predicts a much larger asymmetry. Our data are inconsistent with the predictions given in Ref. 7. The model of Elias, Pati, and Salam[8] uses the $SU(4)^4$ symmetry. With the parameters given in their paper, our data restrict the mass of the neutral boson to be greater than 55 GeV.

Previously a limit on M_Z was set by looking at the energy dependence of the neutrino neutral-current cross section. Using data from a single experiment, this gives $M_Z > 3$ GeV.[9] Comparison between the two experiments is made difficult by problems of relative normalization. The limit placed in this way is $M_Z > 10$ GeV.[10]

In conclusion, we see no asymmetry in the reaction $e^+e^- \rightarrow \mu^+\mu^-$ other than that caused by second-order QED, thus setting a lower limit on the ratio of mass to coupling constant for a neutral vector boson.

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Linearizing the Volkov-Akulov Model

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The nonlinear realization of supersymmetry of Volkov and Akulov is related to a constrained linear realization in two and four dimensions.

One of the earliest realizations of supersymmetry was the nonlinear model of Volkov and Akulov[1]; this was soon followed by the linear scalar multiplet of Wess and Zumino.[2] It has been asserted[3] that the two models are related in the same sense that the nonlinear σ model is related to linear realizations of $O(n)$ symmetry. In this Letter I show that this is indeed the case; specifically, I show that if one applies the correct supersymmetrically invariant constraints to the linear multiplet, one is left with the Volkov-Akulov model.

I first consider the two-dimensional case[4]: The linear scalar multiplet consists of two scalar fields A and F and a Majorana spinor ψ with transformations

$$\delta A = \bar\epsilon\psi, \quad \delta\psi = (F - i\partial\!\!\!/ A)\epsilon, \quad \delta F = -\bar\epsilon i\partial\!\!\!/\psi. \tag{1}$$

A supersymmetric action for these fields is

$$S_L = \tfrac{1}{2}\int d^2x[(\partial A)^2 + \bar\psi i\partial\!\!\!/\psi + F^2]. \tag{2}$$

This can be rewritten in superspace notation by introducing a superfield Φ,

$$\Phi = A + \bar\theta\psi + \tfrac{1}{2}\bar\theta\theta F. \tag{3}$$

Then

$$S_L = \tfrac{1}{4}\int d^2x d\bar\theta d\theta(\Phi\bar P P\Phi), \tag{4}$$

where $P = \partial/\partial\bar\theta - i\partial\!\!\!/\theta$. The supersymmetry transformations (1) are generated by $Q = \partial/\partial\bar\theta + i\partial\!\!\!/\theta$, $\delta\Phi = [\bar\epsilon Q, \Phi]$.

The Volkov-Akulov model in two dimensions consists of a single Majorana spinor λ with transformation

$$\delta\lambda = \epsilon - (\bar\epsilon i\gamma_\mu\lambda)\partial_\mu\lambda \tag{5}$$

and invariant action

$$\begin{aligned} S_{VA} &= +\tfrac{1}{2}\int d^2x \det(\eta_{a\mu} - \bar\lambda i\gamma_a\partial_\mu\lambda) \\ &\equiv \tfrac{1}{2}\int d^2x \det(V_{a\mu}) \\ &= -\tfrac{1}{2}\int d^2x[1 - \bar\lambda i\partial\!\!\!/\lambda - \tfrac{1}{2}\bar\lambda\lambda\epsilon_{\mu\nu}(\partial_\mu\bar\lambda\gamma_5\partial_\nu\lambda)]. \end{aligned} \tag{6}$$

(A dimensioned constant[5] that usually appears in the Volkov-Akulov transformation and action has been set equal to 1.)

