Ion-Mixing Mode and Model for Density Rise in Confined Plasmas

B. Coppi and C. Spight[(a)]
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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A model for the rate of density rise observed when neutral gas is fed into the plasma chamber is presented for regimes where known collisional transport processes do not provide an adequate explanation. A dense layer of cold plasma produced at the edge of the plasma column and the resulting ion-temperature gradient, sharper than the relative density gradient, can lead to the excitation of electron temperature fluctuations driven by ion drift modes. These can induce a net inflow of electrons and ions under realistic conditions.

The possibility of controlling the plasma density in a magnetically confined plasma can be used to realize regimes with different transport properties and, in particular, to select the optimal heating cycle[1] and to deal with the problem of refueling in a thermonuclear reactor.

It has been found experimentally, first in the ST experiment at Princeton University,[2] then to a remarkable degree in the Alcator experiment[3] at Massachusetts Institute of Technology, and by now in a number of other experiments on magnetically confined plasmas, that the peak plasma density can be raised simply by bleeding neutral gas into the plasma chamber during a stable discharge. The only apparent limitation is that the gas injection should not be so rapid as to induce a sharp cooling in a considerable portion of the current channel and consequently the excitation and rapid amplification of a resistive mode[4] with poloidal wave number $m^0=2$. Thus, one method of avoiding a disruption of the plasma column is to set the magnetic field at such high values[3] that the region where the $m^0=2$ is localized falls well within the plasma column. Here, in fact, the temperature remains relatively high and the mode can affect only a relatively small plasma volume. Another method consists in keeping the plasma current rising at an appropriate rate as gas is bled into the plasma chamber. Then a layer of current that maintains the outer part of the current channel, in which the $m^0=2$ tends to develop, can be produced.[5] We notice that $m^0=2$ modes depend on the effects of finite electrical resistivity and tend to be weakened by other kinetic effects such as finite drift-wave frequency, ion gyroradius, and transverse ion viscosity, whose importance increases as the temperature increases.[4] Therefore we argue that in this case the mode excitation is weakened as a result either of a local decrease of the plasma resistivity[5] or of a local redistribution of the current[6] or of both. When the plasma density is increased in a sequence of experiments, the energy replacement time is observed to improve, while the degree of decontamination from impurity ions coming from the metallic surfaces surrounding the plasma column is also improved. Therefore, well-confined hydrogenic (deuterium) plasmas with effective charge number about unity can be produced.[3]

The temperature and density profiles realized, at nearly steady state, in Alcator have been relatively well simulated by the transport model reported by Coppi and Taroni[7] as long as the initial density is peaked at the center of the plasma column. However, when an influx of neutrals of the walls is added in order to simulate the bleeding of gas into the plasma chamber and no provision is made for an anomalous inward transport of cold plasma, the density profile tends to develop a sharp peak at the outer edge of the plasma column. Therefore, we have been led to assume that, in the presence of a vigorous power input, such as Ohmic heating in the case of Alcator, the dense layer of cold plasma that is produced at the outer edge of the column tends to produce a sharp temperature gradient $|d\ln T/dr|$ relative to the

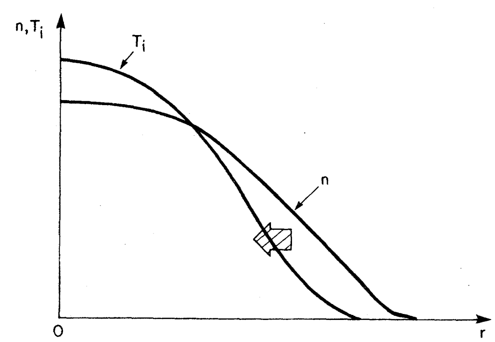

FIG. 1. Type of temperature and density profiles leading to the excitation of ion-mixing modes.

density gradient $|d\ln n/dr|$. (See Fig. 1.) Then ion drift modes which tend to mix the hot and cold ion populations can be excited.[8,9] At the same time the resulting electron temperature fluctuations, associated with the effects of finite longitudinal electron thermal conductivity,[10] can produce a net inward transport of ions and electrons. The rate of this transport is consistent with the needed rise time of the center density. In fact, a numerical transport code[11] incorporating the results presented here has been developed in order to provide a detailed comparison with the experimental observations in Alcator.

