Evidence for Higher-Order Contributions to the Stopping Power of Relativistic Iron Nuclei

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(Received 20 March 1978)

Our measured ranges of 600-MeV/amu ^{56}Fe ions in a variety of substances are shorter than predicted by the standard Bethe stopping-power theory and are consistent with values computed using high-order corrections to the Mott cross section.

Accurate knowledge of stopping power for heavy ions is important for a wide variety of endeavors ranging from cosmic-ray astrophysics to biomedical research to controlled-fusion design. The familiar Bethe expression[1,2] becomes invalid for dealing with particles of high atomic charge because of the failure of the first Born approximation. Although the rigorous extension of the Bethe theory using higher-order Born approximations is a formidable task, considerable success has been achieved with a variety of approximate and semiclassical techniques. Bloch[3] has calculated the close-collision correction to compensate for the assumption of an infinitely broad electron wave packet and Lindhard[4] has summarized the close and distant polarization corrections. Ahlen[5] has emphasized the dominant influence of the deviations of the Mott scattering cross section from the first-Born-approximation cross section on the stopping power of relativistic heavy ions. He has calculated an expression to compensate for this which is accurate for values of $|Z_1|/\beta$ as large as 100, where Z_1 and β are the charge and speed of the ion. Recent data[6] obtained with 1–7-MeV/amu H, He, and Li ions in Al, Cu, Ag, and Au absorbers support the validity of the low-velocity correction of Bloch[3] and Lindhard.[4] We report here the first evidence for higher-order contributions to stopping power which occur in the relativistic regime. This has been accomplished by means of accurate range measurements of 600-MeV/amu ^{56}Fe nuclei obtained at the Lawrence Berkeley Laboratory Bevalac.

Barkas and Berger[7] have tabulated stopping powers and ranges of protons based on the Bethe formula with the shell corrections of Fano and Turner.[8] At energies below 800 MeV/amu, Sternheimer's[9] correction to the range due to the density effect falls below 0.1% for even the lightest of materials.[7] The high-velocity value of the shell correction is included in the value of I_{adj}, the adjusted mean excitation energy used in the Barkas-Berger tables. I_{adj} differs significantly from I, the mean excitation energy used in the Bethe formula, only for materials of high atomic charge.[10] For lead, the difference amounts to 4% in I_{adj}, corresponding to only ~0.6% in total range at 600 MeV/amu. The remainder of the shell correction is negligible at velocities exceeding that of the most tightly bound electrons. At such velocities the residual range is so small that even gross errors in this correction would negligibly affect the total range at 600 MeV/amu.

The usual method of calculating ranges for heavy ions in the absence of high-order corrections is to scale the proton ranges using the equation

$$R(\beta) = MZ^{-2}[\lambda(\beta) + B_Z(\beta)], \quad (1)$$

where M is the mass of the ion in units of the proton mass, Z is the atomic number of the nucleus, $\lambda(\beta)$ is the range of a proton at the same speed β, and $B_Z(\beta)$ is the range extension due to attachment and stripping of atomic electrons.[11] For 600-MeV/amu ^{56}Fe nuclei the range extension is only ~0.4% of the total range for lead and is less than 0.2% for all other substances used in this experiment.

Blocks of various materials used in the range measurements (polyethylene, graphite, aluminum, and lead) were precisely machined and weighted such that 600-MeV/amu ^{56}Fe ions would exit with small residual range and stop in a stack of 125-μm Lexan polycarbonate sheets. In addition, a stack made entirely of Lexan was prepared. Each sheet was cut to a precise size and weighed to provide an accurate measure of the mean path-length (in g/cm^2) for each sheet. During exposure the sheets, but not the absorbers, were tipped at an angle of $30.0° \pm 0.5°$ to facilitate scanning. The uncertainty in the beam direction was less than $\pm 0.2°$. After etching, the mean stopping positions were found by counting the number of nonpenetrating tracks in each sheet. A sample histogram appears in Fig. 1. The distributions are consistent with Gaussian functions plus a small nuclear-interaction background, while the widths can be ascribed to a combination of range straggling,[12] absorber porosity, and energy spread of the beam.