The relation between the two models is

$$A=\tfrac{1}{2}\bar{\lambda}\lambda, \tag{7a}$$

$$\psi=\lambda(1-\bar{\lambda}i\partial\!\!\!/\lambda), \tag{7b}$$

$$F=-\det(V_{a\mu}). \tag{7c}$$

The three constraints (7) are supersymmetrically invariant; that is, they are consistent with the transformations (1) and (5). Upon substituting the constraints (7) into the linear action (2), one discovers that $S_L \propto S_{\rm VA}$.

In superspace the results (7) can be written as

$$\Phi=-\tfrac{1}{2}\det(V_{a\mu})(\bar{\lambda}+\bar{\theta})(\lambda+\theta). \tag{8}$$

These results were obtained in the following manner: The *Ansatz* was made that $\psi=\lambda+\dots$; this, combined with the transformations (1) and (5), leads immediately to $A=\frac{1}{2}\bar{\lambda}\lambda+\dots$ and $F=1-\bar{\lambda}i\partial\!\!\!/\lambda+\dots$; but this, in turn, implies the complete result (7). When I considered another *Ansatz*, such as $\psi=i\partial\!\!\!/\lambda+\dots$, I immediately found a contradiction, suggesting that (7) may well be unique.

It is simple to invert (7b) and find $\lambda=\psi(1+\bar{\psi}i\partial\!\!\!/\psi)$; then (7a) and (7c) express A and F in terms of ψ, and one can ask if there are simple supersymmetrically invariant constraints involving only A, ψ, and F, that lead to (7a) and (7c). In superspace these constraints are

$$\Phi^2=0 \text{ and } \Phi=-\tfrac{1}{2}\Phi\bar{P}P\Phi, \tag{9a}$$

which can be written out in terms of A, ψ, and F:

$$AF=\tfrac{1}{2}\bar{\psi}\psi,\quad A^2=0,\quad A\psi=0,\quad A=AF,$$
$$(F-Ai\partial\!\!\!/)\psi=\psi, \text{ and } F^2-A\Box A+\bar{\psi}i\partial\!\!\!/\psi=F. \tag{9b}$$

Clearly, the six constraints (9b) are not independent.

Finally, note that these constraints (9) imply that any action of the form

$$\tfrac{1}{4}\int d^2x\, d\bar{\theta}\, d\theta[-\tfrac{1}{2}Q(\Phi\bar{P}P\Phi)+V(\Phi)] \tag{10}$$

reduces to

$$\tfrac{1}{4}\int d^2x\, d\bar{\theta}\, d\theta(q_1+v_1)\Phi=(q_1+v_1)S_{\rm VA}$$

upon substitution of (8) into (10). (q_1 and v_1 are the coefficients of the linear terms in the Taylor's expansion of Q and V, respectively.)

The two-dimensional case is simple and suggestive but basically uninteresting from a physical point of view; therefore, I investigated the four-dimensional case. Although the computations are much more complicated than in the two-dimensional case, the results are largely analogous. In four dimensions,[6] the scalar multiplet consists of a Majorana spinor ψ, two scalars A and F, and two pseudoscalars B and G. The transformations are

$$\delta A=\bar{\epsilon}\psi,\quad \delta B=\bar{\epsilon}i\gamma_5\psi,\quad \delta F=-\bar{\epsilon}i\partial\!\!\!/\psi,$$
$$\delta G=\bar{\epsilon}\gamma_5\partial\!\!\!/\psi, \text{ and } \delta\psi=[(F+i\gamma_5G)-i\partial\!\!\!/(A+i\gamma_5B)]\epsilon. \tag{11}$$

An invariant action is

$$S_L=\tfrac{1}{2}\int d^4x[(\partial A)^2+(\partial B)^2+\bar{\psi}i\partial\!\!\!/\psi+F^2+G^2]. \tag{12}$$

