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New Upper Limit for $\mu \to e\gamma\gamma$

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The differential decay rates of $\mu \to e\gamma\gamma$ for the most general local interaction are presented. It is shown that recently published data on $\mu \to e\gamma$ imply an upper limit on the branching ratio for $\mu \to e\gamma\gamma$ of 5×10^{-8} with 90% confidence. This is almost two orders of magnitude better than the existing experimental limit. Gauge models which allow a larger rate for $\mu \to e\gamma\gamma$ than for $\mu \to e\gamma$ are discussed.

Recently there has been a resurgence of theoretical interest[1] and experimental activity[2-4] in the field of rare muon-decay modes. Studies have shown that gauge models of weak interactions do not in general conserve fermion flavors such as muon and electron number. It has been suggested that processes such as $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, $\mu \to 3e$, or μ + nucleus $\to e$ + nucleus may take place at a rate near existing upper limits. The relative rates of different muon-number–nonconserving effects depend on the details of the various possible models. For example, in models where the $\mu - e$ transition proceeds via mixings of charged heavy leptons, it is possible that the decay $\mu \to e\gamma\gamma$ is less suppressed than $\mu \to e\gamma$. It is therefore useful to establish an upper limit for $\mu \to e\gamma\gamma$ even if it is somewhat less stringent than that for $\mu \to e\gamma$. In this note we show that the data published by Depommier *et al.*[2] and Povel *et al.*,[3] which considerably improved the upper limit for $\mu \to e\gamma$, can provide a new upper limit for the decay $\mu \to e\gamma\gamma$.

Since modern theories of muon-number nonconservation typically involve intermediate particles with masses much larger than that of the muon, the amplitude may be described by a local effective Lagrangian density

$$m^3\mathcal{L}_{\rm eff} = \bar\psi_e(a_S + b_S\gamma_5)\psi_\mu F^{\alpha\beta}F^{\alpha\beta} + \bar\psi_e(a_P + b_P\gamma_5)\psi_\mu F^{\alpha\beta}\tilde F^{\alpha\beta} + m^{-1}\bar\psi_e(a_V + b_V\gamma_5)\gamma^\sigma\psi_\mu F^{\alpha\beta}\frac{\partial}{\partial\chi_\beta}F^{\alpha\sigma} + m^{-1}\bar\psi_e(a_A + b_A\gamma_5)\gamma^\sigma\psi_\mu F^{\alpha\beta}\frac{\partial}{\partial\chi_\beta}\tilde F^{\alpha\beta}, \quad (1)$$

where all the fields are evaluated at the same space-time point and m is the muon mass. $F^{\alpha\beta}$ is the electromagnetic field tensor with its dual $\tilde{F}^{\alpha\beta}=\frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F^{\gamma\delta}$. The coupling constants, a_i and b_i, are dimensionless and energy independent. This local Lagrangian is an approximation to the true $\mu\to e\gamma\gamma$ interaction up to small correction terms which are of the order of m/M_i, with M_i being the masses of the intermediate heavy particle. It is not difficult to see that because of the symmetry properties of indices the tensor interactions vanish:

$$\bar{\psi}_e\sigma^{\alpha\beta}\psi_\mu[a(FF)^{\alpha\beta}+a'(F\tilde{F})^{\alpha\beta}]=0.$$

Thus Eq. (1) corresponds to the most general local interaction. Dreitlein and Primakoff[5] have already discussed the case in which a_S and a_P are nonzero.

A straightforward calculation then yields a differential decay distribution:

$$\frac{d^2\Gamma}{dE_1dE_2}=\frac{G(a_i,b_i)}{16\pi^3m^6}E_eE_1^2E_2^2(1-\cos\theta)^2, \tag{2}$$

$E_{1,2}$ are the photon energies; $E_e=m-E_1-E_2$, the electron energy; and the angle between the photons is given by $\cos\theta=(E_e^2-E_1^2-E_2^2)/2E_1E_2$. G is given in terms of the effective couplings in Eq. (1) as

$$G(a_i,b_i)=(4a_S+a_V)^2+(4b_S-b_V)^2+(4a_P+2a_A)^2+(4b_P-2b_A)^2. \tag{3}$$

Equations (2) and (3) immediately allow us to set bounds on parameters in any model which permits $\mu\to e\gamma\gamma$ via an effective local interaction.

