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Unified Approach to Jet Processes in Quantum Chromodynamics

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We have analyzed in the leading-logarithm approximation a closed set of equations, the solution of which describes correctly the processes $e^+ + e^- \to$ anything, $e + h \to e$ + anything, and $e^+ + e^- \to h$ + anything. This approach allows us to justify clearly the application of the renormalization group and the so-called factorization property. Since the technique is basically a skeleton expansion, it should be useful to give a unified description of all jet processes.

The discovery of asymptotic freedom[1] has made it meaningful to calculate perturbatively to investigate the short-distance behavior of physical processes. The most celebrated example is in deep-inelastic leptoproduction, wherein one deploys the full machinery of the renormalization group (RG) and operator product expansion (OPE).[2] The latter is necessary in order to isolate the so far uncalculable long-distance contribution, related to the composite nature of a hadron with the attendant color confinement effects.

There is yet another way to look at high-energy processes in a renormalizable theory. This is to arrange an approximation in such a way that unitarity is enforced to the desired order. It is quite persuasive that the same RG and OPE results can be reproduced.[3] This route, though tedious at times, offers some advantages. Technically, because of its implementation, one does not have to be mired in the deep Euclidean region, which is the proper domain of RG and OPE. Thus, many more physically interesting processes can be investigated. It is physically direct, since one now works with variables which make closer contact with the processes involved and some quantities, such as "anomalous dimensions," are more ready for interpretation.[4,5] We may call this a procedure to unfold the physical content of the RG.

We have launched a program in this direction and this note is a brief summary of our general attitude and procedure and of some results obtained. Briefly speaking, we have succeeded in developing a closed set of equations via skeleton expansion, such that their solution agrees with the results via RG (and OPE) in areas where such a method is applicable. The technique is general enough that it can be applied to jet processes in all kinematically justifiable regions.

Specifically, let us consider the following three processes: (i) $\gamma(q) \to$ anything; (ii) $\gamma(q) \to a(p)$ + anything; and (iii) $\gamma(q) + a(p) \to$ anything, where γ is assumed to couple to a conserved current. $a(p)$ is a hadron of momentum p. In our approach, all these processes are studied simultaneously.

We have investigated this problem in $(\varphi^3)_6$ and quantum chromodynamics (QCD), both of which are asymptotically free. Since the method is much more transparent in $(\varphi^3)_6$, we will begin with it to introduce the subject matter. The essential arguments carry through in QCD and we will indicate the refinements later.

Except for some inessential kinematics, all three processes are embodied in the study of two structure functions of jets. Let $F(k^2)$ be the total probability for a virtual jet φ of squared invariant mass k^2 to decay into anything, and $F(k,p)$ be the probability to find one hadron of momentum p in this jet.

If radiative effects are temporarily set aside, then at the tree level, the diagonal terms give the leading logarithmic contribution as $k^2 \to \infty$ in all orders of the coupling constant g.[6] As emphasized earlier, however, a high-energy approximation makes sense only if unitarity is respected to that same level of accuracy. Unitarity is easily restored here, if one notices that a jet is actually the imaginary part of the φ propagator. Note further that when a cut is made to a self-energy diagram to obtain the corresponding production cross section, there are terms which correspond to radiative correction of the vertices.

Based on this line of argument, we see that the set of simultaneous equations which have to be solved is the following[7]:

$$F(k^2)=2\pi z\delta(k^2-m^2)+\tfrac{1}{2}|G(k^2)|^2\int\frac{d^6r}{(2\pi)^6}F(r^2)F((k-r)^2)|\Gamma(k^2,r^2,(k-r)^2)|^2, \tag{1}$$

$$F(k,p)=2\pi z\delta(p^2-m^2)\delta(p-k)+|G(k^2)|^2\int\frac{d^6r}{(2\pi)^6}F((k-r)^2)F(r,p)|\Gamma(k^2,r^2,(k-r)^2)|^2. \tag{2}$$

Here $G(k^2)$ is the full propagator and we have

$$F(k^2)=2\,\mathrm{Im}G(k^2); \tag{3}$$

z is a wave-function renormalization constant, which is defined off shell to allow $m\to 0$ for F's (the factorization property).[8] We spare writing down the equation for the full vertex Γ, since it is not needed in the subsequent argument. We are interested in the limit of $k^2\to\infty$ with $k\cdot p/k^2\neq 0$ fixed. Now, it is not hard to verify[6] that the important region of integration in Eqs. (1) and (2) is $k^2\gg r^2,(k-r)^2$. Then, one can replace $\Gamma(k^2,r^2,(k-r)^2)$ by $\Gamma(k^2,\mu^2,\mu^2)$, where μ^2 is some finite squared mass, and pull the vertex out of the integrals. In particular, for $\epsilon<1$,

$$F(k,p)\simeq 2\pi z\delta(p^2-m^2)\delta(p-k)+|\Gamma(k^2,\mu^2,\mu^2)|^2|G(k^2)|^2\int^{\epsilon k^2}d\sigma F(\sigma)\int\frac{d^6r}{(2\pi)^6}\delta(\sigma-(k-r)^2)F(r,p). \tag{2$'$}$$

