ergies). Rather, the main difference in the interpretation of the experimental data most likely originates with the fact that the present experimental data reveal the detailed dependence of the multiplicity simultaneously on Z *and* on energy loss.

In summary, we have measured the dependence of the γ multiplicity on energy loss, scattering angle, and on fragment charge for the Kr-Sn and Kr-Er reaction products. The energy loss, as opposed to scattering angle or net mass transfer, appears to be the basic quantity determining M and hence the transfer of angular momentum for these reactions. Our results for M, which indicate quantitative discrepancies with a classical picture of the reaction mechanism, are well explained by a diffusion model.

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Angular-Momentum Dissipation in Heavy-Ion Collisions

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The angular-momentum dissipation in deeply inelastic heavy-ion collisions is described as a transport process. Using transport coefficients which have been evaluated within a single-particle model, mean value and variance of the intrinsic angular momenta of the fragments are calculated. The fluctuations become important already for initial relative angular momenta $l \lesssim \frac{1}{2} l_{\text{grazing}}$. Excellent agreement with γ-multiplicity data for the reaction ^{86}Kr(5.99 MeV/nucleon) + ^{166}Er is obtained.

The dissipation of relative angular momentum into intrinsic angular momentum of the fragments is one of the most interesting relaxation phenomena in reactions between heavy nuclei.[1-9] Experimental information on angular-momentum dissipation has been obtained from γ rays[1-7] and light particles[8] emitted by the fragments and, in addition, from sequential fission of the heavy fragment.[9] The description of angular-momentum dissipation is a sensitive test for any theoretical approach to deeply inelastic heavy-ion collisions. In this Letter we report on a transport-theoretical treatment and its application to the reaction ^{86}Kr(5.99 MeV/nucleon) + ^{166}Er.[1] Our study clarifies the importance of fluctuations which are not accounted for in classical friction models.

From the Fokker-Planck equation, the relations

$$\langle M\rangle = [(\mathscr{G}_1 + \mathscr{G}_2)/\mathscr{G}_{tot}]\, l[1 - \exp(-t/\tau_e)], \tag{1}$$

$$\sigma_M{}^2 = D_{MM}\tau_e[1 - \exp(-2t/\tau_e)], \tag{2}$$

have been derived[10] for the mean value $\langle M\rangle$ and the variance $\sigma_M{}^2$ of the z-component M of the intrinsic angular momentum. The z axis is defined by the initial relative angular momentum $\vec{l}$ of the colliding nuclei. The relaxation time which governs the angular-momentum transfer from relative to intrinsic motion is given (with $\hbar \equiv 1$) by

$$\tau_e = (\mathscr{G}_1 + \mathscr{G}_2)\frac{\mathscr{G}_{rel}}{D_{MM}\mathscr{G}_{tot}}\left(\frac{E_l{}^*}{g_1 + g_2}\right)^{1/2}, \tag{3}$$

with the intrinsic, relative, and total moments of inertia $\mathscr{G}_k$ ($k = 1, 2$), $\mathscr{G}_{rel}$, and $\mathscr{G}_{tot}$, respectively. The local excitation energy is calculated from $E_l{}^* = E_{c.m.} - V_c - l^2/2\mathscr{G}_{rel}$ assuming that the radial kinetic energy is dissipated at the interaction barrier V_c. The single-particle level densities are denoted by $g_k = A_k/12$ and $E_{c.m.}$ is the center-of-mass energy. The angular-momentum diffusion coefficient D_{MM} is evaluated according to Ref. 10 with the excitation energy $E_l{}^*$ and the parameters $\Delta = 2$, $\gamma = 3$, and $\Delta j = 2$. Whereas Δ and γ have been fitted previously to experimental mass distributions, the value for Δj is a theoretical estimate. Form factor effects are included in the calculation as described in Ref. 10.

We calculate $\langle M\rangle$ and $\sigma_M{}^2$ as functions of l by using the mean interaction times $t = \tau_{int}(l)$ which are obtained from the experimental angular distribution.[11] For sufficiently long interaction times $\tau_{int} \gtrsim 2\times10^{-21}$ s, the angular-momentum distribution reaches its equilibrium with $\langle M\rangle_{equ} = l(\mathscr{G}_1 + \mathscr{G}_2)/\mathscr{G}_{tot}$ and $\sigma_{M,equ}{}^2 \approx 110$. The essential difference to classical friction models is the occurrence of a relatively large variance. The mean value $\langle M\rangle_{equ}$ looks identical to the classical sticking limit but the intrinsic moments of inertia are calculated microscopically.[10] We define a total angular momentum for a given l value,

$$I_{tot}(l) \equiv \langle|\vec{I}_1|\rangle_l + \langle|\vec{I}_2|\rangle_l, \tag{4}$$

by the mean absolute values $\langle|\vec{I}_k|\rangle$ of the intrinsic angular momenta of the fragments. This quantity is related to the mean γ multiplicities $\langle M_\gamma\rangle$ and is evaluated in the following way. As suggested by the structure of Eqs. (1) and (2) we assume that mean value and variance of M are split between the two fragments according to the moments of inertia $\langle M_1\rangle/\langle M_2\rangle = \sigma_{M_1}{}^2/\sigma_{M_2}{}^2 = \mathscr{G}_1/\mathscr{G}_2$. Since the initial angular momentum is taken in the z direction, the mean values of the dissipated angular momentum in x and y directions vanish. However, the variances are different from zero. For simplicity we assume equal variances for all three components, although there is experimental evidence for an anisotropy in the reaction

