Zero-Mass Quarks and the U(1) Problem

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The possibility of a zero-mass up or down quark, in broken chiral symmetry, is shown to be in serious contradiction with the nonrenormalization theorem.

The instanton solution[1] of quantum chromodynamics[2] with its associated quantum effects[3] leads to the unpleasent possibility of parity and time-reversal nonconservation in strong interactions. A possible resolution of this puzzle has been advanced recently by Peccei and Quinn,[4] who assume a global U(1) chiral symmetry for the Lagrangian. This possibility, however, requires the presence of a very light pseudoscalar pseudo Goldstone boson, the axion,[5] whose existence has not yet been confirmed by experiment.[5,6] If the axion is not found, then other possibilities should be contemplated in order to account for P and T invariance in strong interactions.

Another alternative would be to require that the mass of the up or down quark be zero. Weinberg[5] has argued against this possibility based on the result for the kaon mass difference induced by the quark masses. In fact, if $m_u=0$ or $m_d=0$, then the K^0-K^+ mass difference turns out to be so large that electromagnetic contributions would hardly compensate the result to bring it into agreement with experiment. This argument, though, is based on the assumption that certain wave-function renormalization factors are equal. Relaxation of this hypothesis could result, *a priori,* in a kaon mass difference of the right sign and magnitude even if $m_u=0$. We shall show here that this alternative contradicts the nonrenormalization theorem[7] thus providing stronger evidence against the possibility of having zero-mass quarks after chiral symmetry is broken.

Let us start by considering the following chiral-symmetry–breaking Hamiltonian

$$H'=\epsilon_0 u_0+\epsilon_8 u_8+\epsilon_3 u_3. \tag{1}$$

Computing the divergences of the axial-vector currents and using the $SU(3)\otimes SU(3)$ algebra in the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ representation[8] one finds

$$\mu_\pi^2 f_\pi = Z_\pi^{1/2}(\sqrt{\tfrac{2}{3}}\,\epsilon_0+\sqrt{\tfrac{1}{3}}\,\epsilon_8), \tag{2}$$

$$\mu_K^2 f_K = Z_K^{1/2}(\sqrt{\tfrac{2}{3}}\,\epsilon_0-\tfrac{1}{2}\sqrt{\tfrac{1}{3}}\,\epsilon_8\pm\tfrac{1}{2}\epsilon_3), \tag{3}$$

where the double sign in front of ϵ_3 in Eq. (3) refers to K^+ and K^0, respectively, and $Z^{1/2}$ are the standard wave-function renormalization factors. The kaon mass difference induced by the u_3 term in the Hamiltonian can be expressed in terms of the quark masses as

$$(\mu_{K^+}^2-\mu_{K^0}^2)_{u_3}=\frac{Z_K^{1/2}}{f_K}\;\frac{m_u-m_d}{2}. \tag{4}$$

Setting $m_u=0$, Eq. (4) can be written as

$$(\mu_{K^+}^2-\mu_{K^0}^2)_{u_3}=-\frac{Z_K^{1/2}}{Z_\pi^{1/2}}\;\frac{f_\pi}{f_K}\,\mu_\pi^2, \tag{5}$$

while for $m_d=0$ Eq. (5) simply changes sign.

If one assumes $Z_K^{1/2}=Z_\pi^{1/2}$ and uses $f_K/f_\pi=1.22$, then Eq. (5) gives $\mu_{K^+}-\mu_{K^0}=\mp 15$ MeV for $m_u=0$ or $m_d=0$, respectively. As has been argued by Weinberg,[5] it is unlikely that the electromagnetic contributions would compensate such a large value to bring it into agreement with the experimental mass difference of -4 MeV. On the other hand, the u_3-induced mass difference can be written as

$$(\mu_{K^+}^2-\mu_{K^0}^2)_{u_3}=\tilde{\epsilon}_3/f_K, \tag{6}$$

where the renormalized parameter $\tilde{\epsilon}_3$ may be obtained from the $\eta\to 3\pi$ decay rate. In fact, a chiral perturbation-theory calculation of the $\eta\to 3\pi$ amplitude together with the experimental rate gives[9]

