of the intermediate region could lead to an increased number of vacancies.[14] This in turn could explain why in the low-density solid layers the ferromagnetic trend dominates over the antiferromagnetic exchange interaction whereas in bulk solid ^{3}He near the melting pressure density a magnetic transition to an antiferromagnetic state apparently takes place[9] in low magnetic fields.

The enhanced susceptibility in the Mylar experiment[1] has been interpreted[15] by attributing properties of a two-dimensional itinerant-electron ferromagnet to the high-density liquid layers next to the solid ^{3}He layer on the surface.

In order to gain further insight into these surface-induced effects, new experiments in which the properties of the surface layers could be studied in a well-defined geometry, as functions of temperature, magnetic field, and ^{4}He impurity level, would be most desirable.

We wish to acknowledge helpful discussions with J. Kurkijärvi, P. Wölfle, and G. Kharadze.

Note added.—After this paper had been submitted, we received a reprint from H. M. Bozler *et al.*[16] describing similar measurements on ^{3}He on Grafoil. They observe a reduction in $\tau_2{}^*$ below ~1.1 mK instead of the sharp increase found in our experiment roughly in the same temperature region. The discrepancy may be due to the presence of superfluid ^{3}He in their geometry.

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Zero-Sound Attenuation from Order-Parameter Fluctuations in Liquid ^{3}He

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We calculate the attenuation of zero sound due to fluctuations of the superfluid order parameter above T_c. The results appear to account for the excess attenuation recently observed by Paulson and Wheatley.

The attenuation of zero sound in liquid ^{3}He increases dramatically just below the superfluid transition temperature. This increased attenuation is now understood to result from coupling between the density oscillations and the collective modes of the order parameter.[1] Recently Paulson and Wheatley[2] have reported an increase in the attenuation of zero sound above T_c, relative to the attenuation expected for a normal Fermi liquid. We have calculated the attenuation of zero sound above T_c due to fluctuations of the order parameter. This contribution to the sound attenuation seems able to account for the excess attenuation found by Paulson and Wheatley.

The zero-sound attenuation coefficient, α_0, is given by[3]

$$\alpha_0 = T\dot{s}/2c_0\,\delta E_{\rm mech}, \tag{1}$$

where c_0 is the zero-sound velocity, $\delta E_{\rm mech}$ is the mechanical energy density in the sound wave, and $\dot{s}$ is the time derivative of the entropy density, which for weakly damped zero sound takes the form[4]

$$T\dot{s} = \frac{1}{4k_BT}\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}\int\frac{d^3p_3}{(2\pi)^3}\,2W\,\delta(\epsilon_1+\epsilon_2-\epsilon_3-\epsilon_4)\{n^0(\epsilon_1)n^0(\epsilon_2)[1-n^0(\epsilon_3)][1-n^0(\epsilon_4)]\}$$
$$\times\left\{\sum_{l\geq 2}\nu_l\left(1+\frac{F_l{}^s}{2l+1}\right)[P_l(\hat{p}_1\cdot\hat{q})+P_l(\hat{p}_2\cdot\hat{q})-P_l(\hat{p}_3\cdot\hat{q})-P_l(\hat{p}_4\cdot\hat{q})]\right\}^2. \tag{2}$$

In this equation $n^0(\epsilon)$ is the equilibrium Fermi distribution, $\vec{p}_4=\vec{p}_1+\vec{p}_2-\vec{p}_3$, and the ν_l are related to the zero-sound solution of the collisionless kinetic equation

$$(\omega-\vec{q}\cdot\vec{v}_p)\nu_{\vec{p}}-\vec{q}\cdot\vec{v}_p\int(d\Omega/4\pi)F^s(\hat{p}\cdot\hat{p}')\nu_{\vec{p}'}=0 \tag{3}$$

