(7) and (9) could also have been obtained without the Bäcklund transformations (2) by subjecting the *one*-soliton solution in $1+1$ dimensions to a Lorentz transformation. The same is true for the α_2 solutions. While neither α_1 nor α_2 is truly a three-dimensional solution of Eq. (3a), as mentioned in connection with Eqs. (7) and (8), it can be shown that there exist an infinite number of multiple solutions α_{2n}, $n=2,3,4,\ldots$, which, for typical values of the Bäcklund parameters (θ,λ), are *genuinely three dimensional*. These, then are *new* solutions, since they *cannot* be derived by a simple rotation from the corresponding multiple solutions in $1+1$ dimensions. (iii) The technique described above enables us to write down new exact solutions of Josephson's equation[7]

$$[\partial_x^2+\partial_y^2-c_0^{-2}(\partial^2/\partial t^2)]\psi(x,y,t) =\lambda_J^{-2}\sin\psi(x,y,t)$$

which describes the propagation of magnetic flux through a Josephson tunneling junction.

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Photon and Pion Emission from the Nucleon-Antinucleon System

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Quantitative estimates of the single-γ and -π emission rates for transitions between bound states of the nucleon-antinucleon system are presented. Quantum-number assignments in the context of potential and baryonium models are suggested for new mesons recently seen in monoenergetic γ emission from the $\bar{p}p$ atom. It is shown that π transitions are important in distinguishing between alternative models.

There has been considerable interest recently in the spectroscopy of mesons whose masses lie in the vicinity of the antinucleon-nucleon ($\bar{N}N$) threshold. Such structures have been seen, for example, in $\bar{N}N$ total and elastic cross sections,[1] $\bar{p}d$ spectator experiments,[2] and πp production experiments,[3] and through the observation of γ rays from the $\bar{p}p$ system.[4]

There exist numerous theoretical predictions for new mesons near the $\bar{N}N$ threshold. Some of these approaches involve the use of $\bar{N}N$ potential models,[5] while others are based on topological expansions[6] or the Massachusetts Institute of Technology bag model extended to the diquark-antidiquark ($Q^2\bar{Q}^2$) sector.[7]

In the present Letter, we adopt the potential approach. We use a model for the intermediate and long-range parts of the strong $\bar{N}N$ potential,

$v_N(r)$, constructed via the G-parity transformation from a *realistic* NN interaction, in this case the Paris potential.[8] For the short-range potential, we use a simple phenomenological cutoff, $V_N(r) = V_N(r_0)$ for $r \leq r_0$. We explore the hypothesis that the energetic γ rays seen experimentally[4] arise from transitions between quasiatomic and quasinuclear eigenstates of the combined potential $V_N(r) + V_C(r)$, where $V_C(r) = -e^2/r$. We present the results of quantitative calculations of the widths Γ_γ associated with such transitions, as well as the widths Γ_π for single-pion emission. The γ-emission widths have been calculated using the standard form of the coupling of electromagnetic field to nucleons and antinucleons.[9] For π emission, we use a nonrelativistic p-wave coupling of Chew-Low type.[10]

The properties of the spectrum of strongly bound states obtained by solving the Schrödinger equation with the Paris potential have been discussed previously.[11] The main features of the spectrum for a wide class of such potential models are the following:

(a) The lowest-lying states of each total angular momentum J form an isospin $I=0$, spin $S=1$ natural-parity band with $J^{PC} = 0^{++}, 1^{--}, 2^{++}$, etc. This is due to the maximum attractive coherence of Wigner, tensor, quadratic spin-orbit, and vector-meson–exchange potentials in these channels.

(b) There are usually some $I=1$ states of low orbital angular momentum L which lie relatively close to threshold. However, there is a large breaking of isospin degeneracy for the natural-parity band; the $I=1$ states typically lie hundreds of MeV above the $I=0$ members.

