viously studied has shown a peak of the type now seen for the first time in a sputtered film. If the peak is impurity related, it remains to be seen whether there is an impurity effect on the superconducting behavior of crystallized films, since the annealing behavior may indicate that rejection of the impurity by precipitation or desorption is complete prior to crystallization.

Finally, and from a more general viewpoint, it can be remarked that the principal significance of the present work lies in the conclusion that a well-defined internal-friction peak associated with a stress-induced ordering mechanism has been identified for the first time in a metallic glass. If the history of internal-friction studies on crystalline materials is used as a guide, it now appears likely that other peaks await discovery in a variety of metallic glasses, and that these will be of considerable use in the characterization of these interesting new materials at an atomic level.

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Resonant Nonlinear Mode Conversion in He II

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The results of an experiment to observe the nonlinear conversion of second sound to first sound within a waveguide show that this resonant process occurs at precisely the predicted frequency. Unique procedures are used to calibrate the first-sound transducers and we find the amplitude of the mode-converted first sound has its predicted value value which is determined principally by $\partial\rho/\partial(w^2)$ where w is the difference between the normal-fluid and superfluid velocities and ρ is the density.

The purpose of this Letter is to describe an experiment which results in the first observation of a propagating first-sound (pressure-density) wave generated by the nonlinear interaction of two second-sound (temperature-entropy) waves. This resonant mode-conversion process is in some ways similar to the generation of a longitudinal sound wave through the interaction of two transverse waves in a solid, in that the difference in the propagation velocities of the two modes places severe restrictions on the frequencies and wave vectors of the interacting waves.[1] This has been observed in solids[2] but no absolute measurements were made which could be used to determine coupling coefficients. In our experiment, a new application of the reciprocity principle[3] allows absolute measurements of the primary and mode-converted wave amplitudes and thus a direct quantitative measurement of the coupling between the mode unique to superfluid (second sound) and the ordinary sound wave (first sound).

In the nondissipative approximation, the general hydrodynamic description of He II is given by the Landau two-fluid equations.[4] When wavelike solutions to the two-fluid equations are sought, retaining terms which are second order, the wave equations for the second-order pressure or temperature variations in the superfluid are not homogeneous but have driving terms which are proportional to quadratic combinations of the first-order quantities. The following is the wave equation for second-order pressure disturbances, p_2, neglecting the isobaric coefficient of thermal expansion which is small in the region of experimental interest and vanishes at 1.17°K:

$$\frac{\partial^2 p_2}{\partial t^2} - u_1{}^2\nabla^2 p_2 = u_1{}^2\left[\frac{\partial^2}{\partial r_i\,\partial r_j}\left(\rho v_i v_j + \frac{\rho_n\rho_s}{\rho} w_i w_j\right) - \frac{1}{2}\frac{\partial^2\rho}{\partial p^2}\frac{\partial^2 p_1{}^2}{\partial t^2} - \frac{1}{2}\frac{\partial^2\rho}{\partial T^2}\frac{\partial^2 T_1{}^2}{\partial t^2} - \frac{\partial^2\rho}{\partial p\,\partial T}\frac{\partial^2 p_1 T_1}{\partial t^2} - \frac{\partial\rho}{\partial(w^2)}\frac{\partial^2(w^2)}{\partial t^2}\right], \tag{1}$$

where the subscript 2 denotes the second-order contribution which, by definition, is proportional to quadratic combinations of first-order terms subscripted with 1 except u_1 which is the speed of first sound. $\vec{v}$ is the center-of-mass velocity and ρ_n and ρ_s are the densities of the normal-fluid and superfluid components with the total fluid density given by $\rho=\rho_s+\rho_n$. $\vec{w}=\vec{v}_n-\vec{v}_s$, where $\vec{v}_n$ and $\vec{v}_s$ are the velocities of the normal-fluid and superfluid components. T and p are the temperature and pressure.

In the linear approximation these modes are uncoupled. In general, these driving terms do not lead to the generation of propagating (first- or second-) sound waves unless the phase velocity of the driving term is equal to the speed of first or second sound. The simplest case would be a first- or second-sound wave which interacts with itself to generate second-harmonic distortion and ultimately a shock for sufficiently large amplitudes or long distances.[5]

In the region of interaction of two second-sound waves, the square of the sum of the waves will have components with phase velocities other than u_2 and for the appropriate angle of intersection can be made equal to u_1; here, u_1 and u_2 are the speeds of first and second sound. For two second-sound waves of equal frequency, ω, and equal amplitude, traveling in different directions indicated by their wave vectors, $\vec{k}$ and $\vec{k}'$, the condition that the phase velocity, u_{ph}, of the component at 2ω be that of first sound is $u_{\mathrm{ph}}=u_1=2\omega/|\vec{k}+\vec{k}'|$ or $\cos(\frac{1}{2}\theta)=u_2/u_1$ where θ is the angle between $\vec{k}$ and $\vec{k}'$. This resonance condition leads to the generation of a propagating first-sound wave at a frequency 2ω, whose amplitude grows linearly with x in the region of interaction if the attentuation of second sound can be neglected, and x is chosen as the direction which bisects θ. This result can be understood physically as an end-fire array antenna which is driven by the nonlinear interaction of the two second-sound waves in analogy with the model used to analyze the "parametric end-fire array" that is used in underwater acoustics to produce highly directional low-frequency sound.[6] We have shown elsewhere[7] that the coupling coefficient for this process should be dominated by $\partial\rho/\partial(w^2)$. The appearance of w^2 as a third Galilean-invariant internal variable in the equation of state is a feature unique to superfluid helium.