We adopt a guiding-center description for both electrons and ions where we neglect at first finite–ion-gyroradius effects and refer to a one-dimensional (x dependent), plane equilibrium configuration (ignoring toroidal and shear effects) in which the magnetic field is in the z direction. We consider electrostatic modes represented by $\vec{E}_1 = -\nabla\tilde{\varphi}$, with $\tilde{\varphi} = \tilde{\varphi}(x)\exp(ik_y y + ik_\parallel z - i\omega t)$. The relevant wavelengths are larger than the Debye length and the quasineutrality condition $\tilde{n}_i = \tilde{n}_e$ is to be satisfied. Since we need to consider the effects of finite longitudinal electron thermal conductivity we refer to the frequency range

$$k_\parallel^2 v_{\text{th}i}^2/\nu_i < \omega \lesssim k_\parallel^2 v_{\text{th}e}^2/\nu_e\,,$$

where $\nu_{e,i}$ are the average collision frequencies, $T_e \sim T_i$, and $k_\parallel^2 v_{\text{th}e}^2/\nu_e^2 \sim k_\parallel^2 v_{\text{th}i}^2/\nu_i^2 < 1$. The electron and ion mass conservation equations, in the guiding-center approximation, are

$$\frac{\partial}{\partial t}\tilde{n}_j + \tilde{v}_{Ex}\frac{dn}{dx} + n\nabla_\parallel \tilde{u}_{j\parallel} = 0, \tag{1}$$

where $\tilde{v}_{Ex} = -ik_y c\tilde{\varphi}/B$. Then Eq. (1) can be rewritten as $\tilde{n}_j/n_j = k_\parallel \tilde{u}_{j\parallel}/\omega - (e_j\tilde{\varphi}/T_j)(\omega_{*j}/\omega)$ and we see that $\tilde{u}_{i\parallel} = \tilde{u}_{e\parallel}$. Here $\omega_{*j} = k_y c T_j (d\ln n/dx)/e_j B$, and $j = e, i$. The perturbed electron-momentum-balance equation along the field, $0 = -\nabla_\parallel(n_e\tilde{T}_e + n\tilde{T}_e - en\tilde{\varphi}) - \alpha_T n\nabla_\parallel \tilde{T}_e$, reduces to

$$\frac{\tilde{n}_e}{n} = \frac{e\tilde{\varphi}}{T_e} - \frac{\tilde{T}_e}{T_e}(1+\alpha_T), \tag{2}$$

where α_T is the thermal force coefficient.[12]

In order to obtain $\tilde{T}_e/T_e$ we refer to the electron thermal-energy-balance equation

$$\tfrac{3}{2}n\left(\frac{\partial}{\partial t}\tilde{T}_e - \frac{c}{B^2}\nabla\tilde{\varphi}\times\vec{B}\cdot\nabla T_e\right) + nT_e\nabla_\parallel \tilde{u}_{e\parallel} = \nabla_\parallel\left(\hat{\chi}_e\frac{nT_e}{m_e\nu_e}\nabla_\parallel \tilde{T}_e\right)$$

where $\hat{\chi}_e$ is the thermal conductivity coefficient as tabulated in Ref. 12. Thus we have,

$$\frac{\tilde{T}_e}{T_e} = \frac{-i}{\omega_\chi - i3\omega/2}\left[\omega_{*e}(\tfrac{3}{2}\eta_e - 1)\frac{e\tilde{\varphi}}{T_e} + \omega\frac{\tilde{n}_e}{n}\right], \tag{3}$$

where $\omega_\chi \equiv \hat{\chi}_e k_\parallel^2 T_e/(m_e\nu_e)$, and $\eta_e = d\ln T_e/d\ln n$. If we consider for simplicity, the limit where $\omega \sim \omega_{*e} < \omega_\chi$ we obtain

$$\frac{\tilde{n}_e}{n} = \frac{e\tilde{\varphi}}{T_e}\left[1 + (1+\alpha_T)\frac{i}{\omega_\chi}(\omega - \omega_{*e} + \tfrac{3}{2}\omega_{*e}\eta_e)\right]. \tag{4}$$

We notice that, according to Eq. (4), the density fluctuations are out of phase with respect to the electric potential fluctuations by the quantity

$$[\mathrm{Re}(\omega/\omega_{*e}) - 1 + \tfrac{3}{2}\eta_e](1+\alpha_T)\omega_{*e}\omega_\chi,$$

that is positive for realistic values of η_e.