It is useful to define complex fields

$$R\equiv\frac{1+\gamma_5}{2},\quad L\equiv\frac{1-\gamma_5}{2},\quad \mathcal{A}\equiv\frac{A-iB}{2},\quad \mathcal{F}\equiv\frac{F+iG}{2}, \tag{13a}$$

$$\delta\mathcal{A}=\bar{\epsilon}R\psi,\quad \delta\mathcal{F}=-\bar{\epsilon}i\partial\!\!\!/R\psi,\quad \delta R\psi=2R(\mathcal{F}-i\partial\!\!\!/\mathcal{A})\epsilon, \tag{13b}$$

$$S_L=\tfrac{1}{2}\int d^4x\{4[(\partial\mathcal{A})(\partial\mathcal{A}^*)+\mathcal{F}\mathcal{F}^*]+\bar{\psi}i\partial\!\!\!/R\psi+\bar{\psi}i\partial\!\!\!/L\psi\}. \tag{13c}$$

In terms of these we can define chiral superfields[7]

$$\Phi_+=\mathcal{A}+\bar{\theta}R\psi+\bar{\theta}R\theta\mathcal{F}+i\bar{\theta}\gamma^\mu R\theta\partial_\mu\mathcal{A}-\tfrac{1}{2}(\bar{\theta}\theta)(\bar{\theta}i\partial\!\!\!/R\psi)-\tfrac{1}{8}(\bar{\theta}\theta)^2\Box\mathcal{A},$$
$$\Phi_-=\mathcal{A}^*+\bar{\theta}L\psi+\bar{\theta}L\theta\mathcal{F}^*+i\bar{\theta}\gamma^\mu L\theta\partial_\mu\mathcal{A}^*-\tfrac{1}{2}(\bar{\theta}\theta)(\bar{\theta}i\partial\!\!\!/L\psi)-\tfrac{1}{8}(\bar{\theta}\theta)^2\Box\mathcal{A}^*. \tag{13d}$$

Then supersymmetry transformations are generated as in two dimensions, and the action can be written as

$$S_L=\tfrac{1}{4}\int d^4x[\int d\bar{\theta}R\,d\theta\Phi_+\bar{P}P\Phi_-+\int d\bar{\theta}L\,d\theta\,\Phi_-\bar{P}P\Phi_+]. \tag{13e}$$

To facilitate calculation, I used a slight generalization of the Volkov-Akulov model with transformation[8]

$$\delta\lambda=\epsilon-i\bar{\epsilon}\gamma^\mu(1+\alpha i\gamma_5)\lambda\partial_\mu\lambda \tag{14}$$

and invariant action

$$S_{\rm VA}=+\tfrac{1}{2}\int d^4x\det[\eta_{a\mu}-\bar{\lambda}i\gamma_a(1+\alpha i\gamma_5)\partial_\mu\lambda]\equiv+\tfrac{1}{2}\int d^4x\det(V_{a\mu}). \tag{15}$$

This is a parity-nonconserving realization of the conventional super-Poincaré algebra. (Note that $[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = 2i\bar{\epsilon}_2\gamma^\mu\epsilon_1\partial_\mu$ as always.)

