Muon Scattering at 219 GeV and the Proton Structure Functions

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Results on the protron structure function, F_2, are presented for $0.3 < q^2 < 80.0$ GeV^2 and $10 < \nu < 200$ GeV. The results support the conclusions of earlier work at 97 and 147 GeV that scaling is violated. A new value for $R = \sigma_S/\sigma_T = 0.44 \pm 0.25$ has been obtained using all the Fermilab proton measurements.

In this Letter we report the most recent measurements at Fermilab of the cross section for scattering of 219-GeV muons from a hydrogen target. The data were taken using the muon-scattering facility, details of which have been published previously.[1,2] The results are based on a total of 7.5×10^{10} incident muons yielding 2.0×10^4 useful events in the kinematic range $0.3 < q^2 < 80$ GeV^2 and $10 < \nu < 200$ GeV, where q^2 is the square of the four-momentum transfer of the muon and ν its laboratory energy loss. There were 1.5×10^4 events with $q^2 > 1.0$ GeV^2.

The cross section for muon inclusive scattering, in the one-photon-exchange approximation, is related to the structure function F_2 by

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{2\pi\alpha^2}{p^2 q^4}\frac{F_2(q^2,\nu)}{\nu}\left[2EE' - \frac{q^2}{2} + \frac{(q^2-2m_\mu{}^2)(1+\nu^2/q^2)}{1+R(q^2,\nu)}\right]$$

where E, p, E', and p' are the incident and scattered muon laboratory energies and momenta, $q^2 = 2(EE' - pp'\cos\theta - m_\mu{}^2)$, $\nu = E - E'$, θ is the muon scattering angle, m_μ the muon mass, R the ratio of the total cross section on protons of scalar and transverse virtual photons, and α is the fine-structure constant

The value of R has been determined by comparing cross sections at the same q^2 and ν using the 219-GeV data and all earlier measurements at 97 and 147 GeV. First, the two sets of measurements made under different conditions at 147 GeV were combined. Then the three different energy measurements were compared and χ^2 minimized as a function of F_2 and R where an overlap occurred. Table I shows the best-fit values of R as a function of $x = 1/\omega$ where $x = q^2/2m_p\nu$ and m_p is the proton mass. The χ^2 distributions of smaller bins were summed and minimized to obtain an average value of R; this average value is 0.44, and the statistical error is ± 0.16 from a $\chi^2 = 159$ for 148 degrees of freedom. The systematic errors in the measurement of R are estimated to be ± 0.05 from the normalization, ± 0.10 from uncertainty in the acceptance, and ± 0.15 from uncertainty in track-finding efficiency. These errors have been combined in quadrature

TABLE I. Values of $R = \sigma_S/\sigma_T$ for bin-centered values of x. The errors are statistical. The systematic error is ± 0.19.

x	q^2 (GeV^2)	R
0.069	2–12.5	$-0.11^{+0.29}_{-0.20}$
0.025	2–6	$0.56^{+0.31}_{-0.23}$
0.009	1–3	$0.44^{+0.27}_{-0.22}$

with the statistical error to give the final result $R = 0.44 \pm 0.25$ for $0.009 < x < 0.1$ and $1 < q^2 < 12$ GeV^2.

Since there are uncertainties in the value of R measured at low energy[3] for $x > 0.1$ and since our data are insensitive to these uncertainties for $x > 0.1$ (as shown in Fig. 2), we use our value to derive our best estimate for $\nu W_2(q^2, \nu) \equiv F_2(x, q^2)$. Figure 1 shows the values of the proton structure function as a function of q^2 for various values of ω. The values are weighted to give correct values at the bin centers. Only statistical errors are shown. The overall systematic error is estimated to be 5%. There is agreement between these new measurements and our previous values which are also shown recalculated for $R = 0.44$. A consistency check shows that the data have a $\chi^2 = 148$ for 126 degrees of freedom. In Fig. 2 we have combined all the data for muon scattering on hydrogen to show the variation of F_2 with x for various q^2 ranges. The Massachusetts Institute of Technology–Stanford Linear Accelerator Center (MIT-SLAC) data[4,5] are also shown. The sensitivity of our results at each point to a variation of R between 0.19 and 0.69 is also indicated. These results exhibit a now well-known pattern of scaling violation[2,6,7] which may be