The selection rules for π transitions between bound states of an $\bar{N}N$ potential labeled by $\{L_i S_i J_i I_i\}$ and $\{L_f S_f J_f I_f\}$ are the following: (a) $L_i + L_f + l + 1$ is even (parity); (b) $G_i = -G_f$, where $G = (-1)^{L+S+I}$; (c) $C_i = C_f$, where $C = (-1)^{L+S}$, only for π^0 emission from the $\bar{p}p$ or $\bar{n}n$ system; (d) the coupling $\Delta(J_i\, l\, J_f)$ is allowed, where l is the orbital angular momentum of the pion with respect to the $\bar{N}N$ center of mass. Note also that $\pi^\pm$ emission from $\bar{p}p$ ($\bar{n}n$) populates only $I_f = 1$ states, π^0 emission from $\bar{p}p$ ($\bar{n}n$) involves $|I_i - I_f| = 1$, and π^0 emission from $\bar{p}n$ ($\bar{n}p$) has $I_i = I_f = 1$. In these calculations, we have assumed that the isospin of the quasiatomic $\bar{p}p$ wave function at short distances corresponds to the channel for which the nuclear potential is most attractive ($^{13}S_1$ and $^{31}S_0$).[12] This "isospin dominance" hypothesis is expected to be a reasonable first approximation.[13] The pion rates are more sensitive to isospin admixtures than those for γ emission, which does not have isospin selection rules. For the dominant $^{13}S_1$ and $^{31}S_0$ quasiatomic states, G-parity conservation requires that the final state in pion emission have $G = +1$.

Typical results for γ and π transitions from quasiatomic S states and ($^{33}P_1$, $^{31}P_1$) quasinuclear states near threshold to deeply bound quasinuclear states are shown in Figs. 1 and 2.[12] We have used a cutoff $r_0 = 0.82$ fm; the spectrum is sensitive to the choice of r_0, and so the results should be regarded as representative rather than predictive.

Estimates of the yields Γ_γ/Γ_T and Γ_π/Γ_T are shown in Table I, for transitions from the quasiatomic S states to the four most deeply bound states. For the total width Γ_T, we use a rough estimate $\Gamma_T = \Gamma_A + \Gamma_\pi$, where the annihilation width Γ_A into multimeson channels (without $\bar{N}N$ in final state) is obtained from $\Gamma_A = \nu\sigma_A \int_0^\infty dr \times f(r) u^2(r)$. Here $f(r)$ is a short-ranged annihilation function,[5] $u(r)$ is the radial wave function of the initial state, and $\nu\sigma_A = 8$ mb is an effective annihilation cross section. This value of $\nu\sigma_A$ has been chosen to yield widths of the order of 10–20 MeV for the three lowest $\bar{N}N$ states, i.e., a bit less than the instrumental widths (20–35 MeV)

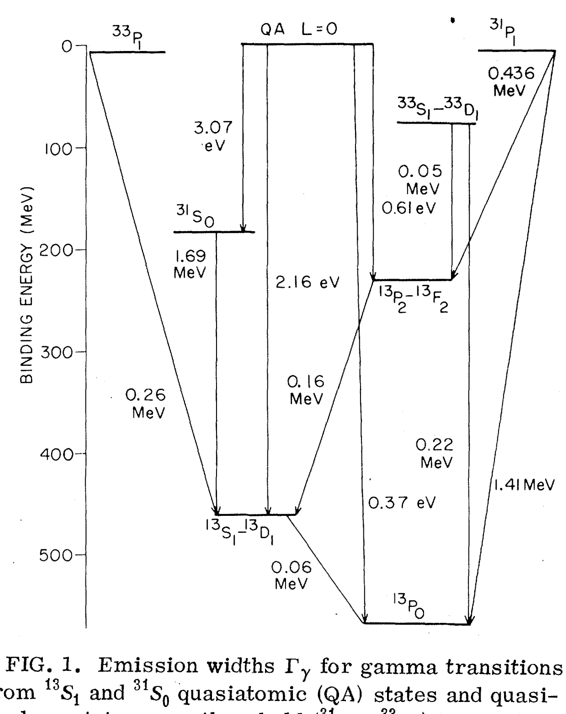

FIG. 1. Emission widths Γ_γ for gamma transitions from $^{13}S_1$ and $^{31}S_0$ quasiatomic (QA) states and quasinuclear states near threshold ($^{31}P_1$, $^{33}P_1$) to deeply bound configurations of a potential model.

