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Ion-Ring Igniter for Inertial Fusion

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This paper outlines a scheme employing magnetically compressed ion rings for the transport of energy to a deuterium-tritium pellet.

Several candidates, e.g., lasers,[1] relativistic electrons beams,[2] intense beams of light ions in the 10-MeV range,[3] and 25–100-GeV heavy-ion beams,[4] have been suggested for the high-powered energy source or "ignitor" required for pellet fusion. In this Letter I present a scheme that employs ion rings for energy compression and transport to the pellet.

The basic idea is to inject a pulse of ions into a magnetic mirror, trap these ions in the form of an ion ring, and magnetically compress the ion ring to increase its energy and reduce its dimensions. The compressed ion ring is accelerated axially to impact a D-T pellet. Figure 1 shows a schematic of the proposed system. A pulse of ions from a magnetically insulated annular ring diode[5] is injected through a cusp-shaped magnetic field. By virtue of the conservation of the canonical angular momentum the ions begin to rotate and give rise to a θ current. Previous experimental work[6] has demonstrated that (i) intense ion beams are charge neutralized by electron flow along field lines, (ii) such electrons are created at nearby boundary surfaces, (iii) charge-neutralized intense ion beams propagate across the field in ballistic cyclotron orbits, and (iv) magnetic neutralization of the circulating ion current by azimuthally drifting electrons does not take place because the radial electric field required for this drift is shorted out by electron flow along field lines. The rotating beam is expected to be trapped between mirrors M_1 and M_2 because of axial momentum loss by dissipation induced in surrounding structures.[7-9]

Magnetic compression of the trapped ring is achieved in two stages. The first stage utilizes superconducting coils that generate a spatially increasing field from, say, 10–20 kG to approximately 100 kG determined by the state of the art. However, pulsed coils will be required to trans-

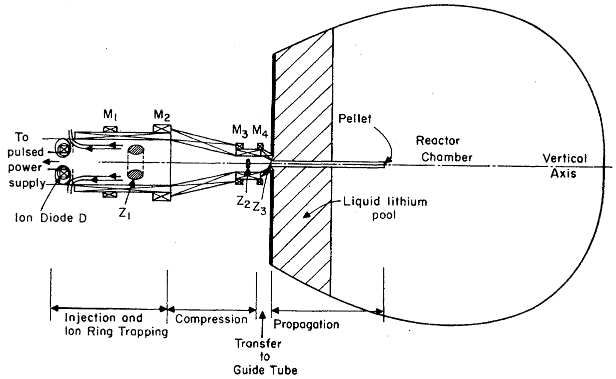

FIG. 1. Schematic of ion-ring–pellet fusion reactor.

port the ion ring to regions of high magnetic field. The second state employs a pulsed system to increase the field to, say, between 0.5 and 1 MG. One possible reactor configuration[10] is to employ the LINUS[11] scheme which depends upon an imploding, rotating, liquid lithium liner to compress magnetic flux and also to absorb the products of nuclear fusion. For sufficiently high-gain (Q) pellets it may not be necessary to recover the expansion energy of the lithium liner which in any case is Rayleigh-Taylor unstable during this phase. Furthermore, the LINUS system in this application would be much smaller in power and energy requirements than a system for magnetic confinement of the reacting plasma.

An alternative reactor configuration (shown in Fig. 1) is to propagate the compressed ring to the pellet by opening mirror M_4. The outward radial expansion of the ring due to the decrease of the axial field is prevented by making the wall radius decrease with axial distance in such a manner that the image currents at the wall provide the radial ring equilibrium. As the ring moves across field lines, electrons on open field lines flow readily to the wall while new electrons flow towards the ring to maintain space-charge neutralization. The pellet in Fig. 1 is located at the end of a guide tube fabricated out of solid lithium which protrudes some distance above a pool of lithium on the floor of a lithium-fall type of reactor chamber. Presumably the guide tube is destroyed in each shot and becomes part of the lithium pool, and a new tube is inserted by appropriate machinery. This scheme has the advantage that the ring propagates in vacuum while any ambient environment suitable for reactor operation can be maintained in the chamber. Furthermore, a single ion ring can be designed to have enough energy for pellet ignition so that the synchronous firing of many beams is not required.

I now give some quantitative estimates of the energy and power density that can be delivered to the pellet in this scheme.