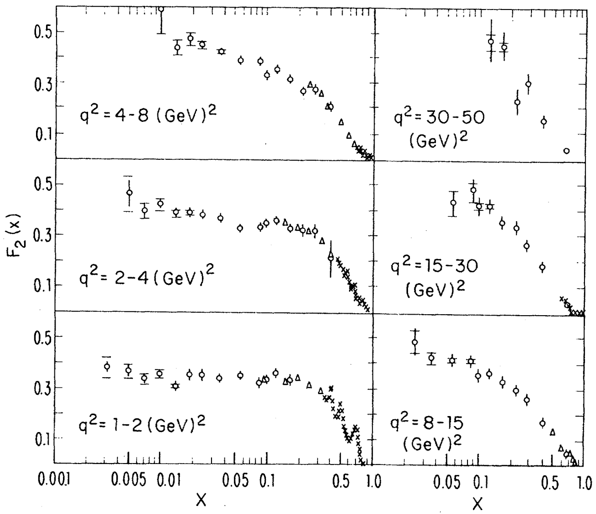

FIG. 2. $F_2(x, q^2)$ for hydrogen as a function of x for various q^2 bins for $R = 0.44$ using all Fermilab data (open circles) and the MIT-SLAC data (open triangles, Ref. 4; crosses, Ref. 5). The upper and lower horizontal bars indicate the change in F_2 for $R = 0.69$ and 0.19. The errors shown are statistical only.

characterized by a power-law dependence of F_2 on q^2 for various x values where

$$F_2(x, q^2) = F_2(x, q_0^2)(q^2/q_0^2)^b. \tag{1}$$

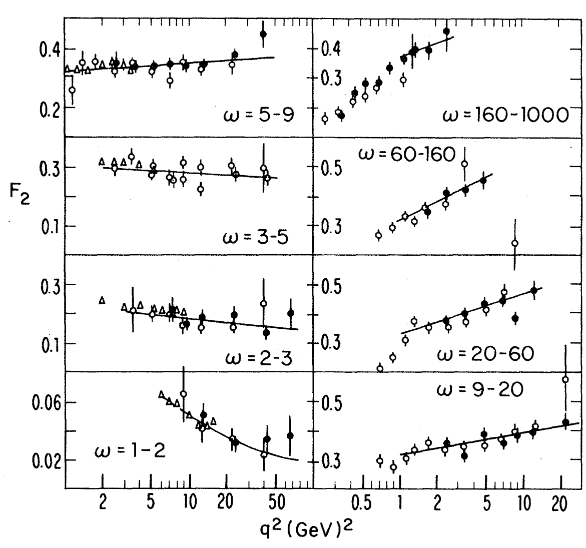

FIG. 1. F_2 for hydrogen as a function of q^2 for various ω bins for $R = 0.44$ showing the 219-GeV data (closed circles), the earlier Fermilab data (Ref. 2) recalculated for $R = 0.44$ (open circles), and the MIT-SLAC data (Ref. 4) (open triangles). Note the difference in scales and the suppressed zeroes for the ordinate. The solid lines are the fits with Eq. (1) for all Fermilab data.