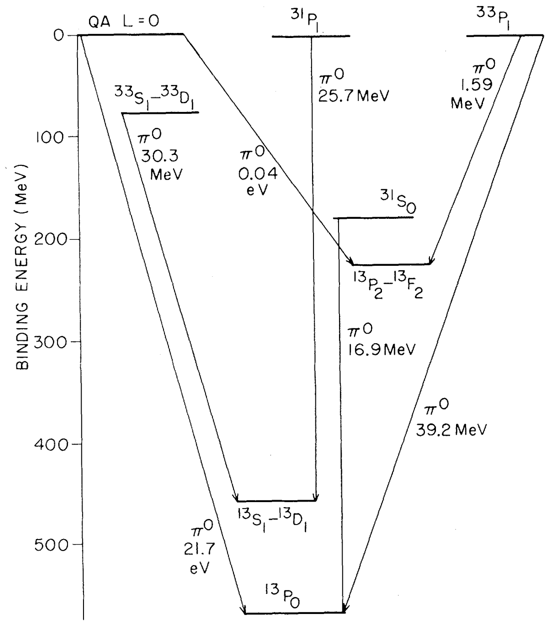

FIG. 2. Widths Γ_π for single-pion transitions connecting the levels of the $\bar{p}p$ system in a potential model.

quoted in Ref. 4.

Since it is difficult to predict *absolute* energies with any confidence in the $\bar{N}N$ potential model, several scenarios are possible for the quantum numbers of the states seen in the γ-ray experiment.[4] The spin assignments are not as aribtrary as they may seem, however, since many features of the *level order* are preserved in a variety of potential models.[5,11] In Table I, we present a reasonable set of spin assignments for these states, *if* they can be understood in the context of the potential description. Several considerations have gone into this assignment:

(a) The states at 183 and 216 MeV are much too close in energy to be successive members of the $I=0$ natural-parity band; so one of them is likely to be an $I=1$ state.

(b) If we assign the 420-MeV state to be $^{13}P_0$, then the 216-MeV state would be $^{13}S_1$-$^{13}D_1$, and the 183-MeV state $^{31}S_{01}$- or $^{33}S_1$-$^{33}D_1$. However, many calculations[5,11] suggest that $^{13}S_1$-$^{13}D_1$ should lie *much* deeper than other configurations containing $L=0$. We predict an additional 0^{++} state below 500-MeV binding. The values of Γ_γ/Γ_T are in accord with experimental values, although we have not attempted a best fit to the data. Note that the γ transitions to the 0^{-+} and 1^{--} states are $M1$ rather than $E1$. These $M1$ transitions are comparable to the $E1$ transitions to the 0^{++} and 2^{++} states because we are well above the region where the long-wavelength limit applies.

Single-pion emission from quasiatomic $L=0$ levels to quasinuclear states, when it is enegetically allowed, is generally considerably stronger than γ emission, unless the selection rules prohibit $l=0, 1$ (as for the 2^{++} final state). The π^0 transitions would present greater experimental difficulties than $\pi^\pm$, but are crucial in populating states where $\pi^\pm$ transitions are forbidden or small. We note also from Fig. 2 that *very strong* pion transitions between loosely bound quasinuclear states ($^{33}P_1$, $^{31}P_1$, and $^{33}S_1$-$^{33}D_1$ in our model) and deeply bound states are predicted. In some cases, the single-pion transitions dominate the multimeson-decay mode. This may preclude having certain narrow states near threshold if narrow deeply bound states also exist.