The relation between the two models is

$$
\begin{aligned}
\mathcal{A} &= \tfrac{1}{2}\bar{\lambda}R\lambda[1 - i(1+i\alpha)\bar{\lambda}\not{\partial}\lambda] - \tfrac{1}{4}(\bar{\lambda}\lambda)^2(1+i\alpha)^2(\partial_a\bar{\lambda}R\sigma^{ab}\partial_b\gamma),\\
\mathcal{F} &= \tfrac{1}{2} - i\bar{\lambda}\not{\partial}R\lambda - (1+i\alpha)(\bar{\lambda}L\lambda)(\partial_a\bar{\lambda}R\sigma^{ab}\partial_b\lambda)\\
&\quad + (1-i\alpha)[(\bar{\lambda}\gamma_a L\lambda)(\partial^a\bar{\lambda}\not{\partial}R\lambda) + (\bar{\lambda}\gamma_a R\partial_b\lambda)(\bar{\lambda}\gamma^b L\partial^a\lambda) - \tfrac{1}{4}(\bar{\lambda}L\lambda)\Box(\bar{\lambda}R\lambda)]\\
&\quad + (1+\alpha^2)(\bar{\lambda}L\lambda)\, i[(\bar{\lambda}\not{\partial}L\lambda)(\partial_a\bar{\lambda}R\sigma^{ab}\partial_b\lambda) - 2(\bar{\lambda}\gamma_a L\partial_b\lambda)(\partial^a\bar{\lambda}R\sigma^{bc}\partial_c\lambda)],\\
R\psi &= R\Big(\lambda[1 - i(1+i\alpha)\bar{\lambda}\not{\partial}L\lambda - (1+i\alpha)^2(\bar{\lambda}L\lambda)(\partial_a\bar{\lambda}R\sigma^{ab}\partial_b\lambda)] + (1-i\alpha)\gamma^a\lambda(\bar{\lambda}R\partial_a\lambda)\\
&\quad + \tfrac{1}{2}(1+\alpha^2)\{(\bar{\lambda}L\lambda)[\tfrac{1}{2}\Box\lambda(\bar{\lambda}R\lambda) - \tfrac{1}{2}\lambda(\partial_a\bar{\lambda}R\gamma^a\not{\partial}\lambda) - 2\sigma^{ab}\lambda(\partial_a R\sigma_{bc}\partial^c\lambda)]\\
&\quad - (\bar{\lambda}R\lambda)[\gamma^a\lambda(\partial_a\bar{\lambda}\not{\partial}R\lambda) + \gamma^a\partial_b\lambda(\bar{\lambda}\gamma^b R\partial_a\lambda)]\}\\
&\quad - \tfrac{1}{4}i(1+i\alpha)(1+\alpha^2)(\bar{\lambda}\lambda)^2[\not{\partial}\lambda(\partial_a\bar{\lambda}R\sigma^{ab}\partial_b\lambda) - 2\gamma^a\partial_b\lambda(\partial_a\bar{\lambda}R\sigma^{bc}\partial_c\lambda)]\Big). \qquad (16)
\end{aligned}
$$

Although it is not obvious in this form, $\mathcal{F}$ is proportional to $\det(V_{a\mu})$ plus extra total derivative terms. When the result (16) is substituted into the linear action (13c), as in the two-dimensional case, $S_L \propto S_{VA}$. Again, the result (16) can be derived from simple supersymmetrically invariant constraints

$$\Phi_+{}^2 = \Phi_-{}^2 = 0 \text{ and } \Phi_\pm = -\Phi_\pm\bar{P}P\Phi_\mp. \qquad (17)$$

Some components of these are

$$\mathcal{A}^2 = 0,\quad \mathcal{A}R\psi = 0,\quad \mathcal{A}\mathcal{F} = \tfrac{1}{4}\bar{\psi}R\psi,\quad \mathcal{A} = 2\mathcal{A}\mathcal{F}^*,\quad R\psi = 2R(\mathcal{F}^* - i\mathcal{A}\not{\partial})\psi,\quad \mathcal{F} = 2\mathcal{F}^*\mathcal{F} + i\bar{\psi}\not{\partial}L\psi - 2\mathcal{A}\Box\mathcal{A}^*. \qquad (18)$$

Although the four-dimensional calculations are far more tedious than the two-dimensional calculations, the method is essentially the same: The *Ansatz*, $\psi = \lambda + \ldots$, and the requirement of supersymmetry, imposed order by order in λ, lead unambiguously to the result (16).

These results make explicit the connection between linear and nonlinear realizations of supersymmetry. It should now be possible to investigate aspects of supersymmetry breakdown in greater detail; furthermore, since the coupling of the linear multiplet to supergravity is known,[9] it should be possible to use these results to couple the Volkov-Akulov model to supergravity and to study the super-Higgs phenomenon.

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