Clearly any model which permits $\mu\to e\gamma$ will also permit $\mu\to e\gamma\gamma$, if nothing else, as bremsstrahlung from external muon and electron lines. In general one would expect $\mu\to e\gamma\gamma$ to be suppressed further by a factor of (α/π). However, there are gauge models which, contrary to this expectation, can yield a larger rate for $\mu\to e\gamma\gamma$ than for $\mu\to e\gamma$. In particular this is the case for all gauge theories in which the $\mu\to e$ transition is GIM (Glashow-Iliopoulos-Maiani) suppressed and the mediating heavy leptons are *charged*. A simple example in this class of models is the SU(2)⊗U(1) theory of Wilczek and Zee,[1] which we shall present to illustrate our point:

$$\begin{pmatrix}\nu_e\\ e\\ h_e\end{pmatrix}_L,\quad \begin{pmatrix}\nu_\mu\\ \mu\\ h_\mu\end{pmatrix}_L,$$

with

$$h_e=\cos\theta h_1+\sin\theta h_2,\quad h_\mu=-\sin\theta h_1+\cos\theta h_2. \tag{4}$$

The $h_{1,2}$ are two doubly charged heavy leptons with masses $m_{1,2}$. This model predicts the $\mu\to e\gamma$ and $\mu\to e\gamma\gamma$ amplitudes respectively to be

$$T_{\mu e\gamma}\sim eG_Fm\cos\theta\sin\theta(m_1^2/M_W^2-m_2^2/M_W^2)[\bar{\psi}_e(1+\gamma_5)\sigma^{\lambda\rho}\psi_\mu]k^\lambda\epsilon^\rho, \tag{5}$$

and

$$T_{\mu e\gamma\gamma}\sim e^2G_F\cos\theta\sin\theta[f(m_1^2/k_1\cdot k_2)-f(m_2^2/k_1\cdot k_2)][\bar{\psi}_e(1+\gamma_5)\gamma^\lambda\psi_\mu][(k_1\cdot k_2)\epsilon^{\alpha\rho\sigma\lambda}(k_1-k_2)^\alpha +k_1^\sigma\epsilon^{\alpha\beta\rho\lambda}k_1^\alpha k_2^\beta-k_2^\rho\epsilon^{\alpha\beta\sigma\lambda}k_1^\alpha k_2^\beta]\epsilon_1^\rho\epsilon_2^\sigma, \tag{6}$$

where k_i,ϵ_i are photon momenta and polarizations. The amplitude in Eq. (6) just corresponds to the a_A, b_A local interactions in Eq. (1). The $f(X_i)$ in Eq. (6) are rather complicated functions and they can be evaluated numerically.[6] The important feature is that, for certain range of values of $X_i\equiv m_i^2/k_1\cdot k_2$, they can be of the order of unity. In other words the Glashow-Iliopoulos-Maiani suppression[7] for $\mu\to e\gamma\gamma$ may be much less severe than the factor m^2/M_W^2 which applies to $\mu\to e\gamma$ in Eq. (5). Consequently, there is the possibility that $\Gamma(\mu\to e\gamma\gamma)/\Gamma(\mu\to e\gamma)$ is comparable to $(\alpha/\pi)M_W^4(m_1^2-m_2^2)^{-2}$. Namely, $\mu\to e\gamma\gamma$ may be the more favorable decay process in this model. This is reminiscent of the situation in strangeness-changing neutral-current processes[6] with $\Gamma(K_L\to\gamma\gamma)\gg\Gamma(K_L\to\mu^+\mu^-)$, even though simple counting of coupling constants would lead one to conclude that they should be of comparable magnitudes.

Other terms in Eq. (1) can also be realized in modern theories of weak interactions. For example the "scalar" and "pseudoscalar" interactions (a_S, a_P, b_S, b_P) will be nonzero in models (e.g., that of Bjorken and Weinberg[1]) where the $\mu \to e$ transition is mediated by Higgs bosons (with $m_{\rm Higgs} \gg m$ as is expected).

The experiments[2,3] which searched for $\mu \to e\gamma$ were also sensitive to the $\mu \to e\gamma\gamma$ decay. Each employed two large sodium iodide crystal spectrometers which viewed a stopping μ target at 180°. Both experiments accepted events in which a neutral particle deposited energy in one crystal and a charged particle in the other. A photon accompanying either the charged or neutral partner would simply increase the energy detected without being distinguished. According to the angular distribution in Eq. (2), events where the second photon accompanies the electron are much more probable than those in which the two photons enter a single crystal.