In some potential models,[5] all four $L=0$ $\bar{N}N$ channels ($^{11}S_0$, $^{31}S_0$, $^{13}S_1$, $^{33}S_1$) possess a strongly bound state. In this case, the $2P\to 1S$ quasiatomic transitions will have a transition energy close to the unperturbed Coulombic value of 9.4 keV, within 1 keV or 2.[5,14] However, in other models,[11] one of these states is not strongly bound. It would thus be very interesting to look for additional *low-energy* γ's (10 keV to few MeV range) well above the unperturbed $2P\to 1S$ energy, indi-

TABLE I. Possible quantum-number assignments in a potential model for states seen in energetic γ emission from $\bar{p}p$ system.[a]

E_γ (MeV)	Γ_γ/Γ_T (expt)	J^{PC} (I^G)	Γ_γ/Γ_T	Γ_{π^0}/Γ_T	Γ_{π^-}/Γ_T
183	7×10^{-3}	0^{-+} (1^-)	8×10^{-3}	NO	NO(*)
216	6×10^{-3}	2^{++} (0^+)	2×10^{-3}	2×10^{-4}	NO
420	9×10^{-3}	1^{--} (0^-)	1×10^{-2}	NO(*)	NO
(550–600)	···	0^{++} (0^+)	1×10^{-3}	1×10^{-1}	NO

[a]Includes γ or π from $^{13}S_1$ and $^{31}S_0$ $\bar{p}p$ quasiatomic states only. Transitions labeled NO are forbidden for π^- if we neglect isospin mixing in the final state. Transitions labeled NO(*) are forbidden if we assume isospin dominance in the initial state.

TABLE II. Possible quantum-number assignments in a baryonium model for states seen in γ emission.[a]

E_γ^{expt} (MeV)	E_γ^{th} (MeV)	J^{PC} (I^G)	$\pi^0(\bar{p}p)$	$\pi^\pm(\bar{p}p)$	$\pi^0(\bar{p}n)$	$\pi^\pm(\bar{p}n)$
183	150	1^{--} (1^+)	YES	YES	YES	NO
216	230	1^{--} (0^-)	NO(*)	NO	NO	YES
420	510	0^{++} (0^-)	NO(*)	NO(*)	NO	NO
...	610	0^{++} (0^+)	YES	NO	NO	YES

[a] $\pi^0, \pi^\pm$ from quasiatomic $^{13}S_1$ or $^{31}S_0$ states of the $\bar{p}p$ systems or π^0, $\pi^\pm$ from loosely bound $^{31}S_0$ or $^{33}S_1$ quasinuclear $\bar{p}n$ states, as predicted in Ref. 7. Transitions labeled NO(*) are forbidden because we assume isospin dominance in quasiatomic states, while transitions labeled NO are forbidden if quasinuclear states have good isospin.

cating the presence of an s state in the transition region between a quasiatomic and a deeply bound quasinuclear configuration.

It is also possible that the mesonic states seen in the γ-ray experiment[4] correspond more closely to exotic quark configurations of type $Q^2\bar{Q}^2$. Jaffe has recently[7] provided detailed estimates of the energies of such states and their relative coupling strength to the $\bar{N}N$ channel. Based on these results, we suggest a possible (but not unique) identification of these "baryonium" states with experiment in Table II. A critical feature of this approach is approximate *isospin degeneracy* for most trajectories. Thus it is natural to interpret the states at 183 and 216 MeV as an isospin doublet. The 1^{--} doublet, with the $I=0$ partner lying lowest, is predicted[7] to lie close to the observed energies. The 2^{++} doublet is predicted above the $\bar{N}N$ threshold in the mass region of 1900–2000 MeV. This is consistent with a recently proposed spin assignment[15] of $2^{++}(1^-)$ for the $S(1930)$ meson.[1] The presence of a doublet in this region also provides an isospin interference mechanism which could explain the absence of a sharp peak in the $\bar{p}p\rightarrow\bar{n}n$ cross section[1] near 1930 MeV. The problem here is that there is yet no evidence for an isospin partner of the 420-MeV state. Thus, as for the potential model, we must assume that the "ground state" of baryonium ($^{13}P_0$) lies below 500 MeV binding energy.