There are now two observations to make: (1) We can replace $\delta(\sigma-(k-r)^2)$ by $\delta(\lambda^2-(k-r)^2)$, where λ is again some finite mass. The correction in each order is at least one power of $\ln k^2$ down. (2) $G(k^2)$ and $F(k^2)$, because of the relation in Eq. (3), can be calculated through RG as k^2 gets large. Furthermore, in $(\varphi^3)_6$, the asymptotic form factor $\Gamma(k^2,\mu^2,\mu^2)$ can be obtained through the same RG technique.[8] We have the combination

$$\lim_{k^2\to\infty}|G(k^2)|^2|\Gamma(k^2,\mu^2,\mu^2)|^2\int^{\epsilon k^2}d\sigma F(\sigma)\simeq\left(\frac{1}{k^2}\right)^2g^2(k^2), \tag{4}$$

where $g(k^2)$ is the effective running coupling constant. All in all, Eq. (2′) becomes

$$F(k,p)=2\pi z\delta(p^2-m^2)\delta(p-k)+\frac{g^2(k^2)}{(k^2)^2}\int\frac{d^6r}{(2\pi)^6}\delta(\lambda^2-(k-r)^2)F(r,p). \tag{5}$$

This is easily shown to be the scaling master equation,[4,5] *in which we have justified the replacement of g by $g(k^2)$*. The solution of this equation not only reproduces the known results for leptoproduction (iii), but also covers the inclusive annihilation process (ii). Details will be published elsewhere.[9]

We turn now to QCD. It is quite obvious what the strategy is from the above. We should look for a choice of gauge such that at the tree level, the diagonal terms dominate in a jet. The next crucial point to check is that in such gauges the form factors in the appropriate Sudakov regions can be calculated via RG. This we have done. Once these main steps are taken, the analysis follows that of $(\varphi^3)_6$ closely. Some details are now in order.

To be concrete, let us look at the flavor nonsinglet piece of the structure function. The basic integral equation may be represented as in Fig. 1. The spin-averaged structure function is

$$W_{\mu\nu}(q,p)=\mathrm{Tr}(\not{p}T_{\mu\nu}(q,p)). \tag{6}$$

Out of all the gauges, a particular convenient one (with $\omega=2p\circ q/q^2$) is $(q-p/\omega)^\mu A_\mu{}^a=0$, where the superscript a is a color index. This gauge is almost lightlike, i.e., $(q-p/\omega)^2=p^2/\omega^2\simeq 0$, upon neglecting quark mass in the numerators. By going to various frames, this can still represent a wide class of gauges. The advantages of this choice are many fold. Firstly, the criterion of diagonal term dominance is met. Secondly, because of the almost lightlike nature, the polarization sum of a gluon is simplified, which facilitates calculation considerably. It may be ob-

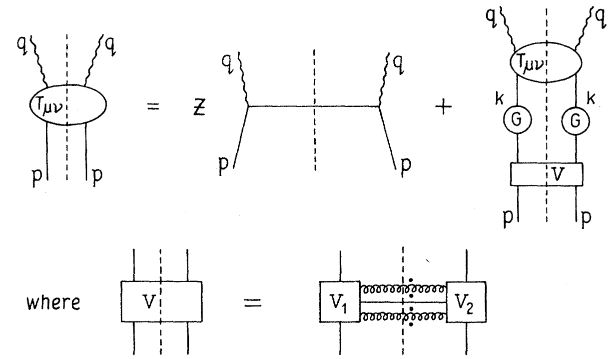

FIG. 1. Basic integral equation to be solved for the spin-dependent structure function $T_{\mu\nu}$ in leptoproduction and e^+e^- inclusive annihilation. G is the full quark propagator, with Z as its off-shell wave-function renormalization constant, V is the full two-particle irreducible kernel, and V_1 and V_2 are generalized vertices. The curly lines are gluons.

