 2F(\pi^+)] = \epsilon_3/\epsilon_8\sqrt{3}$. Our value of ϵ_3/ϵ_8 then yields for the SU(2)-breaking parameter $\tilde{\delta} = [F(K^0) - F(K^+)]/F(K^+)$ value $\tilde{\delta} = 0.014$, a value substantially smaller than the SU(2) splitting of $F/\sqrt{Z}$, $\delta = 0.036$.

Does one expect first-order perturbation theory in the difference $m_a - m_b$ to be reliable? Consider, for instance, the dynamical quark masses $M_a(P^2)$ defined in Eq. (2). The approximate SU(3) symmetry of low-energy hadron physics suggests that the differences $\langle M_a\rangle - \langle M_b\rangle$ are small, where $\langle\ \rangle$ indicates an average over a range of $P^2 \sim 1$ GeV. Furthermore, first-order perturbation theory in these differences may be valid for, say, baryon masses. One expects that the $\langle M_a\rangle - \langle M_b\rangle$ would be zero if the $m_a - m_b$ were zero. Thus one can hope that an approximate linear relation between these quantities holds. On this assumption, the ratio $\langle\epsilon_3\rangle/\langle\epsilon_8\rangle \equiv 3^{1/2}[\langle M_d\rangle - \langle M_u\rangle]/[2\langle M_s\rangle - \langle M_d\rangle - \langle M_u\rangle]$ will be equal to the value of $\epsilon_3(P^2)/\epsilon_8(P^2)$ as $P^2 \to \infty$, the corresponding ratio of m's. However, there is a lot of unknown physics between 1 GeV and ∞, and we know of no evidence that confirms this hope.[13,14]

To summarize, the comparison of the $m_u = 0$ model with experiment using first-order perturbation theory is not favorable. One might hope to be able to rule out the model on this basis, but we are reluctant to place such a burden on a method of unproven reliability.

We now turn to a nonperturbative test of the $m_u = 0$ hypotheses. We seek to rule out the hypothesis by deriving a consequence known to be false. The motivation for this test is that the value

0.036 obtained for the parameter δ in the $m_u=0$ model seems a bit large. We will argue that if δ were suitably large, while m_d-m_u were suitably small, the meson in the 0^+ octet with the flavor quantum numbers of the pion would be almost a Goldstone boson. Its mass could then be estimated using a "partially conserved vector current hypothesis" and the assumption that the operator normalization factors $\sqrt{Z}$ that appear are all equal. We therefore carry out this calculation. If $\mu_{\rm calc}$ were less than, say $2\mu(\pi^+)$, one would argue that the $m_u=0$ model predicts a 0^+, isotriplet, almost-Goldstone boson. Since the lightest actual 0^+ isotriplet boson is the $\delta(970)$, which is evidently quite heavy, one would be able to rule out the $m_u=0$ model.

To carry out this calculation, we need some current-algebra equations of a standard type. We begin with

$$\begin{aligned} -2F(K^+)\sqrt{Z(K^+)} &\approx \langle 0|\bar{s}s|0\rangle + \langle 0|\bar{u}u|0\rangle , \\ -2F(K^0)\sqrt{Z(K^0)} &\approx \langle 0|\bar{s}s|0\rangle + \langle 0|\bar{d}d|0\rangle . \end{aligned} \tag{9}$$

These equations are most simply obtained[15,6] by taking the vacuum expectation value of the commutation relations between the axial currents and the axial densities and assuming that the single 0^- meson states saturate the sum over intermediate states. We will also assume that the constants $\sqrt{Z(K^+)}$, $\sqrt{Z(K^0)}$, and also $\sqrt{Z(\pi^+)}$ are more nearly SU(3) symmetric[16] than are the meson-current coupling constants. Thus we will approximate

$$\sqrt{Z(K^+)} \approx \sqrt{Z(K^0)} \approx \sqrt{Z(\pi^+)} \equiv \sqrt{Z(0^-)} . \tag{10}$$

Notice that under this assumption

$$\langle 0|\bar{d}d|0\rangle - \langle 0|\bar{u}u|0\rangle = -2[\sqrt{Z(0^-)}]F(K^+)\delta . \tag{11}$$

Thus δ measures the amount of SU(2)-symmetry breaking in the vacuum. If δ is large enough compared to the amount of SU(2) breaking in the Lagrangian, one expects the δ meson to be an almost-Goldstone boson.

We next consider the δ meson. Taking a matrix element of the vector current, one obtains

$$F(\delta^+)\mu(\delta^+)^2 = (m_u - m_d)\sqrt{Z(\delta^+)} , \tag{12}$$

where $\langle 0|\bar{d}\gamma^\mu u|\delta^+(q)\rangle = q^\mu\sqrt{2}F(\delta^+)$ and $\langle 0|\bar{d}u|0\rangle = \sqrt{2}\sqrt{Z(\delta^+)}$. Taking the vacuum expectation value of the commutation relation between $\bar{d}\gamma^\mu u$ and $\bar{u}d$, one obtains

$$2F(\delta^+)Z^{1/2}(\delta^+)[1+C(\delta^+)] = \langle 0|\bar{d}d|0\rangle - \langle 0|\bar{u}u|0\rangle . \tag{13}$$

In the limit of a Goldstone δ meson, only the one-δ-meson intermediate state would contribute to the commutator and one would obtain (13) with $C(\delta^+)=0$. [That is, the one-meson pole would dominate $\int d^4x\langle 0|T\{\bar{d}u, \bar{u}d\}|0\rangle$, which one relates to Eq. (13) by taking the divergence of $\langle 0|T\{\bar{d}\gamma^\mu u, \bar{u}d\}|0\rangle$ at $q^\mu \to 0$.] Thus $C(\delta^+)$ represents the correction to δ-meson pole dominance. We will assume $C(\delta^+)\approx 0$ later, but we retain it now so that we can exhibit how the final result depends on it.