by

$$\nu_{\vec{p}}=\sum_l\nu_lP_l(\hat{p}\cdot\hat{q}). \tag{4}$$

Order-parameter fluctuations contribute to the zero-sound attenuation through $2W$, the scattering rate for the process $\vec{p}_1,\vec{p}_2\to\vec{p}_3,\vec{p}_4$. This scattering rate diverges as $T\to T_c{}^+$ and the total momentum $\vec{K}=\vec{p}_1+\vec{p}_2\to 0$, signaling the formation of stable $\vec{K}=0$ pairs for $T\leq T_c$. For $l=1$ pairing the singular contribution to the scattering rate is[5]

$$2W_{\rm sing}(\vec{k},\vec{k}';\vec{K})=6\pi[4\pi/N(0)]^2|\textstyle\sum_m t_m(K,2\epsilon_k)Y_{1m}(\hat{k})Y_{1m}{}^*(\hat{k}')|^2, \tag{5}$$

$$t_m(K,\omega)=(\theta+\xi_{1m}{}^2K^2+i\pi\beta\omega/8)^{-1} \tag{6}$$

where $N(0)=m^*k_F/2\pi^2$, $\theta=\ln(T/T_c)$, and the coherence lengths are

$$\xi_{10}{}^2=\tfrac{9}{5}\xi_{00}{}^2,\quad \xi_{1\pm1}{}^2=\tfrac{3}{5}\xi_{00}{}^2,\quad \xi_{00}{}^2=[7\zeta(3)/12\pi^2k_F{}^2](T_F/T_c)^2. \tag{7}$$

The z axis for the spherical harmonics in Eq. (5) is parallel to $\vec{K}$, and the momenta $\vec{k}$, $\vec{k}'$, and $\vec{K}$ are related to the momenta $\vec{p}_i$ in Eq. (2) by

$$\vec{p}_1=\vec{k}+\tfrac{1}{2}\vec{K},\quad \vec{p}_2=-\vec{k}+\tfrac{1}{2}\vec{K},\quad \vec{p}_3=\vec{k}'+\tfrac{1}{2}\vec{K},\quad \vec{p}_4=-\vec{k}'+\tfrac{1}{2}\vec{K}. \tag{8}$$

We next make the approximation of keeping only the $l=2$ term in Eq. (2) (the corrections are of order $v_F{}^2/c_0{}^2\ll 1$), and integrate over ϵ_k, $\hat{k}$, and $\hat{k}'$, following Emery's calculation of the transport coefficients.[5] The rate of entropy production then becomes

$$T\dot{s}=(3/80\pi k_BT)\nu_2{}^2(1+\tfrac{1}{5}F_2{}^s)^2\int_0^\infty dK\,K^2\int_{-\infty}^\infty d\epsilon\,\mathrm{sech}^4(\beta\epsilon/2)$$
$$\times\int d\Omega_k\int d\Omega_{k'}|\textstyle\sum_m t_m(K,2\epsilon)Y_{1m}(\hat{k})Y_{1m}{}^*(\hat{k}')|^2[1-P_2(\hat{k}\cdot\hat{k}')]. \tag{9}$$

Although the integrals in (9) all converge with $t_m(K,2\epsilon)$ given by (6), the resulting attenuation is several times larger than observed, presumably because (6) is correct only to lowest order in $\xi_{1m}K$ and $\beta\omega$. To simulate the missing $\xi_{1m}K$ and $\beta\omega$ dependences, we introduce a cutoff in the K integrals, requiring that $K\xi < x_c$ (here ξ is either one of the ξ_{1m} or an appropriate average). For values of x_c consistent with experiment, it is a good approximation to put $\mathrm{sech}^4(\beta\epsilon/2)=1$ in (9); all the integrals can then be done analytically. Using this approximation for $T\dot{s}$, and keeping only the $l=0,1$ components of ν_p in $\delta E_{\rm mech}$ (which is again correct to leading order in $v_F{}^2/c_0{}^2$), we find that the zero-sound attenuation due to order-parameter fluctuations is

$$\delta\alpha_0=\frac{\pi}{5}\frac{v_F{}^2}{mk_Fc_0{}^3}\left(1+\frac{F_2{}^s}{5}\right)^2\frac{a_1}{\xi_{00}{}^3}\left(x_c-\theta^{1/2}\tan^{-1}\frac{x_c}{\theta^{1/2}}\right), \tag{10}$$