$$|\tilde{\epsilon}_3/f_\pi|=0.012\ \text{GeV}^2, \tag{7}$$

$$|(\mu_{K^+}^2-\mu_{K^0}^2)_{u_3}|=0.0098\ \text{GeV}^2, \tag{8}$$

with large errors. An extended partially conserved axial-vector current[10] and current-algebra calculation of the same process, which predicts much improved values for the $\Delta I=1$ baryon mass differences, gives instead[11]

$$|\tilde{\epsilon}_3/f_\pi|=0.0056\pm 0.0005\ \text{GeV}^2, \tag{9}$$

$$|(\mu_{K^+}^2-\mu_{K^0}^2)_{u_3}|=0.0046\pm 0.0004\ \text{GeV}^2. \tag{10}$$

The use of any of the above-mentioned methods shows then that the alternative $m_u=0$ or $m_d=0$ is very likely ruled out provided $Z_\pi^{1/2}=Z_K^{1/2}$.

Let us study then the case $Z_\pi^{1/2}\neq Z_K^{1/2}$. Demanding that the left-hand side of Eq. (5) be equal to Eq. (8), one finds

$$|Z_K^{1/2}/Z_\pi^{1/2}| = 0.66, \tag{11}$$

while using Eq. (10) this ratio becomes

$$|Z_K^{1/2}/Z_\pi^{1/2}| = 0.31. \tag{12}$$

Clearly, these results are in conflict with chiral perturbation theory which predicts[12] $Z_K^{1/2}/Z_\pi^{1/2} \approx 1.025$. However, there is a much more fundamental reason to reject the solutions (11) and (12), i.e., they are in serious contradiction with the nonrenormalization theorem,[7] as will be shown below using the techniques of Auvil and Desphande.[13]

Let us consider a three-point function involving two pseudoscalar fields φ_i, φ_j and one scalar field σ_k, i.e.,

$$G_{ijk}(p^2,p'^2,q^2) = \frac{(p^2-m_i^2)(p'^2-m_j^2)(q^2-m_k^2)}{Z_i^{1/2}Z_j^{1/2}Z_k^{1/2}} \times \iint d^4x\, d^4y\, e^{ip\cdot x} e^{-ip'\cdot y} \langle 0|\, T[\varphi_i(x)\varphi_j(y)\sigma_k(0)]\,|0\rangle, \tag{13}$$

where $q = p - p'$ and $i,j,k = 1,\ldots,7$. For $k = 4,5,6,7$, G may be related to a three-point function of two pseudoscalar fields and a vector current defined by

$$i(p+p')_\mu F_{ijl}^{+}(p^2,p'^2,q^2) + iq_\mu F_{ijl}^{-}(p^2,p'^2,q^2) = -\frac{(p^2-m_i^2)(p'^2-m_j^2)}{Z_i^{1/2}Z_j^{1/2}} \iint d^4x\, d^4y\, e^{ip\cdot x} e^{-ip'\cdot y} \langle 0|T[\varphi_i(x)\varphi_j(y)V_{\mu,l}(0)]|0\rangle. \tag{14}$$

In fact, using $\partial^\mu V_{\mu,l} = \epsilon_8 f_{8lk}\sigma_k$ and integrating by parts, one finds

$$G_{ijk}(p^2,p'^2,q^2) = \left(\frac{q^2-M_k^2}{Z_k^{1/2}\epsilon_8 f_{8lk}}\right)\{(p^2-p'^2)F_{ijl}^{+}(p^2,p'^2,q^2) + q^2F_{ijl}^{-}(p^2,p'^2,q^2) - f_{ijl}[(Z_j/Z_i)^{1/2}(p^2-m_i^2) - (Z_i/Z_j)^{1/2}(p'^2-m_j^2)]\}. \tag{15}$$