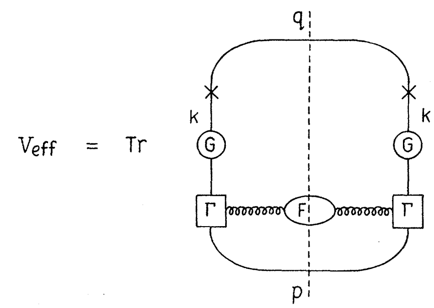

FIG. 2. In the leading-logarithm approximation, $V_{\rm eff}$ is the effective kernel. The crosses indicate where the external photons act. F is the inclusive gluon decay function (i.e., imaginary part of its self-energy).

served that in order to facilitate calculation a change in gauge to evaluate radiative corrections is permissible provided at the end we properly renormalize the wave functions of all the external lines in conformity with the gauge $(q-p/\omega)^\mu A_\mu^a=0$. This is important, because it is only then that we can have the well-known infrared cancellation.

Taking advantage of this last feature, and of the electromagnetic gauge invariance, we can reduce the spinorial equation of Fig. 1 completely down to that for the invariant structure functions W_1 and νW_2 in the leading-logarithmic approximation, collectively denoted by W. This reads

$$W(q,p)=W_0(q,p)+\int\frac{d^4k}{(2\pi)^4}W(q,k)V_{\rm eff}(q,k,p), \qquad (7)$$

where $W_0(q,p)$ is the single-particle driving term (see Fig. 1). Note that the off-shell wave-function renormalization constant Z is infrared divergent, which is needed to ensure the finiteness of W's. $V_{\rm eff}$ is the effective scalar kernel, given by

$$V_{\rm eff}(q,k,p)={\rm Tr}(\not{q}GV_1\not{p}V_2G)/4q\cdot k. \qquad (8)$$

Implementing the ladder nature of the leading diagrams, $V_{\rm eff}$ is represented graphically in Fig. 2.

Now, we are in a position to complete the analysis in parallel to that of $(\varphi^3)_6$ and we quote a sample of results.[9] (i) In this gauge, the driving term and $V_{\rm eff}$ precisely give the factors needed to reproduce the known anomalous dimensions for moments of the structure function in leptoproduction. For the inclusive e^+e^- annihilation, the corresponding moments are shifted by one unit, exhibiting the Gribov-Lipatov[10] relation. (ii) G, Γ, and F combine in the same way as in $(\varphi^3)_6$ to give rise to the running coupling constant. (iii) As in other renormalizable field theories, each $\ln q^2$ can be identified to arise from the lack of a sharp transverse-momentum cutoff. The mechanism works in such a way that the invariant mass, which is proportional to the square of the transverse momentum of the daughters, degrades in each sequential decay as a jet is finally hadronized. It is suggestive that one may unravel some of the confinement physics by studying the momentum composition of the produced hadrons as a function of the rapidity distance from the jet vertex.

We thus establish that the scaling master equation[4,5] holds for e^+e^- inclusive annihilation as well as for deep inelastic leptoproduction. For the former process, this reads ($t=\ln q^2/\mu^2$)

$$t\frac{\partial}{\partial t}W(t,\omega)=\frac{\alpha(t)}{2\pi}\int_\omega^1\frac{d\xi}{\xi}C_2(R)\left[\frac{\xi}{\omega}-1+\frac{2}{(1-\omega/\xi)_+}+\frac{3}{2}\delta\left(1-\frac{\omega}{\xi}\right)\right]W(t,\xi) \qquad (9)$$

for the flavor nonsinglet case. $C_2(R)$ is the Casimir coefficient for the quarks.[5] A slight extension[9] will cover the flavor-singlet situation as well. Note that the scale of q^2 is μ^2, which indicates the finiteness of parton distribution functions as the quark mass goes to zero. Thus we have demonstrated the factorization property[11] simultaneously for processes (ii) and (iii).

In conclusion, we would like to make these remarks: (a) Strictly speaking, the significance of the leading-logarithm calculation performed here for e^+e^- annihilation becomes clear only if the nonleading logarithms can be shown to sum to a power series of the running coupling constant (or some small parameter). In view of the fact that up to the level of our analysis no essential difference has been observed between inclusive annihilation and leptoproduction, we conjecture that this is likely so, since the latter process does possess such an expansion.[2] (b) The skeleton expansion nature of our approach should be commented on. It is believed that a hard process which involves large momentum scale(s) has two components,[12] one being the impact region where jets are formed. The description here presumably should correspond to some simple skeleton exchange. The second component is the structure of jets, the study of which we have outlined in this note. We are working to explore the validity of this picture in the context of QCD.

During the preparation of this manuscript, an interesting article[13] was brought to our attention. Although the details in areas which overlap ours differ somewhat, the conclusion is on the whole in agreement.

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