where $a_1=3.08$. As $T\to T_c$, the excess attenuation has the limiting form

$$\delta\alpha_0=A-B(T/T_c-1)^{1/2}, \tag{11}$$

with

$$B=\frac{4\pi^5 a_1}{5}\left(\frac{12}{7\zeta(3)}\right)^{1/2}\frac{m^*}{m}\left(1+\frac{F_2^{\,s}}{5}\right)^2\left(\frac{k_BT_c}{\hbar c_0}\right)^3\frac{1}{k_F^{\,2}}, \tag{12}$$

$$A=(2x_c/\pi)B. \tag{13}$$

As previously noted by Emery,[5] the coefficient B is independent of the cutoff x_c. Furthermore, one can easily show that both the limiting form (11) for $\delta\alpha_0$ and the value for B given in (12) follow directly from (6) and (9), with no additional approximations [in particular, without replacing $\mathrm{sech}^4(\beta\epsilon/2)$ by 1]. Hence the functional form of $\delta\alpha_0$ for $T\to T_c$ and the value of the coefficient B are the two theoretical results which can be tested unambiguously against experiment.

For $(T-T_c)/T_c<0.02$, the 30.87-bar data of Paulson and Wheatley are well represented by

$$\delta\alpha_0^{\,\exp}=0.123-0.526(T/T_c-1)^{1/2}. \tag{14}$$

If in (12) we take $T_c=2.69$ mK, $F_2^{\,s}=0$, and use for the remaining constants the values tabulated in Ref. 6 for 30 bar, we find $B=0.651$. Given the uncertainties in the experimental values of m^*/m and T_c, and our neglect of $F_2^{\,s}$, the agreement between theory and experiment seems to be satisfactory. For $l=3$ pairing, a_1 is replaced in (12) by $a_3=11.25$, which gives a value for B that is clearly inconsistent with experiment. If we use the experimental value for B to calculate A from (13), the cutoff needed to fit the measured attenuation at T_c is $x_c=0.37$, which also seems reasonable. However, our cutoff procedure is apparently too crude to account for the behavior of the excess attenuation farther above T_c. To see this, we note that the full result (10) for $\delta\alpha_0$ can be written in terms of the coefficients A and B of the limiting form (11) as

$$\delta\alpha_0=A-(2B/\pi)\theta^{1/2}\tan^{-1}(x_c/\theta^{1/2}), \tag{15}$$

where $x_c=\pi A/2B^\circ$ In Fig. 1 we show this function evaluated with the experimental values of A and B, together with data of Paulson and Wheatley, for temperatures where the data agee well with (11). The discrepancy between Eq. (15) and experiment is substantial. We emphasize, however, that this discrepancy in no way diminishes the significance of the agreement of (11) and (12) with experiment, because these results are cutoff independent. The processes inadequately represented in (15) by the cutoff all lead to corrections to (11) of higher order in θ, and the failure of (15) means only that a simple cutoff overestimates these corrections.

We can also compare Eq. (11) with the frequency and pressure dependences reported by Paulson and Wheatley. Finite-frequency corrections to $\delta\alpha_0$ should be unimportant for $\hbar\omega\ll k_BT_c$, and hence we expect little frequency dependence over the range of 5–25 MHz, in agreement with experiment. For a rough check of the pressure dependence we fix $F_2^{\,s}=0$ and $x_c=0.37$, independent of pressure. We then find $\delta\alpha_0(T_c)/\alpha_0(T_c)=0.03$ at 0.05 bar, where Paulson and Wheatley report $[\delta\alpha_0(T_c)/\alpha_0(T_c)]_{\exp}=0.02$.