An accurate knowledge of beam energy is es-

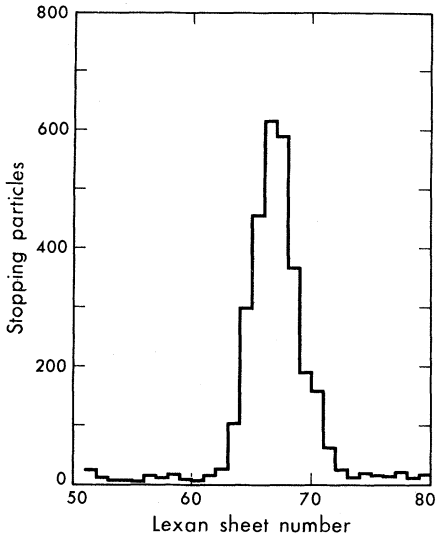

FIG. 1. Histogram of stopping particles as a function of Lexan sheet number for the aluminum absorber.

sential to any stopping-power measurement. The Bevalac provides an excellent spectrometer for this purpose. From measurements of the magnetic field and radio frequency made by the Bevalac operators at the time of this experiment, it was determined[13] that the extracted beam energy was 599.9 ± 2.1 MeV/amu. This uncertainty reflects a natural energy width of ± 0.75 MeV/amu due to phase oscillations, as well as an additional uncertainty determined by known variations of magnetic field and beam radius consistent with successful extraction. An upper limit on the pulse to pulse variation in energy can be set at ~± 1 MeV/amu from the full widths of the straggling distributions. The actual variation is certainly less than this as measurements with other beams indicate.[14] Upon extraction and before entering the experimental area, the beam passes through a variety of matter used in beam monitoring. Table I contains a list of this matter along with estimates of the uncertainties involved.

TABLE I. Material in the beam line.

Description	Composition	Thickness (g/cm^2)
37 SEM Foils	Al	0.201 ± 0.005
21-in. air gap	Air	0.0645 ± 0.0002
	Kapton	0.0356 ± 0.002
Wire chamber	CO_2	0.0029 ± 0.0006
	Ar	0.0105 ± 0.0006
Window	Al	0.0343 ± 0.0034
Window	Mylar	0.046 ± 0.010
	Total	0.395 ± 0.012

A summary of the measured ranges appears in Table II. For each absorber, the mean range is the sum of the absorber thickness (column 5) plus the mean distance traveled in the Lexan stack in terms of an equivalent range in that absorber. Since the Lexan accounts for only a small part of the total range, the equivalent range can be computed with low error using standard range-energy relations. Adjustments due to corrections to stopping power enter only in second order and thus are negligible. Corrections to the range for multiple Coulomb scattering can be found by scaling proton corrections[15] by M^{-1}. For the lead, this correction is only ~0.03% and is much smaller for the other absorbers.

Comparison of these measured ranges with theory requires an independent measure of I_{adj}. Early measurements with high-energy protons[16,17] yielded smaller values for I_{adj} than those presently accepted, which are based on low-energy measurements. This discrepancy has been attributed to an improper deconvolution of the Bragg curve for the high-energy results.[18] This explanation has been recently verified experimentally by Zrelov *et al.*[19] Hence I_{adj} is not a function of energy, and the commonly accepted values are appropriate to the present experiment. Recommended values of I and I_{adj} have been presented by

TABLE II. Absorber data.

Absorber material	I_{adj}	$\langle A/Z \rangle$	ρ (g/cm^3)	Thickness (g/cm^2)	Mean Lexan range (g/cm^2)
Polyethylene CH_2	53.4 ± 5	1.753	0.925	10.81 ± 0.01	0.713 ± 0.004
Lexan $C_{16}H_{14}O_3$	66.9 ± 5	1.896	1.204	•••	12.615 ± 0.066
Graphite (carbon)	78.0 ± 4	2.002	1.749	12.57 ± 0.02	0.961 ± 0.005
Aluminum (2024)	167.9 ± 2	2.080	2.789	13.94 ± 0.02	1.167 ± 0.006
Lead	826.0 ± 25	2.527	11.329	20.68 ± 0.06	1.311 ± 0.007

many authors over the past twenty years.[2,20-24] A review of this literature has persuaded us to use the values listed in Table II. Pieces of the aluminum, lead, and graphite were spectroscopically analyzed to determine the level of impurities. The lead and graphite were better than 99.9% pure so that adjustments in I_{adj} were unnecessary. Bragg's rule was used to compute I_{adj} and $\langle A/Z\rangle$ for the aluminum absorber (which contained 4% Cu by weight and several other impurities). Values of I_{adj} for Lexan and polyethylene were computed by using the values of Ref. 20 which are based on the work of Thompson[25] involving the deviations from the Bragg rule ($\lesssim 1\%$ in stopping power) for condensed organic compounds. The errors in I_{adj} in Table II reflect observed experimental fluctuations in measurement and interpretation of stopping-power data.

Measured values for total ranges are compared in Table III with theoretical values computed in three different ways.