For ions the perturbed momentum balance equation is

$$m_i n\frac{\partial}{\partial t}\tilde{u}_{i\parallel} = -\nabla_\parallel(n\tilde{T}_i + \tilde{n}_i T_i) + \alpha_T n\nabla_\parallel \tilde{T}_e - en\nabla_\parallel\tilde{\varphi}, \tag{5}$$

and for the considered frequency range the relevant thermal-energy-balance equation reduces to an adiabatic in the sense that $(nd/dt)_i T_i = (2T_i/3)\times(d/dt)_i n_i$. Here $(d/dt)_i \equiv \partial/\partial t + \tilde{v}_{Ex}\partial/\partial x$. Thus, we have, for $\eta_i \equiv d\ln T_i/d\ln n$,

$$\frac{\tilde{n}_i}{n} = -\frac{e\tilde{\varphi}}{T_i}\left\{\frac{\omega_{*i}}{\omega}\left[1 + \frac{k_\parallel^2 T_i}{m_i\omega^2}(\eta_i - \tfrac{2}{3})\right] + \frac{k_\parallel^2 T_i}{m_i\omega^2}\left(\alpha_T\frac{\tilde{T}_e}{e\tilde{\varphi}} + 1\right)\right\}\left(1 - \frac{5}{3}\frac{k_\parallel^2 T_i}{m_i\omega^2}\right)^{-1}. \tag{6}$$

We consider the resulting dispersion relation in the limit

$$\omega/k_\parallel v_{\text{th}i} > 1 \text{ and } \eta_i \sim \eta_e > 1. \tag{7}$$

Then we obtain

$$1 + iA + k_\parallel^2\frac{T_e}{m_i}\frac{\omega_{Ti}}{\omega^3} - \frac{\omega_{*e}}{\omega} = 0, \tag{8}$$

where $\omega_{Ti}/\omega_{*e} < 0$ and $A = \tfrac{3}{2}\eta_e(\omega_{*e}/\omega_\chi)(1+\alpha_T)$. We notice that Eq. (8) has three roots, of which one, that is found when $(k_\parallel^2 T_e/m_i)\omega_{Ti}/\omega^3$ is neglected, is the known electron-drift mode. This is stabilized by the finite electron thermal conductivity term A. The other two roots arise from the term in ω_{Ti}. In particular, when the term in ω_{*e}/ω is neglected we obtain

$$\omega = -[(k_\parallel^2 T_e/m_i)\omega_{Ti}/(1+A^2)]^{1/3}(1 - iA)^{1/3}. \tag{9}$$

The lack of phase synchronization between density and potential fluctuations gives rise to the quasilinear particle flow

$$\Gamma_x = \langle \tilde{n}\tilde{v}_{Ex}\rangle = -(2c/B)\,\mathrm{Im}\left(\sum_k k_y \tilde{\varphi}_k \tilde{n}_k\right). \tag{10}$$

$$\Gamma_x = 3(1+\alpha_T)D_B{}^i n \sum_k k_y{}^2 \left|\frac{e\tilde{\varphi}}{T_i}\right|^2 \frac{1}{\omega_\chi}\frac{cT_i}{eB}\frac{d\ln T_e}{dx}, \tag{11}$$

where $D_B{}^i \equiv cT_i/(eB)$. Therefore, $\Gamma_x < 0$ for $dT_e/dx < 0$. Notice that if we write $\partial n/\partial t + \partial \Gamma_x/\partial x = 0$ we do not obtain a diffusion equation from this in the sense that the particle inflow is not driven by the density gradient. In order to have a rough estimate for Γ_x we may assess the saturation level of these modes as[10] $\tilde{v}_{Ex} \sim [c/(eB)]\,dT_i/dx$. This leads to $e\tilde{\varphi}/T_i \sim 1/(k_y r_T)$, where $r_T = -(d\ln T_i/dx)^{-1}$, and we have, leaving numerical factors aside,

$$\Gamma_x \sim \frac{n}{r_T} D_B{}^i \frac{1}{\omega_\chi}\left(\frac{c}{eB}\frac{cT_e}{dx}\right)\frac{T_i}{T_e r_T}. \tag{12}$$