The allowed pion transitions for the baryonium model are indicated by YES in Table II. As we note by a comparison of Tables I and II, the detection of single-pion transitions is important in distinguishing between these models.

Transitions from the $\bar{p}n$ system are also of considerable interest. Such systems can be formed in $\bar{p}d\rightarrow p_s+(\bar{p}n)$ spectator experiments.[2] Further experiments of this type, particularly the detection of a γ or π in coincidence with the spectator proton or neutron, should be very revealing. By this means, one may be able to detect γ's or π's which connect two quasinuclear states. These transitions are predicted to be quite strong, as per Figs. 1 and 2.

Another possibility is γ or π emission from the continuum to discrete bound states of the $\bar{N}N$ system. In particular, in an $\bar{N}N$ collision at the energy of the narrow $S(1930)$ resonance, we have the possibility of enhancing the contribution of a particular initial channel. The selection rules will favor different final states than those populated from quasiatomic $L=0$ states. One may also be able to see γ rays to quasinuclear states very close to threshold. Such states are predicted in both potential[5,11] and baryonium[6,7] models.

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New Upper Limit for $\mu \to e\gamma\gamma$

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The differential decay rates of $\mu \to e\gamma\gamma$ for the most general local interaction are presented. It is shown that recently published data on $\mu \to e\gamma$ imply an upper limit on the branching ratio for $\mu \to e\gamma\gamma$ of 5×10^{-8} with 90% confidence. This is almost two orders of magnitude better than the existing experimental limit. Gauge models which allow a larger rate for $\mu \to e\gamma\gamma$ than for $\mu \to e\gamma$ are discussed.

Recently there has been a resurgence of theoretical interest[1] and experimental activity[2-4] in the field of rare muon-decay modes. Studies have shown that gauge models of weak interactions do not in general conserve fermion flavors such as muon and electron number. It has been suggested that processes such as $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, $\mu \to 3e$, or μ + nucleus $\to e$ + nucleus may take place at a rate near existing upper limits. The relative rates of different muon-number–nonconserving effects depend on the details of the various possible models. For example, in models where the $\mu \to e$ transition proceeds via mixings of charged heavy leptons, it is possible that the decay $\mu \to e\gamma\gamma$ is less suppressed than $\mu \to e\gamma$. It is therefore useful to establish an upper limit for $\mu \to e\gamma\gamma$ even if it is somewhat less stringent than that for $\mu \to e\gamma$. In this note we show that the data published by Depommier *et al.*[2] and Povel *et al.*,[3] which considerably improved the upper limit for $\mu \to e\gamma$, can provide a new upper limit for the decay $\mu \to e\gamma\gamma$.

Since modern theories of muon-number nonconservation typically involve intermediate particles with masses much larger than that of the muon, the amplitude may be described by a local effective Lagrangian density

$$m^3\mathcal{L}_{\rm eff} = \bar\psi_e(a_S + b_S\gamma_5)\psi_\mu F^{\alpha\beta}F^{\alpha\beta} + \bar\psi_e(a_P + b_P\gamma_5)\psi_\mu F^{\alpha\beta}\tilde F^{\alpha\beta} + m^{-1}\bar\psi_e(a_V + b_V\gamma_5)\gamma^\sigma\psi_\mu F^{\alpha\beta}\frac{\partial}{\partial\chi_\beta}F^{\alpha\sigma} + m^{-1}\bar\psi_e(a_A + b_A\gamma_5)\gamma^\sigma\psi_\mu F^{\alpha\beta}\frac{\partial}{\partial\chi_\beta}\tilde F^{\alpha\beta}, \quad (1)$$

where all the fields are evaluated at the same space-time point and m is the muon mass. $F^{\alpha\beta}$ is the