We have made a Monte Carlo calculation of the acceptance of the two apparatus for $\mu \to e\gamma\gamma$ events. Dimensions were scaled from the diagrams given in Refs. 2 and 3. The quoted energy resolution was used. We reproduced the quoted $\mu \to e\gamma$ geometrical acceptances of 3.8% for Depommier *et al.* and 1.2% for Povel *et al.* to within 10%. Figure 1 shows a scatter plot of the energy deposited in each detector for 6×10^5 pseudorandom $\mu \to e\gamma\gamma$ events in the experiment of Depommier *et al.* There is a distinct peak at the same position as the $\mu \to e\gamma$ peak would appear. This peak results from the absorption of all three particles in the two crystals. The energy sharing is equal because the detector geometry constrains the three particles to be nearly collinear. The continuum results from events where one photon escapes detection. We calculated the ratio, η, of $\mu \to e\gamma\gamma$ events to $\mu \to e\gamma$ falling into the acceptance regions defined by $E_e < |52.8 \text{ MeV} - \sigma_e|$ and $E_\gamma < |52.8 \text{ MeV} - \sigma_\gamma|$ where E_e (σ_e) and E_γ (σ_γ) are the energy (resolution) of the electron and photon side, respectively. We found that η was 0.072 for Depommier *et al.* and 0.018 for Povel *et al.* The value of η for Depommier *et al.* was larger than for Povel *et al.* because the solid angles subtended by the crystals were larger. While η depends sensitively on the solid angle, it is insensitive to other details of the experiment such as spot size and energy cuts. Dividing the measured upper limit of 3.6×10^{-9} and 1.1×10^{-9} by η, we obtain the new upper limit for $\mu \to e\gamma\gamma$ as 5.0×10^{-8} and 6.1×10^{-8} for Depommier *et al.* and Povel *et al.*, respectively. This result is to be compared with the existing limits of 1.6×10^{-5} set by Frankel *et al.*[8] and 4×10^{-6} by Poutissou *et al.*[9] From this new limit on the $\mu \to e\gamma\gamma$ branching ratio we derive a limit on the constant $G(a_i, b_i)$ by integrating Eq. (2) and dividing by the expression for the decay rate[10] of $\mu^+ \to e^+\bar{\nu}_\mu \nu_e$ and obtain

$$G(a_i, b_i) < 4\times 10^{-6} G_{\rm F}{}^2 m^4,$$

or

$$G(a_i, b_i) < 7\times 10^{-20},$$

where $G_{\rm F}$ is the Fermi constant.

We have only made a brief study for situations that do not correspond to local interactions of Eq. (1).[11] One possibility is that one of the photons is a soft emission off the external lines. The "hard" photon has an energy of ~52.8 MeV/c. It is not difficult to deduce a limit of $\sim 1\times 10^{-9}$ from the experiments of Refs. 2 and 3. However, as already discussed, one would *a priori* expect for such a process that the branching ratio is less than $(\alpha/\pi)R(\mu \to e\gamma\gamma)$ or $\sim 1\times 10^{-11}$. We have also looked into the possibility that $\mu \to e\gamma\gamma$ is mediated by an extremely light scalar boson.[12] Again, the collinearity of the decay products leads us to deduce a limit of $\sim 1\times 10^{-9}$. Thus in the two nonlocal models which we have considered the derived $\mu \to e\gamma\gamma$ branching-ratio limit is more stringent than

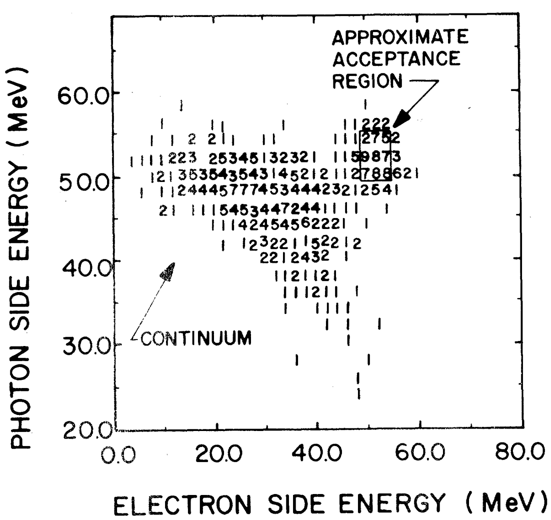

FIG. 1. Hypothetical two-dimensional energy spectrum in the apparatus of Ref. 2 expected for 6×10^5 $\mu \to e\gamma\gamma$ decays according to Eq. (2). Those events in which all three particles are detected cluster around 53 MeV in each detector. The broad continuum contains those events in which an electron and one photon are detected. The contribution of this continuum to the region where we searched for $\mu \to e\gamma\gamma$ (see text) is less than 0.1%. Multiplying the number in the scatter plot by 17 gives the approximate number of counts in the bin.

for a local interaction. However, we believe that the useful limits are those based on the local interactions.

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Inclusive Hadron Production in e^+e^- Annihilation at $\langle s\rangle = 53$ GeV^2

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We report on inclusive hadron production in e^+e^- annihilation at $\langle s\rangle = 53$ GeV^2, using a small solid-angle magnetic spectrometer with good particle identification at 90° to the beams at SPEAR II. The cross sections of $\pi^\pm$ and $K^\pm$ when compared with data at $s = 23$ GeV^2 exhibit scaling in $(s/\beta)d\sigma/dx$ with $x = 2E/s^{1/2}$. The invariant cross section depends on the momentum as p^{-4}.

We have measured the inclusive hadronic cross section with a small solid-angle spectrometer at the highest SPEAR II energies between $s = 49$ and 58 GeV^2. This was an extension of a previous experiment at SPEAR I.[1]

The single-arm magnetic spectrometer used in this experiment was similar to that used in our earlier experiment.[1,2] It was situated at $(90 \pm 13)°$