The excess zero-sound attenuation calculated here has the same origin as the fluctuation-induced decrease in the viscosity predicted by Emery[5]: Emery's results imply that $\delta\eta(T_c)/\eta(T_c)$ should be comparable to $\delta\alpha_0(T_c)/\alpha_0(T_c)$. The only existing viscosity measurements which might test this prediction are those of Parpia *et al.*[7] Unfortunately, near to T_c these measurements show an unexplained deviation from the normal–Fermi-

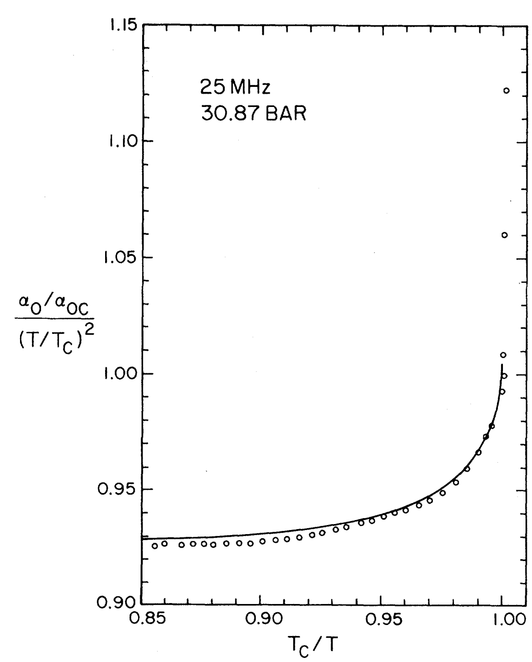

FIG. 1. Comparison of the theoretical zero-sound attenuation (dashed line) calculated from Eq. (15) with the experimental attenuation from Ref. 2 (circles).

liquid T^{-2} behavior. This deviation is several times larger than the expected fluctuation effect, and does not appear to be related to the superfluid transition. As a result, one cannot make a reliable background subtraction to isolate the effects of superfluid fluctuations.[8]

We thank Doug Paulson and John Wheatley for informing us of their preliminary results, and for permission to quote their results prior to publication. We are grateful to V. J. Emery for helpful discussions, and for pointing out an error in a preliminary version of this Letter. This work was supported in part by the National Science Foundation through Grant No. PHY 76-15328.

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Application of the Real-Space Renormalization Group to Dynamic Critical Phenomena

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We present a general approach for applying real-space renormalization-group methods to dynamic critical phenomena. In particular, we discuss the two-dimensional kinetic Ising model treating the interaction between blocks of spins as a small parameter.

We present here a general formulation for applying the real-space renormalization-group (RG) method[1] to critical dynamics. While the analysis will be in terms of two-dimensional kinetic Ising (KI) models,[2,3] we believe the approach to be applicable to higher dimensions and models with order parameters with continuous symmetries.

Consider a set of spins $\{\sigma\}$ on a two-dimensional triangular lattice. We assume that these spins interact via the nearest-neighbor Ising Hamiltonian

$$-H[\sigma]=K\sum_{\langle i,j\rangle}\sigma_i\sigma_j,$$

and the equilibrium probability of having a particular spin configuration $\{\sigma\}$ is given by $P[\sigma]=e^{H[\sigma]}/Z$, where Z is the partition function. In dynamical problems we are interested in the time-correlation functions that can be constructed from the operator

$$G_{\sigma,\sigma'}(t)=\exp(D_\sigma t)\delta_{\sigma,\sigma'}P[\sigma], \tag{1}$$

where $\delta_{\sigma,\sigma'}$ sets $\sigma_i=\sigma_i'$ on all lattice sites, t is time, and $-iD_\sigma$ is the Liouville operator constructed from H for fully microscopic models. If we are dealing with an Ising model, then D_σ must be constructed by hand. We can construct, for example, the spin-spin correlation function from $G_{\sigma,\sigma'}(t)$ by multiplying by σ_i and σ_j' and summing over all σ and σ'.

In this paper we will study the KI model.[2,3] In this case the two primary properties satisfied by D_σ (or $D[\sigma|\sigma']$ in matrix notation) are

$$\mathrm{Tr}^{\sigma'}D[\sigma|\sigma']P[\sigma']=0 \tag{2a}$$

(where $\mathrm{Tr}^{\sigma'}$ means to sum over all spins σ') to