In order to assess the range of values which A can take we notice that it can be rewritten as

$$A = \frac{3}{\tilde{\chi}_e}\left(\frac{T_e}{T_i}\frac{m_e}{m_i}\right)^{1/2}\frac{1}{k_\parallel \lambda_e}\left(\frac{\omega_{Ti}}{k_\parallel v_{\mathrm{th}\,i}}\right)(1+\alpha_T),$$

where $k_\parallel \lambda_e < 1$, $\lambda_e = v_{\mathrm{th}\,e}/\nu_e$ being the electron mean free path, and the meaningful root of Eq. (8) is found for $\omega_{Ti} > k_\parallel v_{\mathrm{th}\,i}$. Thus we estimate A to be well in excess of $(m_e/m_i)^{1/2}$.

We observe that if $\eta_i{}^2/|A| > 1$, the rate at which the ion thermal energy can be carried out tends to be larger than the rate of inward particle transport. However, we argue that a strong heat source (Ohmic heating in the case of Alcator) is present at the center of the plasma column while the injected neutral gas has a cooling effect on the outer edge (of the plasma column). Thus, a finite value of $|d\ln T_i/d\ln r|$ can be maintained in spite of the enhancement of ion thermal conductivity that is produced by the considered modes. This observation has been confirmed in fact by the results of the appropriate transport code developed by Englade.[11]

In the simplified model presented, the effects of magnetic shear have been neglected, while only the collisional limit for the modes of interest has been considered. Referring to the first point we notice the following: (a) The influence of shear on temperature-gradient-driven "quasi flute modes" that can be reproduced in slab geometry[13] was examined in Ref. 9. There it was found that absolute modes existed (in the presence of magnetic shear) in contrast with the case of density-gradient-driven electron-drift modes that exist only as convective modes[14] in the same configuration with magnetic shear. (b) In toroidal geometry the effect of magnetic shear can be minimized when "disconnected ballooning" modes of the type presented in Ref. 13 are considered. The dispersion relation we have derived can be extended approximately to these modes by taking $k_\parallel \simeq 1/qR$, where R is the torus major radius, $q = 2\pi/\iota(r)$, $\iota(r)$ is the rotational transform, and r is the minor radial coordinate.

Referring to the second point we recall that the analysis of collisionless temperature-gradient-driven modes is well known.[8,9] Consideration of the collisionless limit is important when discussing the process for particle inflow in the center of the plasma column. In this limit the instability condition requires $\eta_i \geq 1$ for relatively short wavelengths, corresponding to $b_i = k^2 T_i/m_i \Omega_i{}^2 \gtrsim 1$ or $\eta_i \gtrsim 2$ for $b_i < 1$, while $\omega \lesssim k_\parallel v_{\mathrm{th}\,i}$.

Recalling the results of Coppi and co-workers,[15] we notice that when $\eta_i > 1$, as we assume to be the case, the accumulation of impurity ions at the center of the plasma column can be prevented by the onset of impurity-driven modes with $\omega/k_\parallel \lesssim v_{\mathrm{th}\,i}$. Thus, we may argue that the formation of a cold plasma layer at the outer edge can, at the same time, favor the inward transport of cold plasma and ensure a high degree of decontamination. In the experiments that have been simulated numerically,[11] the assumed rate of inflow of the the neutral gas was 1.6×10^{16} particle/(cm^2 sec) while the rate of rise of the center density was about 8×10^{13} particle/(cm$^3\times$ 10 msec), for a plasma column radius of 10 cm, a Gaussian temperature profile, a center temperature of about 1 keV, and a peak density in the range $(3-23)\times 10^{13}$ particle/cm^3. The relevant inward flow of cold plasma can be evaluated on the basis of the estimate (12) and of the expression $D_B = 2\times 10^6[T_i/(100\ \mathrm{eV})][B/(10\ \mathrm{kG})]^{-1}$ cm^2/sec. In the experimental situation and in the relevant numerical simulation[11] additional transport processes producing an outflow of particles and thermal energy are taken into account[7] in order to reproduce the equilibrium state that is reached when the injection of gas ceases. Thus, during the gas injection phase the rate of inflow due to the ion-mixing mode has to be sufficiently high in order to prevail over the outflow due to other processes. When injection ceases, the values of η_i and η_e at the edge decrease and the residual effect of the ion-mixing mode is no longer sufficient to produce a net particle inflow.