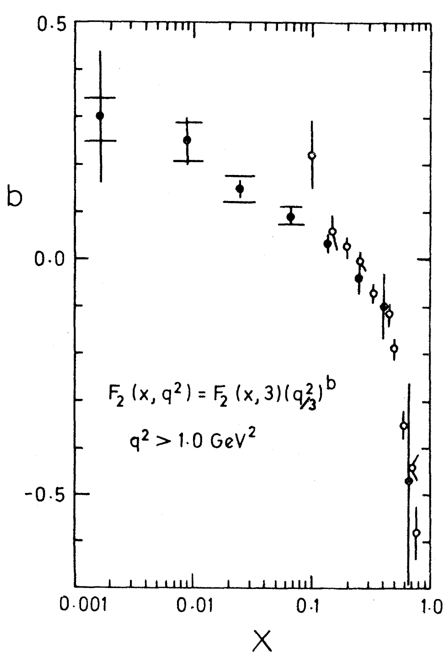

FIG. 3. The scaling violation parameter b of Eq. (1) as a function of x for all Fermilab data (closed circles) and for the MIT-SLAC data (Ref. 4) (open circles). The upper and lower horizontal bars indicate the change in b for $R = 0.69$ and 0.19. The errors are statistical only.

Figure 3 shows the variation of b as a function of x for the combined data for various assumed values of R for $q^2 > 1.0$ GeV^2. The rise of F_2 with increasing q^2 at small x may be reduced with a smaller value of R but will not disappear for any reasonable variation of R as a function x and q^2.

We have attempted to fit the combined Fermilab data to a global scale-noninvariant structure function suggested by one of us,[8]

$$F_2^p(x,q^2) = A(2+\epsilon_0)x(1-x)^{1+\epsilon_0} + B\frac{14(4+\epsilon_1)}{9(5+\epsilon_1)}(1-x)^{3+\epsilon_1}\frac{q^2}{q^2+m_0^2}, \quad (2)$$

where $\epsilon \equiv \kappa \ln[(q^2+m_0^2)/m_0^2]$, $\epsilon_0 \equiv G_{00}+\epsilon$, $\epsilon_1 \equiv G_{10}+\epsilon$.

The functional form chosen is suggested by the simple phase-space approach of Bjorken[9] combined with an explicit q^2-dependent factor for the approach to scaling in the second term from Devenish and Schildknecht.[10] The first term can be loosely associated with a bare photon interacting with a three-valence-quark (plus gluons) configuration. The second term is a diffractive component in which a quark-antiquark sea (plus gluons) is added to the valence quarks in the generalized-vector-dominance spirit. These forms were first suggested empirically by the scale-noninvariant properties of the data, but we note that our formula bears a resemblance to the one arrived at by Buras and Gaemers who have parametrized the results of asymptotically free gauge-theory predictions.[11] In particular, we differ principally in our parameter ϵ which we find varying approximately as the cube root of a logarithm, while Buras suggests a log-log behavior. The values of the parameters determined from a fit to the data are given in Table II.

Our new data enable us to make an improved calculation of the integral of the structure function $\int F_2^p(x,q^2)dx$ and the equivalent Nachtmann moment[12] ($N=2$) in different q^2 bands. The procedure used has been modified from that described in Ref. 2 by a numerical integration of the MIT-SLAC data including the resonance contributions. The results are given in Table III which replaces Table III of Ref. 2, and are given with and without the elastic scattering contribution. The uncertainty in R is included in the systematic error. These integrals show a slight decrease with increasing q^2.

TABLE II. Values of parameters for global fit [Eq. (2)] to $F_2^p(x,q)$ for all Fermilab data.

Parameter	$q^2 > 2.0$ GeV^2	$q^2 > 0.2$ GeV^2
A	0.656 ± 0.039	0.661 ± 0.035
B	0.395 ± 0.021	0.357 ± 0.009
κ	0.27 ± 0.20	0.33 ± 0.17
G_{00}	2.00 ± 0.72	1.69 ± 0.68
G_{10}	7.7 ± 1.9	7.2 ± 1.7
m_0^2 (GeV^2)	1.02 ± 0.22	0.573 ± 0.052
χ^2/NDF	46/38	96/64

The results of this work reinforce the conclusions suggested by the earlier muon-scattering data that $F_2(x,q^2)$ increases as x goes to zero for fixed q^2 greater than 2 GeV^2 and increases with q^2 for fixed x less than 0.1. This behavior combined with the decrease of F_2 with increasing q^2 for x greater than 0.2 is consistent with that predicted by quantum chromodynamics for a scale parameter $\Lambda = 500-700$ MeV.[13] The integrals of F_2^p show that the quarks carry about half of the total energy-momentum of the proton, the fraction falling slightly with q^2.[13] Unfortunately the present statistical and systematic uncertainties prevent any meaningful comparison of R with quantum-chromodynamic predictions.[14]