Combining Eqs. (3c), (11), (12), and (13) one obtains

$$\frac{\mu(\delta^+)^2}{\mu(\pi^+)^2} = \frac{1}{\delta}\frac{F(\pi^+)}{F(K^+)}\frac{m_d-m_u}{m_d+m_u}\frac{Z(\delta^+)}{Z(0^-)}[1+C(\delta)]. \tag{14}$$

We now set $m_u=0$ and thus take $\delta=0.036$. We also assume that the correction $C(\delta^+)$ can be neglected. We lack any precise information about the ratio $Z(\delta^+)/Z(0^-)$ of operator normalization coefficients, but we expect this ratio to be order 1 [as it is to lower order in the SU(3) Σ model]. Taking $Z(\delta^+)/Z(0^-)=1$, one obtains[17] $\mu(\delta^+) = 4.7\mu(\pi^+)$. Thus the value of the parameter δ in the $m_u=0$ model is not so large as to lead to the false prediction of an almost-Goldstone δ meson.

The numerical estimate of the δ-meson mass can be improved by dividing Eq. (14) by the corresponding equation for the $K^+(1250)=\bar{s}u$ meson:

$$\frac{\mu(K^+)^2}{\mu(K^+)^2} = \frac{F(K^+)}{F(K^0)-F(\pi^+)}\frac{m_s-m_u}{m_s+m_u}\frac{Z(K^+)}{Z(0^-)}[1+C(K)]. \tag{15}$$

Now the uncertainty about the Z's is alleviated since one can reasonably assume that $Z(\delta^+)/Z(K^+)\approx 1$ on grounds of approximate SU(3) invariance. Furthermore, the approximation $[1+C(\delta^+)]/[1+C(K^+)] \approx 1$ will be more accurate than the approximation $1+C(\delta^+)\approx 1$ in Eq. (14) if $C(\delta^+)\approx C(K^+)$. The resulting equation [with δ obtained from Eq. (2)] is

$$\frac{\mu(\delta^+)^2}{\mu(\pi^+)^2} = \mu(K^+)^2\left[\frac{F(K^0)}{F(K^+)} - \frac{F(\pi^+)}{F(K^+)}\right]\left(\frac{\mu(K^0)^2}{\mu(K^+)^2}\right)R_{su}\left\{\mu(\pi^+)^2 - \left(\frac{F(K^+)}{F(\pi^+)}\right)[\mu(K^0)^2-\mu(K^+)^2]R_{du}\right\}^{-1}, \tag{16}$$

where $R_{du} \equiv (m_d + m_u)/(m_d - m_u)$, $R_{su} = (m_s + m_u)/(m_s - m_u)$. In the $m_u = 0$ model, one obtains $\mu(\delta^+) \approx 5.9\mu(\pi^+) = 800$ MeV, to be compared with the measured value $\mu(\delta^+)_{\rm expt} = 970$ MeV. Given the approximate nature of the calculation, we regard this as remarkably good agreement.

We conclude that the value $m_u = 0$, $\delta = 0.036$ is not unreasonable. Thus if the axion is not found or a suitable theoretical alternative worked out, we see little reason from within the framework of current algebra to reject the possibility that $m_u = 0$. On the other hand, we do not believe that the arguments presented here are sufficiently exact to enable one to extract a reliable value for m_u/m_d, although $m_u/m_d < \frac{1}{2}$ seems to be favored. If m_u/m_d *is* zero (or small), the result (16) for the δ-meson mass is a curiously accurate current-algebra prediction that is interesting in its own right.

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Isomeric States in ^{212}Bi

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Two new alpha activities with half-lives of 25 and 9 min have been observed in reaction of heavy ions with a variety of heavy targets. The 25-min activity was found by combined radiochemical methods and mass separations to be an isomer in ^{212}Bi; the 9-min activity is also likely to be an isomer in ^{212}Bi.

During a search for superheavy elements via the reaction of ^{48}Ca with ^{248}Cm, we observed several alpha lines around 10 MeV.[1] In addition a rather intense line at 11.66 MeV was observed and attributed to the known isomer in ^{212}Po.[2] A closer investigation of the 11.66-MeV peak through analysis of its decay curve revealed a longer-lived component of 9 ± 1 min, in addition to the expected 45-sec half-life. The half-life of the group of lines at 10 MeV was determined to be 25 ± 1 min. We eliminated the possibility that these activities could be associated with the decay of superheavy elements when these activities were found in a bombardment of ^{208}Pb with ^{40}Ar ions.

After preliminary experiments showed that the activities were coprecipitated with CuS from an acidic solution and taking into account the high energy associated with their decay, these activities were presumed to be nuclides in the lead re-