Finally, we notice that, after the model we presented here was formulated, relatively large fluctuations have been observed at the edge of the plasma column of Alcator by CO_2-laser scattering experiments[16] during the rising phase of the particle density as well as during the time where the density is nearly stationary. Although it is premature to compare these observations with the theory presented here, it is evident that they are not inconsistent with each other.

[(a)]Permanent address: Morehouse College, Atlanta, Ga. 30314.

[1]B. Coppi, Comments Plasma Phys. Controlled Nucl. Fusion 3, 47 (1977).
[2]J. C. Hosea, private communication.
[3]G. Boxman, B. Coppi, L. deKock, B. Meddens, A. Oomens, L. Ornstein, D. Pappas, R. Parker, L. Pieroni, S. Segre, F. Schüller, and R. Taylor, in *Proceedings of the Seventh European Conference on Controlled Fusion and Plasma Physics* (Ecole Polytéchnique Fédérale, Lausanne, Switzerland, 1975).
[4]B. Coppi, Phys. Fluids 7, 1501 (1964), and 8, 2273 (1965).
[5]B. Coppi, in *Tokamak Reactors for Breakeven*, edited by H. Knoepfel (Pergamon, New York, 1976), p. 5.
[6]A. H. Glasser, H. P. Furth, and P. H. Rutherford, Phys. Rev. Lett. 38, 234 (1977).
[7]B. Coppi and A. Taroni, Massachusetts Institute of Technology Report No. PRR76/12, 1976 (unpublished) and in *Proceedings of the 1976 Varenna Conference on Heating of Thermonuclear Plasmas* (Editrice Compositori, Bologna, Italy, 1976), p. 192.
[8]L. Rudakov and R. Z. Sagdeev, Dokl. Akad. Nauk SSSR 138, 581 (1961) [Sov. Phys. Dokl. 7, 417 (1961)].
[9]B. Coppi, M. N. Rosenbluth, and R. Z. Sagdeev, International Center for Theoretical Physics, Internal Report No. IC/66/24, 1966 (unpublished), and Phys. Fluids 10, 582 (1967).
[10]B. Coppi and E. Mazzucato, Phys. Fluids 14, 134 (1971).
[11]R. Englade, unpublished.
[12]S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultant Bureau, New York, 1966), Vol. I, p. 205.
[13]B. Coppi, Phys. Rev. Lett. 39, 939 (1977).
[14]B. Coppi, G. Laval, R. Pellat, and M. Rosenbluth, Nucl. Fusion 6, 261 (1966).
[15]B. Coppi, Phys. Ref. Lett. 29, 1076 (1973); B. Coppi, G. Rewoldt, and T. Schep, Phys. Fluids 19, 1144 (1976). In the presence of magnetic shear, unstable normal modes (that is, nonconvective) driven by impurity density gradients are still found as shown in B. Coppi, unpublished.
[16]R. E. Slusher and C. M. Surko, Bull. Am. Phys. Soc. 22, 1091 (1977).

Megagauss Magnetic Field Profiles in Laser-Produced Plasmas

A. Raven[(a)] and O. Willi
Department of Engineering Science, Oxford University, Oxford, United Kingdom

and

P. T. Rumsby
Rutherford Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom
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Measurement of spatial profiles of self-generated magnetic fields in Nd:glass-laser–produced plasmas have been made with high temporal and spatial resolution (30 ps, 1.5 μm) by simultaneous Faraday rotation and interferometry using a Raman-shifted second-harmonic probe beam.

The generation of spontaneous magnetic fields in laser-produced plasmas due to thermoelectric effects,[1,2] absorption,[3] and target discontinuities,[4] have been theoretically predicted and imply substantial modifications to plasma transport properties. Measurements of thermoelectric[5] and resonance-absorption–generated[5,6] field profiles have been made in microwave-plasma interactions, and thermoelectric fields in laser-produced plasmas have been measured by Faraday rotation[7] and probes.[1] In this Letter we present the first published measurements of magnetic field profiles in laser-produced plasmas obtained by simultaneous Faraday rotation and interferometric measurements with good spatial and temporal resolution.

The experiment arrangement is shown in Fig. 1. The target was irradiated through an $f/1.0$ lens with a single beam of the Rutherford Laboratory neodymium:glass laser. The pulse length was 100 ps and the energy varied over the range 5 to 40 J. The focal-spot diameter was measured