TABLE III. Values of the energy-momentum sum rule.

q^2 (GeV^2)	x_{min}	$\int_{x_{min}}^{0.25} F_2^p dx$ Fermilab data only	$\int_0^1 F_2^p dx$ No elastic contribution	$\int_0^1 F_2^p dx$ Including elastic contribution	$N=2$ Nachtmann moment No elastic contribution	$N=2$ Nachtmann moment Including elastic contribution	Errors for integrals Statistical	Errors for integrals Systematic
1–2	0.0025	0.0673 ± 0.0020	0.1876	0.2150	0.1831	0.2095	± 0.0004	± 0.0060
2–4	0.0042	0.0840 ± 0.0018	0.1892	0.1952	0.1870	0.1919	± 0.0008	± 0.0070
4–8	0.0083	0.0838 ± 0.0014	0.1861	0.1868	0.1843	0.1849	± 0.0016	± 0.0070
8–15	0.0222	0.0815 ± 0.0018	0.1716	0.1717	0.1708	0.1709	± 0.0030	± 0.0070
15–30	0.0500	0.0768 ± 0.0027	0.1681	0.1682	0.1676	0.1676	± 0.0049	± 0.0090
30–50	0.1111	0.0512 ± 0.0050	0.1661	0.1661	0.1657	0.1657	± 0.0149	± 0.0140

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Large Parity Admixture in ^{21}Ne?

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In ^{21}Ne there are two levels of the same spin and opposite parity separated by 8 keV. This suggests that a parity-nonconserving nucleon-nucleon potential will give rise to large parity admixtures in these states. In our calculation the standard Cabibbo parity-nonconserving potential gives a circular polarization of at least 10^{-3} for the 2.79-MeV transition $\frac{1}{2}^- \to \frac{3}{2}^+$ ground state. Strong neutral-current effects would lead to P_γ in excess of 1% in contradiction with a recent experiment.

A parity-nonconserving (PNC) interaction between nucleons will manifest itself in nuclei as a circular polarization of the electromagnetic radiation emitted during a transition between nuclear states. Since the PNC potential, $V_{\rm PNC}$, is weak in comparison with the usual (parity-conserving) strong interaction, the circular polarization is usually very small and it is necessary to consider transitions in nuclei for which the effect of $V_{\rm PNC}$ is enhanced. ^{21}Ne is a particularly favorable case, in that it has $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states separated by only 8 keV.

The $J^\pi = \frac{1}{2}^-$ state in ^{21}Ne at 2.79 MeV can go via an $E1$ or an $M2$ transition to the $\frac{3}{2}^+$ ground state (g.s.). Admixtures of the $\frac{1}{2}^+$ state at 2.80 MeV through the PNC potential will enable "irregular" $M1$ and $E2$ transitions to occur as well (other $\frac{1}{2}^+$ states are too far from the $\frac{1}{2}^-$ state to contribute). Interference between the regular and irregular multipole transitions causes the circular polarization, which for randomly oriented ^{21}Ne nuclei is given by[1]

$$|P_\gamma| = \frac{2}{1+\delta^2}\frac{\langle \frac{1}{2}^+(2.80)|V_{\rm PNC}|\frac{1}{2}^-(2.79)\rangle}{|\Delta E|}\frac{\langle {\rm g.s.}\|M1\|2.80\rangle}{\langle {\rm g.s.}\|E1\|2.79\rangle}\left(1+\delta\frac{\langle {\rm g.s.}\|E2\|2.80\rangle}{\langle {\rm g.s.}\|M1\|2.80\rangle}\right), \tag{1}$$

$$\delta \equiv \frac{\langle {\rm g.s.}\|M2\|2.79\rangle}{\langle {\rm g.s.}\|E1\|2.79\rangle}, \tag{2}$$