For a magnetically insulated ion diode with a plasma anode emitting space-charge-limited ion current I can easily establish that the number of ions produced is

$$N = 7.23\times 10^{18}(\eta K_1\rho_1/K_2^2)A^{1/2}(V/Z)^{3/2}\tau, \qquad (1)$$

where $K_1 = 1-5$ is an enhancement factor[12] over the Child-Langmuir current, $K_2 \sim 2$ is the factor by which the magnetic field has to be above the critical field for electron turn around, $\rho_1 = r_1/R_1$ is the inverse aspect ratio of the ring at z_1 (Fig. 1), the anode emitting area $\sim 4\pi\rho_1 R_1^2$ approximates the ring dimensions, η is the trapping efficiency, Ze and A are the charge and atomic mass number of the ion, V is the diode voltage in megavolts, and τ is the pulse time in microseconds.

The adiabatic compression of charge-neutralized ion rings has been treated by Sudan and Ott[10] and later by Weibel[13a] who employed a Lagrangian formalism for including the self-fields and more recently by Lovelace[13b] using the Vlasov equation. Here I follow a simple development along the lines of Refs. 10 and 13a. The conservation of canonical angular momentum furnishes

$$R^2\Omega\left\{1+\frac{Nr_i}{\pi R}\left[\ln\left(\frac{R}{a}\right)-\frac{1}{2}\right]\right\}-\tfrac{1}{2}\omega_c R^2 = P_\theta \equiv \text{const}, \qquad (2)$$

where N is the number of ions in the ring of major radius R and minor radius r, $a = r/\nu$ ($\ln\nu = \ln 8 - \frac{5}{4}$), $\omega_c = qB/mc$, B is the external field, $q = Ze$, $r_i = q^2/mc^2$ the classical ion radius, and Ω is the rotational frequency of the ions. The radial force balance of the ring leads to

$$\omega_c = \Omega[1+(Nr_i/2\pi R)\ln(R/a)]. \qquad (3)$$

The balance of ring thermal pressure and the self-pinching magnetic force leads to

$$Ia = (Nq\Omega/2\pi c)a = \text{const}, \qquad (4)$$

where it has been assumed that the two-dimensional poloidal compression of the ring is governed by an adiabatic exponent $\gamma = 2$. The total ring energy W consists of the kinetic energy of rotation, $W_r = \frac{1}{2}NmR^2\Omega^2$, the thermal energy, $W_t = NkT$, and the self-magnetic energy, $W_m = \frac{1}{2}LI^2$ where L is the ring inductance. It is straightforward to show from Eqs. (2)–(4) that

$$\frac{W}{B} = \frac{1}{2}\frac{q}{c}R^2\Omega\frac{[1+(Nr_i/\pi R)\ln(R/a)]}{[1+(Nr_i/2\pi R)\ln(R/a)]}, \qquad (5a)$$

$$W_m/W_r \simeq [1+(Nr_i/\pi R)\ln(R/a)], \quad W_t/W_r \simeq Nr_i/2\pi R. \qquad (5b)$$

For weak rings $(Nr_i/2\pi)\ln(R/a) \ll 1$, and I note that $\omega_c \simeq \Omega$, and that R^2/a, $R^2\Omega$, and W/B are approximately constant. For strong rings $(Nr_i/2\pi)\times\ln(R/a) \gg 1$ we obtain $R\Omega\ln(R\Omega/a_0\Omega_0) \simeq$ const, $(R/a)\ln(R/a) \simeq$ const, $R^2\omega_c \simeq$ const, and $W \propto \Omega$.

A critical issue is the transfer of the ion ring from being held by the external field to being supported by the wall image currents. A "back of the envelope" calculation of this stage proceeds along the following lines. After the ring is fully compressed, the mirror field M_4 is switched off. Let us assume that the ring is adiabatically transferred by a set of pulsed coils from z_2 to z_3 where the external field is negligible but the confining conducting walls come much closer to the ring. The radial force balance at z_3 where $B \simeq 0$ is given by

$$\frac{Nm}{2\pi}\Omega^2 = -\frac{I^2}{4\pi R}\frac{dL_3}{dR}, \tag{6}$$

where the ring inductance at z_3 is approximated by $L_3 \simeq 8\pi R \ln(b/a)$ where $b = \alpha(R_W - R)$, $\ln\alpha = \ln 2 + \frac{1}{8}$, and R_W is the wall radius. From (6) we obtain

$$b = (Nr_i/\pi)[1 - (b/R)\ln(b/a)]. \tag{7}$$

The conservation of angular momentum and total energy lead to

$$R_2^2\Omega_2\{1 + (Nr_i/\pi R_2)[\ln(R_2/a_2) - \tfrac{1}{2}]\} - \tfrac{1}{2}\omega_{c2}R_2^2 = R_3^2\Omega_3[1 + (Nr_i/\pi R_3)\ln(b_3/a_3)], \tag{8}$$

and

$$V_{z2}^2 + R_2^2\Omega_2^2[1 + (Nr_i/\pi R_2)\ln(R_2/a_2)] = V_{z3}^2 + R_3^2\Omega_3^2[1 + (Nr_i/\pi R_3)\ln(b_3/a_3)], \tag{9}$$

where V_{z2} is any axial velocity imparted to the ring by the pulsed coils. For $R_2 \simeq R_3$ we obtain from (8) and (3)

$$\Omega_3 = \tfrac{1}{2}\Omega_2 \frac{1 + (3Nr_i/2\pi R_2)\ln(R_2/a_2)}{1 + (Nr_i/\pi R_2)\ln(b_2/a_2)}. \tag{10}$$

From Eqs. (1) and (7) we observe that a_3 increases over a_2.