In order to exercise precise control over the intersection angle, θ, of the two second-sound waves and to achieve an interaction region of maximum length within the confines of a 6-in.-diam Dewar vessel, the experiment was performed in a spiral waveguide of rectangular cross section. The bottom and two sides of the waveguide are a groove machined into a single block of aluminum in the form of a spiral (Fig. 1). The top surface is a $\frac{1}{2}$-mil- (13-μm-) thick sheet of Teflon, aluminized on the bottom side and given a quasipermanent charge.[8] The Teflon adheres to a 1.27-cm-thick aluminum top plate which has several insulated "buttons" that are situated over the groove. These buttons and the aluminized Teflon constitute pressure transducers (electret microphones).[9] A thin graphite resistor[10] sprayed on the top plate is used to measure the temperature amplitude of the second-sound waves. The top plate is bolted to the spiral to complete the waveguide.

The waveguide is operated in its lowest non-plane-wave second-sound mode. The cutoff frequency for this mode is given by $\omega_{\mathrm{co}}=\pi u_2/l$, where l is the depth of the waveguide (1.467 cm at $T=0$°K). This mode is equivalent to two plane waves whose angle of intersection, θ, is given by $\cos(\frac{1}{2}\theta)=[1-(\omega_{\mathrm{co}}/\omega)^2]^{1/2}$. The frequency of the mode-converted first sound ω_{mc} is twice the frequency of the second sound. If we let $\omega_{\mathrm{mc}}/2=\omega_{\mathrm{co}}+\Delta\omega$ then $\Delta\omega/\omega_{\mathrm{co}}\simeq u_2^2/u_1^2\simeq 0.35\%$. The second-sound mode is selectively excited by a pair of heaters that are driven by currents which are 90° out-of-phase and have a dipolar heat distribution that is proportional to $\cos k_y y$ where y is the direction along the height of the waveguide

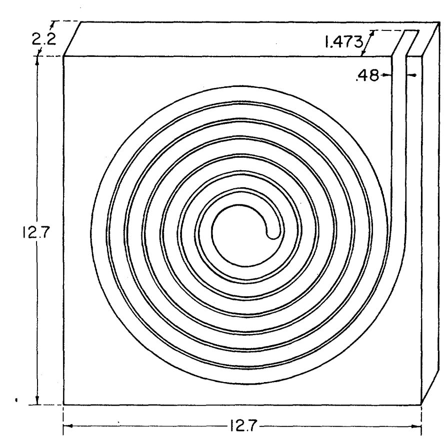

FIG. 1. Schematic representation of the spiral waveguide of rectangular cross section. The total length is 150 cm, the last 60 cm of which were used as an absorber. All dimensions are in centimeters at room temperature.

with $y=0$ at the bottom and $k_y=\pi/l$.

Figure 2 shows first- and second-sound response taken at 1.31°K. The second-sound pickup transducer is 7.4-cm downstream from the dipolar source. The cutoff is not infinitely steep because the group velocity is zero at cutoff and there are viscous losses at the waveguide walls. The finite value of the second-sound signal at cutoff is due to the exponentially decaying tail which penetrates the waveguide by increasing numbers of waveguide depths as cutoff is approached from below.[11] The first-sound signal generated by mode conversion is superimposed on the second-sound data. The full width of the resonance at the half-power points is 3.4 Hz corresponding to a $Q=360$. The frequency scale in Fig. 2 should be doubled when referring to the first-sound data. The appearance of this singular, high-Q, first-sound resonance is a striking confirmation of the existence of this nonlinear interaction in a quantum fluid.

Mode-conversion resonance with Q's ranging from 300 at the lowest T to 1600 at the highest T were observed ans their frequencies as a function of temperature are plotted in Fig. 3. The 55 solid circles represent anywhere from one to fourteen separate determinations of a measured mode-conversion resonance peak at the indicated temperature with the average number of independent measurements per point being 2.7. The experimental error in determining the frequency or temperature is about the size of the circles. The solid line represents the predicted frequencies using published values of u_1 and u_2[12] and the measured depth of the waveguide corrected for thermal contraction. The dotted portion of the line used values of u_2 extrapolated beyond the temperature range listed in Ref. 12. There are no adjustable parameters. The displacement of the resonance above twice the cutoff frequency was checked against theory by calculating $(\omega_{mc}^* - 2\omega_{co})/2\Delta\omega$ where ω_{mc}^* is the measured mode-conversion frequency. The average of this quantity over the points shown in Fig. 3 is 1.17. This is good agreement since both numbers in the numerator are about 1300 Hz and their difference is typically 5 Hz. This precision can be directly traced to the efficacy of using a waveguide to guarantee the plane-wave character of the sound and the frequency of the higher-order mode to control the phase velocity.