The structure functions $F^{\pm}$ on the mass shell are identified with the weak and electromagnetic form factors. Taking the limit $q^2 \to 0$ in Eq. (15) using the smoothness condition[14] for G and F^+ one obtains[13]

$$Z_i = Z_j, \tag{16}$$

$$F_{ijl}^{+} = f_{ijl}, \tag{17}$$

$$G_{ijk} = \frac{m_k^2}{Z_k^{1/2}\epsilon_8 f_{8lk}} f_{ijl}(m_j^2 - m_i^2). \tag{18}$$

Therefore, smoothness leads to results consistent with the nonrenormalization theorem[7] which states that F_{ijl}^{+} at $q^2 = 0$ must remain unrenormalized up to second order in SU(3)-symmetry breaking. Equation (18) is also consistent with the fact that G_{ijk} is a first-order symmetry-breaking effect.

If one relaxes the smoothness condition for G and F^+ in order to accommodate the values of $Z_K^{1/2}/Z_\pi^{1/2}$ given by Eqs. (11) and (12), then it follows from Eq. (15) that there will be huge renormalization effects induced in F_{ijl}^{+}. It should be added that in the case of symmetries realized via Nambu-Goldstone bosons, e.g., SU(2)⊗SU(2) or SU(3)⊗SU(3), the renormalization of F_{ijl}^{+} at $q^2 = 0$ comes in first order in the symmetry breaking[15] rather than in second as for SU(3) or SU(2). However, even in this case the renormalization effects amount only to a few percent.[15]

Recently Zepeda[16] has argued that Eq. (12) has the effect of bringing the ratio ϵ_8/ϵ_0 closer to its SU(2)⊗SU(2)-symmetry-limit value of $-\sqrt{2}$. However, this implies that the ratio of strange- to nonstrange-quark masses must be unusually large; in other words, the improvement of SU(2)⊗SU(2) is achieved at the expense of SU(3). My arguments then show that such a large breaking of SU(3) is inconsistent with the nonrenormalization theorem.

In conclusion, the alternative of zero-mass quarks should be ruled out; and if future experimental evidence does not confirm the existence of the axion, then other possibilities should be studied[17] in order to avoid P and T nonconservation as induced by instanton effects.

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Note added.—After this work was completed I received a preprint by Desphande and Soper[18] who also studied the hypothesis $m_u = 0$. These authors conclude that $m_u = 0$ by allowing SU(2) to be much more broken than what is suggested by first-order perturbation theory. This situation is completely analogous to the one just discussed in connection with SU(3) since the nonrenormalization theorem also applies to SU(2). Regarding the validity of first-order perturbation-theory calculations with a chiral Lagrangian we feel that the results obtained in Ref. 11 show that is is perfectly legitimate. The author wishes to thank A. Zepeda for bringing this problem to his attention.

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Polarization of Λ's and $\bar{\Lambda}$'s Produced by 400-GeV Protons

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We have measured the polarization of 3×10^6 Λ^0 and 2.5×10^5 $\bar{\Lambda}^0$ hyperons produced by 400-GeV protons on a beryllium target. The hyperons were detected at a fixed angle of 7.2 mrad and in the momentum range from 50 to 300 GeV/c for Λ^0 and 50 to 200 GeV/c for $\bar{\Lambda}^0$. The Λ^0 polarization agrees with that measured at 24 and 300 GeV and is -0.24 ± 0.04 at $p_T=2.1$ GeV/c. The $\bar{\Lambda}^0$ polarization is zero up to $p_T=1.2$ GeV/c.

The discovery[1] that Λ^0 hyperons are polarized when produced by the interaction of 300-GeV unpolarized protons with an unpolarized target implies, contrary to early expectations, that spin effects are important in high-energy particle production. Where measured, the polarization has the following properties[1-3]: (i) The polarization direction is perpendicular to the production plane as required by parity conservation. (ii) The magnitude increases monotonically with transverse momentum from 0 at $p_T=0$ to over 20% at $p_T=1.6$ GeV/c. (iii) The polarization does