Because of the close proximity of the ring to the walls of the guide tube during the propagation phase, resistive losses from the image currents are to be expected. A simple calculation which assumes that the image currents flow in a skin depth in the wall leads to

$$W_L = 7.0\times 10^{-4}\frac{R_W}{\alpha R}\left(\frac{\rho}{\alpha R V_z}\right)^{1/2}\lambda^2 I^2 \text{ J/cm}, \tag{11}$$

where I is the ring current in amperes, R_W is the tube radius in centimeters, ρ is the wall resistivity in Ω-cm, λ is of order $\ln(b/a)$, V_z is in cm/sec; αR is the axial extension of the image currents with $\alpha \sim 1$. The pressure on the guide tube wall in the neighborhood of the ring is $\propto I^2$ and indeed this may be of the order of $B^2/8\pi$ where B is the compressed value of the field at z_2 (Fig. 1). This pressure may exceed the bursting strength of the lithium guide tube. Therefore, the ring axial velocity V_z must exceed the sound velocity in lithium in order that the mechanical failure of the tube occurs after the ring has impacted the pellet. The ring axial velocity is adjustable by choosing R_3/R_2. Thus the ring is confined "inertially" during the propagation phase through the tube.

Since the particle thermal velocity V_T can be greater than the θ velocity $V_\theta = R\Omega$ for the compressed ion ring, a pellet of radius r_p smaller than the ring minor radius r will be uniformly illuminated by the ions. The characteristic time for energy delivery is estimated to be $\tau_E^{-1} \simeq (r_p^2/2\pi R a^2)V_T$, and the terminal axial ring velocity V_z should be such that $V_z\tau_E < r_p$. There will be no buildup of space charge at the pellet because the ion ring is charge neutralized by the background electrons. However, as the ions are lost to the pellet an induction electric field builds up to maintain the magnetic flux which accelerates the remaining ions. Under the reasonable assumption that the particle mechanical momentum is absorbed by the pellet but the electromagnetic momentum is shared by the remaining ions, we obtain

$$\tau_e(V_\theta)^{-1}\frac{dV_\theta}{dt} = \frac{2(Nr_i/\pi R)\ln(R/a)}{1 + (Nr_i/\pi R)\ln(R/a)} \simeq 2,$$

during most of ring-pellet interaction phase because $(Nr_i/\pi R)\ln(R/a) > 1$. Thus the e-folding time for V_θ is around τ_E and since $V_T > V_\theta$ initially, our previous estimate for τ_E is still reasonable. There will, however, be an increase with time in the power delivered to the pellet and the effective ion kinetic energy E_{eff} during pellet interaction can be approximated by W/N.

Table I gives a sample calculation for α particles. The range for 30-MeV α particles is ~ 0.14 g, and for a 3-mm-radius pellet at 20 MJ/g we need about 2.3 MJ. The energy delivery time is $\tau_E \simeq 20$ nsec and the wall loss $W_L \lesssim 100$ kJ/m. The figures in Table I show that a reasonable case could be made for ion rings as a technique for energy compression and propagation to the pellet target. Heavier ions can also be employed but they must be highly stripped so that Z/A is not much smaller than for α particles.

There are several advantages that may be claimed in favor of ion rings for energy compression and transport:

TABLE I. Sample parameters for ion-ring compression.

	Initial state	Compressed state
Number of α particles, N	6×10^{17}	6×10^{17}
Magnetic field, B	18.33 kG	0.66 MG
Total ring energy, W	30.2 kJ	$\sim$ 3.0 MJ
Effective ion energy, W/N	3.15 MeV	30 MeV
Major radius, R	18 cm	3 cm
Minor radius, r	4.1 cm	0.6 cm
Circulating frequency, Ω	3.2×10^{7} rad/sec	0.26×10^{9} rad/sec
Mean θ velocity, $R\Omega$	0.58×10^{9} cm/sec	0.78×10^{9} cm/sec
Thermal velocity, V_T	0.52×10^{9} cm/sec	1.53×10^{9} cm/sec

(1) The system has high efficiency because most of the ring energy is derived from magnetic compression. The technology to deliver ion beams of the requisite parameters for injection is available and experiments to trap and compress ion rings are already under way at Cornell University and the U. S. Naval Research Laboratory.