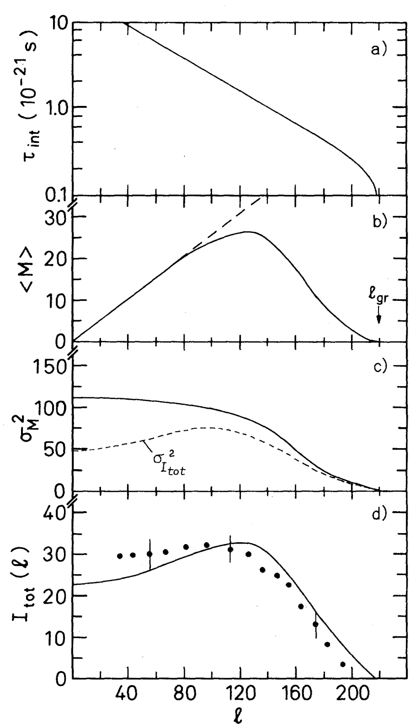

FIG. 1. Angular-momentum dissipation in the reaction ^{86}Kr(5.99 MeV/nucleon) + ^{166}Er. As functions of initial relative angular momentum l are shown (a) the mean interaction time, (b) the mean value of the z-component M of intrinsic angular momentum, (c) the variance of M, (d) the total angular momentum defined by Eq. (4) and corresponding experimental points from γ-multiplicity measurements (Ref. 1). According to Ref. 4 the mean γ-multiplicities $\langle M_\gamma\rangle$ are converted to angular momenta by $I_{tot} = 2(\langle M_\gamma\rangle - 5)$. For completeness we have included in (c) the variance $\sigma I_{tot}{}^2$ (dashed curve).

plane.[9] Such an anisotropy has only a minor effect on $I_{tot}(l)$.[10] The mean values and variances of the intrinsic angular momenta $\vec{I}_1$, $\vec{I}_2$ of the fragments define Gaussian distributions. These are used for calculating $I_{tot}(l)$ as shown in Fig. 1. Without adjusting any parameter for the angular-momentum dissipation the agreement with the experimental result is surprisingly good. In particular, the point where $I_{tot}(l)$ saturates with decreasing l value is predicted correctly. Towards smaller l values $I_{tot}(l)$ is increasingly due to fluctuations. Thus, even in a central collision ($l=0$) a substantial amount of intrinsic angular momentum is generated. This phenomenon bears some similarity to the production of angular momentum in low-energy fission.

The element distribution $d\sigma/dZ_1$ is obtained with the solutions $P(Z_1,t)$ of the Fokker-Planck equation[11] for $t=\tau_{int}(l)$ as

$$\frac{d\sigma}{dZ_1}=\frac{2\pi}{K^2}\int_{l_{cr}}^{l_{max}} l\,P(Z_1,\tau_{int}(l))\,dl\,, \tag{5}$$

with the asymptotic wave number K. The upper limit of the integral is determined by the total cross section. We choose $l_{max}=215$ according to the experimental cut in the Q value ($-Q=\Delta E=20$ MeV). The lower limit is $l_{cr}=0$ in the reaction

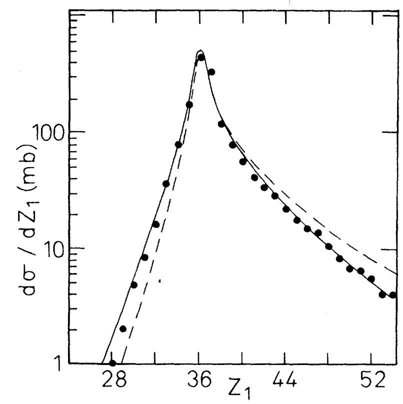

FIG. 2. Calculated element distributions for the reaction ^{86}Kr(5.99 MeV/nucleon) + ^{166}Er compared with experimental data (Ref. 1). The dashed curve corresponds to theoretical values of drift and diffusion coefficients, the solid curve to a fit by adjusting the drift coefficient.

under consideration. The dashed curve in Fig. 2 shows a calculation with theoretical values[11] for the drift and diffusion coefficients $v_Z=0.17\times10^{22}$ s^{-1}, $D_Z=0.32\times10^{22}$ s^{-1} whereas the solid curve is a fit to the data[1] by adjusting $v_Z=0.11\times10^{22}$ s^{-1}.