(a) Using standard range-energy relations as elaborated in Ref. 7, the energy of the beam after passage through the material in the beam line in front of the absorbers (given in Table I) was calculated to be 589.9 ± 2.1 MeV/amu, and ranges were computed using this energy as the starting point. See column 3.

(b) Using only the Mott corrections to stopping power in Ref. 5, the 10.0 MeV/amu lost in the upstream matter was corrected to 10.3 MeV/amu yielding an energy of 589.6 ± 2.1 MeV/amu going into the absorbers. A range $R(E)$ at this energy was computed for each absorber using the standard range-energy relations. Then a fractional correction factor,

$$f = 1 - \frac{\int_{E_0}^{E} dE\ (dE/dx)'/(dE/dx)^2}{R(E)}, \qquad (2)$$

was computed and applied to each calculated range. Here dE/dx is the stopping power computed according to the Bethe expression, $(dE/dx)'$ is the additional stopping power due to only the Mott corrections in Ref. 5, and E_0 is an energy small enough (~ 50 MeV/amu) so that the absolute correction at this energy is small compared to that at ~ 600 MeV/amu. See column 4.

(c) The same calculation in (b) was repeated using both the Mott and Bloch corrections of Ref. 5. See column 5.

In (b) and (c) the values used for ϵ_0, the square-root average excitation energy in Ref. 5, were 88, 112, 122, 325, and 1180 eV for the polyethylene, Lexan, graphite, aluminum, and lead, respectively. These were computed using the oscillator strengths and energy levels of Sternheimer.[9] Calculations show that f is extremely insensitive to large variations in ϵ_0.

Errors in calculated range in Table III were computed by adding in quadrature the errors in range caused by the uncertainty of 2.1 MeV/amu in beam energy and those caused by errors in I_{adj} from Table II. Errors in measured ranges were dominated by angular uncertainties.

The measured ranges are inconsistent with ranges computed using only the Bethe expression with corrections for shell effects and electron attachment and stripping. We conclude that the Mott correction to the Bethe expression is important for fast heavy ions. The polarization correction of Lindhard[4] improves agreement for lead insofar as it contributes $\sim 0.9\%$ to the range deficit. This contribution is less than 0.1% for the other absorbers. Inclusion of the nonrelativistic Bloch correction worsens the agreement between calculated and measured ranges. This disagreement is slightly less than two standard deviations and thus we are unwilling to reject this correction on the basis of these data.

We wish to thank J. Cooper and D. Schmidt for their assistance in scanning and S. P. Ahlen and P. B. Price for helpful discussions and for a critical reading of this manuscript. We thank K. Crebbin for performing calculations regard-

TABLE III. Measured and calculated ranges.

Material	Measured	Range (g/cm^2) Bethe	Mott	Mott + Bloch
Polyethylene	11.447 ± 0.010	11.76 ± 0.16	11.43 ± 0.16	11.55 ± 0.16
Lexan	12.615 ± 0.066	13.04 ± 0.13	12.68 ± 0.13	12.82 ± 0.13
Graphite	13.601 ± 0.021	14.05 ± 0.11	13.64 ± 0.11	13.80 ± 0.11
Aluminum	15.397 ± 0.022	15.96 ± 0.09	15.46 ± 0.09	15.66 ± 0.09
Lead	23.371 ± 0.062	24.42 ± 0.18	23.55 ± 0.18	23.93 ± 0.18

ing the Bevalac beam energy. This work was supported by the National Aeronautics and Space Administration, Grant No. NGR 05-003-376.

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Strain-Coupled Charge-Density Waves in a Linear Conductor

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X-ray studies of tetrathiafulvalenium-thiocyanate [$TTF(SCN)_{0.588}$] show a Peierls instability with one-dimensional charge-density waves near 300 K followed by three-dimensional order at $T_c = 180$ K. Below T_c, an interchain competition between both elastic forces and Coulomb interactions drives a temperature-dependent monoclinic strain. A Landau theory including both charge-density and strain order parameters describes the relative TTF chain slip and the superlattice phase variation.

Two classes of quasi one-dimensional (1D) conductors are unstable against a Peierls distortion.[1-3] The first[4] has conducting chains, such as the Pt-atom chain in $K_2[Pt(CN)_4]Br_{0.3}\cdot 3H_2O$ (KCP), from which charge is transferred to nonconducting chains (e.g., Br in KCP). The second, exemplified by the tetracyanoquinodimethane salt TTF-TCNQ (tetrathiafulvalene-tetracyanoquinodimethane),[5] has both donor and acceptor stacks which are conducting.

There is another class of linear organic compounds which is similar to KCP in that they consist of a single set of conducting chains with charge transfer to a set of nonconducting chains