(2) The system is straightforward in principle and should cost substantially less and is also much smaller than elaborate high-energy accelerators for heavy ions.

(3) It is easier to concentrate the energy into a smaller volume by means of magnetic compression than by focusing of ion beams.

(4) Transport is realized in a good vacuum and plasma collective effects should therefore be reduced to a minimum.

There is of course the problem of providing a lithium guide tube after every shot or resorting to a LINUS-type compression scheme. But in any case, this is much less onerous and expensive than the recent concept[14] of propagating electromagnetic energy to the pellet in a vacuum waveguide that is replaced after each shot. Questions of ring stability[15,16] during compression, transfer to guide tube, and propagation phases are important and the theoretical framework for examining them has been recently established. However, it must be recognized that these problems are less severe here than in schemes in which a field-reversed ion ring has to magnetically confine a thermonuclear plasma for a Lawson time.[10,17]

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Positive-Ion Structure in a Tricritical ^{3}He-^{4}He Mixture

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The mobilities of positive and negative ions have been measured in the normal phase of a ^{3}He-^{4}He mixture near the tricritical point. Whereas for negative ions nearly the same weak temperature dependence in observed as in pure ^{3}He, the mobility of positive ions in the mixture drops sharply by almost a factor of 2 when phase separation is approached. This is attributed to condensation of ^{4}He onto the positive-ion surface.

Ions have served as a very useful probe to study the properties of liquid helium, like recently the superfluid phases of ^{3}He.[1] The structure of these ions, both negative ("electron bubbles") and positive ("snowballs"), seems to be well understood.[2] Yet for a particular case, namely the mobility of positive ions in ^{3}He, experimental data have shown a surprisingly large scatter at temperatures around 0.2 K.[3-5] Roach, Ketterson, and Roach[6] and Bowley[7] ascribed this lack of reproducibility to minute traces of ^{4}He atoms, which condense on the snowball at low temperatures, leading to an increase in the radius of the positive ion and hence to a lower ion mobility. This modified structure of the positive ions in very dilute mixtures of ^{4}He in ^{3}He was investigated in more detail by Sluckin.[8] According to his theory phase separation occurs in the mixture in the vicinity of the ion, with a halo of ^{4}He-rich phase around the ion, separated from the surrounding ^{3}He-rich phase by an additional phase boundary. The halo radius R_H is predicted to grow as the temperature is lowered, until it reaches its final value at a temperature where macroscopic phase separation sets in the bulk liquid.

In a tricritical ^{3}He-^{4}He mixture, where the concentration of ^{4}He is considerably higher ($x_4 = 0.325$), one would expect a similar behavior. Since phase separation in such a mixture occurs at a much higher temperature ($T_t = 0.867$ K) than for the dilute mixtures studied so far, the halo too should form at higher temperatures, thus lowering the mobility of the positive ion in a temperature range where until now no anomalies have been observed.

We have investigated the ion mobility μ in such a tricritical mixture between 0.5 and 1.1 K. A time-of-flight method was used similar to that described by Ahonen *et al.*[1] The ion current from an ^{241}Am source was shaped into pulses by means of a gate grid, and the velocity of these ion packets in a drift space was determined from the arrival time at the collector. The ion pulses were sampled by means of a fast electrometer and a signal averager. Since the drift space was located in the upper half of the sample cell, all data points were taken in the *normal* phase of the mixture; the ^{4}He concentration x_4 was constant above the tricritical point, and decreased along the ^{3}He-rich side of the phase diagram below T_t to $x_4 = 0.09$ at 0.5 K.

The results for the mobilities in this tricritical mixture are plotted in Figs. 1 and 2. For positive ions (Fig. 1), a prominent drop of μ_+ by 40% is observed as the tricritical point is approached from above. Below T_t the mobility shows but a small further decrease down to 0.5 K. In contrast, for negative ions no such step near T_t was found, as the data in Fig. 2 indicate. The measured mobilities were independent of the applied electric drift field E_D in the studied range, $5 \leq E_D \leq 80$ V/cm, and there was no indication for several species of positive charge carriers with different mobilities.[7,9]

A comparison of our data with earlier results for pure ^{3}He, and also for dilute mixtures of ^{4}He in ^{3}He (which do not deviate from pure ^{3}He for $T > 0.5$ K) lead to the following conclusions: For *negative* ions the mobility in the tricritical mixture—like in pure ^{3}He[2,10]—can be well described in terms of Stokes's law for temperatures above