The total intrinsic angular momentum $I_{tot}(Z_1)$ as function of the fragment charge number Z_1 is then determined by

$$I_{tot}(Z_1)=\frac{2\pi}{K^2}\left(\frac{d\sigma}{dZ_1}\right)^{-1}\int_{l_{cr}}^{l_{max}} l\,I_{tot}(Z_1,\tau_{int}(l))\,P(Z_1,\tau_{int}(l))\,dl\,. \tag{6}$$

Here we have considered the dependence of the moments of inertia on the fragment mass number to obtain $I_{tot}(Z_1,l)$. This accounts for the essential part of the coupling between mass and angular-momentum transfer due to the Z_1 dependence of the angular-momentum drift coefficient.[10] The result for $I_{tot}(Z_1)$ is compared with the data[1] in Fig. 3. The fragments close to the projectile are predominantly produced in collision with large l values where $I_{tot}(l)$ is small (cf. Fig. 1). This leads to a pronounced dip in $I_{tot}(Z_1)$. Sufficiently far away from the projectile-charge number $I_{tot}(Z_1)$ becomes rather flat. These fragments are mainly populated in collisions with l values which lead to sticking. Nevertheless, $I_{tot}(Z_1)$ does not rise with decreasing Z_1 as expected from the variation of the moments of inertia in Eq. (1). This is the result of two effects which counteract this Z_1 dependence:

(i) The larger the distance from the projectile-charge number, the smaller the mean l value of the collisions which contribute to the production of the fragment. Thus the mean value $\langle M(Z_1)\rangle$ de-

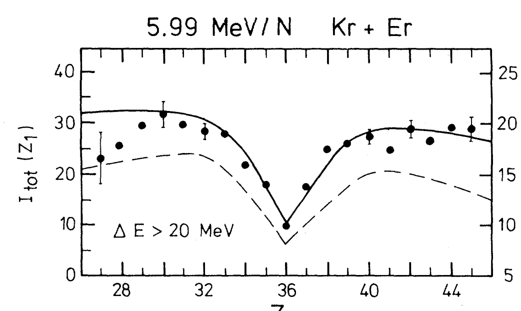

FIG. 3. The total angular momentum $I_{tot}(Z_1)$ defined by Eq. (6) as function of the fragment charge number and corresponding γ-multiplicity data from Ref. 1 for the reaction ^{86}Kr(5.99 MeV/nucleon) + ^{166}Er. The dashed curve is obtained by neglecting fluctuations.

creases with decreasing Z_1 (dashed curve in Fig. 3). This is due to the monotonic increase of the width of the element distribution with decreasing l value. The effect from the l dependence of the charge drift coefficient is negligible.

(ii) The fluctuations in $I_{tot}(l)$ which become more and more important for decreasing l value have a much weaker dependence on Z_1 and l as compared to $\langle M\rangle^2$. The solid curve in Fig. 3 includes the variances.

Beyond the results presented here, our model provides a simple explanation of the puzzling differences in measured γ multiplicities for light[3,4] and heavy[5] systems: Whereas the γ multiplicities at fixed angle are rather constant as function of fragment charge number Z_1, in the latter case they rise with decreasing Z_1 for light systems at angles far from grazing. We interpret this phenomenon in the following way. For Kr + Er and for other heavy systems[5] the angular distributions are sideward peaked and hence many l values contribute at the same scattering angle. Consequently, $I_{tot}(Z_1)$ is rather independent of Z_1 for the same reasons as discussed above for the angle-integrated quantities. For the light systems studied in Refs. 3 and 4, the angular distributions are forward peaked and hence the deflection angle is a monotonic function of the initial relative angular momentum over a certain range of l values. The deflection angle becomes negative for long interaction times.[7] For scattering angles where the mean interaction time is larger than τ_l we have $\langle M\rangle \approx \bar{l}(\mathcal{J}_1 + \mathcal{J}_2)/\mathcal{J}_{tot}$. Here $\bar{l}$ denotes the mean l value which contributes at the scattering angle. The mean dissipated angular momentum dominates $I_{tot}(Z_1)$ for not too small values of l. Since the small l values lead to fusion it is the dependence of $\langle M\rangle$ on Z_1 at a given $\bar{l}$ which essentially determines the Z_1 dependence of $I_{tot}(Z_1)$ for light systems at negative deflection angles.

To summarize, we have shown that the angular-momentum dissipation observed in collisions between heavy nuclei can be quantitatively understood in a transport-theoretical approach based on a Fokker-Planck equation. The calculated intrinsic angular moment of the fragments are in good agreement with experimental data once the fluctuations are taken into account. In addition, the dependence of the mean value and the fluctuations of the intrinsic angular momentum on l provides a clue for understanding the polarization of the fragments and their misalignment measured by the circular polarization of γ rays[7] and by the angular distribution of sequential-fission fragments.[9]

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