The dependence of the first-sound amplitude on second-sound amplitude was found to be quadratic over 1.3 decades. The dynamic range was limited at the lower end by the sensitivity of the detector, and at the upper end by critical velocities at the source.

The value of the coupling constant for this process was determined experimentally by making absolute measurements of the first- and second-sound amplitudes. The calibration of the second-sound (temperature) transducer is straight forward. The first-sound transducers are calibrated using a new application of the reciprocity tech-

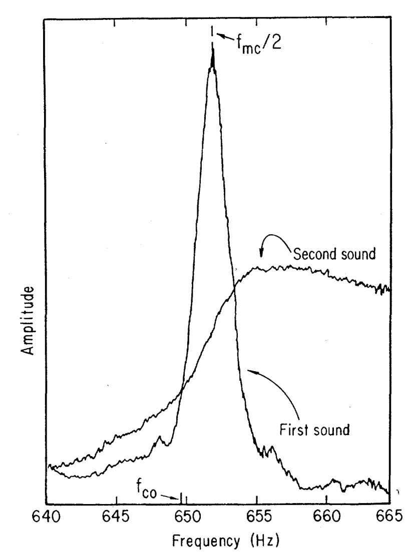

FIG. 2. First- and second-sound data taken at 1.31°K. f_{co} is the cutoff frequency of second sound within the waveguide. The frequency scale should be doubled when referring to the first-sound trace (i.e., the peak occurs at f_{mc}).

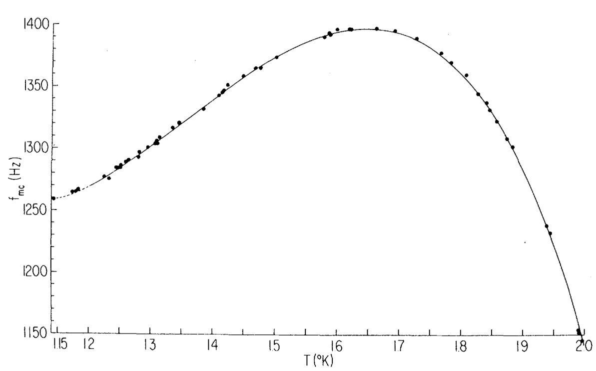

FIG. 3. Measured mode-conversion frequencies. The solid line is the theoretical value based on independent measurements of u_1 and u_2 (Ref. 12) and the measured depth of the waveguide. There were no adjustable parameters. The dotted portion of the line used extrapolated values of u_2.

nique that will be described in detail elsewhere (Ref. 3) but can be motivated here by the following argument: Calibration amounts to determining the electroacoustic transfer function of a transducer. If we have two identical, reversible, linear transducers which obey the reciprocity relation,[13] they can be coupled together by a purely acoustical network whose acoustical transfer impedance can, in principle, be calculated. Purely electrical measurements on the terminals of the two transducers, e.g., the current into the first transducer acting as the speaker and the voltage generated by the second transducer acting as the microphone, will then yield the electroacoustic transfer functions of the identical transducers. The restriction that the transducers be identical and that both are reversible can be lifted in a simple way by exposing both transducers to an identical sound field and measuring the ratio of their responses. This technique had the advantage in the present experiment that it could be applied *in situ* and does not require a primary acoustical standard.

Absolute magnitudes of the coupling constant have been measured for several points between 1.26 and 1.41°K (see Fig. 4) and are found to be in agreement with the theoretical values[7] to within 16% on the average.

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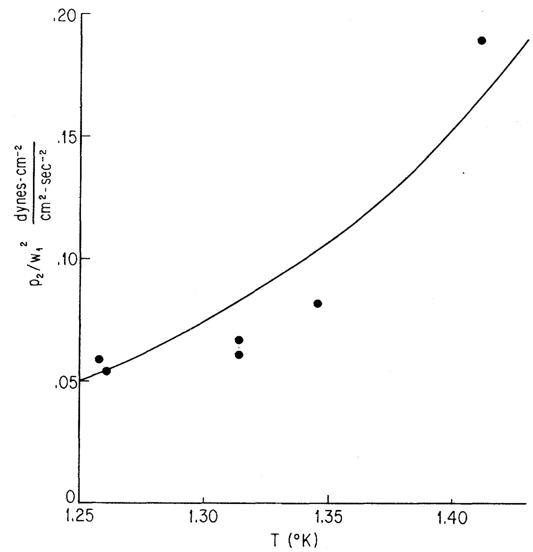

FIG. 4. Mode-conversion coupling constant p_2/w_1^2. Points are absolute measurements and the line is based on Eq. (1) and the value of thermodynamic quantities in Ref. 12.

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