

ionization from excited states was evaluated by the formula of Lotz,<sup>8</sup> its effect on  $R_0$  being negligible. Recombination to excited levels was also neglected.

It is apparent from the above analysis that Mo XV is a rather special case, because the ground configuration is a closed shell  $3d^{10}$  ( $J=0$ ). For other molybdenum ions (Mo XVI to Mo XXIV), the ground configurations  $3d^n$  ( $n=1$  to 9) possess many levels, and hence there are relatively few levels of  $3d^{n-1}4p$  which cannot decay radiatively to the ground configuration. However, an even more important phenomenon is the appearance of configurations  $3p^5 3d^{n+1}$  which lie below  $3d^{n-1}4s$  and have opposite parity.  $E1$  transitions between these two configurations are allowed by configuration mixing of the type  $3p^6 3d^{n-1}4s + 3p^5 3d^n 4p$ . These turn out, in general, to be strong enough to dominate  $E2$  transitions. As an example, we computed the simple case of Mo XXIV (ground configuration  $3p^6 3d$ ). Using the same methods as for Mo XV we obtain  $R_0 = 0.008$  for  $n_e = 5 \times 10^{13}$   $\text{cm}^{-3}$  and  $T_e = 400$  eV (corresponding to TFR conditions at the location of this ion). Thus, we conclude that there is little chance of observing  $E2$  lines for ionization stages other than Mo XV. One possible exception may be Mo XVI, where some levels of  $3p^6 3d^8 4s + 3p^5 3d^9 4p$  cannot decay to the intermediate configuration  $3p^5 3d^{10}$  because of the  $\Delta J$  selection rule and are thus metastable.

We conclude that computed  $E2$  transitions of Mo XV  $3d^{10} - 3d^9 4s$  agree well, both in wavelength and intensity, with the previously unknown lines observed at 57.927 and 58.832 Å in the TFR spectrum. We have also shown that one should not

expect to detect many lines of this type for higher ionization stages of Mo. In any case, they will be unimportant for radiative energy losses.

Forbidden lines of other type, such as magnetic dipole ( $M1$ ) of highly ionized iron<sup>9</sup> observed in solar flares, may also appear in tokamak spectra. The  $E2$  lines, like the  $M1$ , are density sensitive and could be used for density diagnostics in plasma as suggested by Doschek and co-workers.<sup>9,10</sup>

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## Virial Theory of Direct Langmuir Collapse

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A general virial theorem is proved without any assumption of special symmetries and used to calculate the threshold and time for direct collapse of a two-dimensional Langmuir wave packet. The analysis is shown to apply to "bump-on-tail" instabilities, when the "bump" is broad, low, and at high velocity. The example of the type-III solar radio-burst problem is treated both numerically and analytically.

A large-amplitude wave packet of Langmuir waves in a homogeneous nonmagnetic plasma is capable of a wide variety of distinct nonlinear evolutions, depending on its mean energy, wave

vector, and spread in wave numbers.

Often, the wave packet drives secondary Langmuir wave instabilities such as induced scattering off ions and modulational instabilities.<sup>1-3</sup>

These secondary instabilities are driven by the ponderomotive force  $-\nabla(\vec{\mathcal{E}}_0^* \cdot \vec{\mathcal{E}}_1 + \vec{\mathcal{E}}_0 \cdot \vec{\mathcal{E}}_1^*)$ , where  $\vec{\mathcal{E}}_0$  is the envelope field of the primary packet, and  $\vec{\mathcal{E}}_1$  that of the small (noise) perturbation. The perturbation  $\vec{\mathcal{E}}_1$  grows as  $e^{\gamma_1 t}$ , where<sup>3,4</sup>  $\gamma_1/\omega_p \propto W \equiv \langle \vec{\mathcal{E}}_0^2 \rangle / 4\pi m \theta$ . After many  $e$  foldings, a new field configuration  $\vec{\mathcal{E}}$  results, and in regions of space where the ponderomotive force  $-\nabla|\vec{\mathcal{E}}|^2$  is large, a process of Langmuir collapse may occur in which the localization region shrinks to dimensions of several Debye lengths. We shall refer to this scenario as indirect collapse. Indirect collapse is always preceded by the exponentiation of background noise. There are numerous examples in the plasma physics literature of indirect collapse after modulational instability<sup>5-7</sup> and after backscattering from ions.<sup>8,9</sup>

A distinct competing process is direct collapse.<sup>7</sup> This cannot be described as a linear instability. The time,  $\tau_c$ , for direct collapse is often much faster than  $\gamma_1^{-1}$ , and is proportional to  $\sqrt{W}$ , rather than  $W$  [see Eq. (10b)]. Direct collapse is driven by the ponderomotive force,  $-\nabla|\vec{\mathcal{E}}_0|^2$ , associated with the primary wave packet, rather than with the noise-interference terms.

Direct collapse is often confused with the linear modulational instability, first predicted by Vedenov and Rudakov.<sup>1</sup> One reason for this confusion seems to be that both thresholds are proportional to the square of the Fourier-space spectral width,  $\Delta k$ , of the primary field  $\vec{\mathcal{E}}_0$ . However, the processes are generally distinguishable by their different time scales. Direct collapse is usually faster, except when  $W$  is orders of magnitude above threshold. The situation is familiar from nonlinear optics. Here, self-focusing, or "nonlinear lensing," of a coherent laser beam is an example of direct collapse.<sup>10</sup> A competing modulational instability may cause the beam to break up into filaments,<sup>10,11</sup> which then proceed to self-focus separately (indirect collapse). When  $W$  is only a few times threshold this "filamentation" instability does not operate, and only self-focusing of the primary beam occurs.<sup>10</sup> In this Letter we are concerned with the Langmuir wave analog of this direct process.

After proving a general virial theorem which makes no assumption of special symmetry (as in the spherically-symmetric virial theorem of Zakharov<sup>7</sup>), but does assume a certain ordering of wave-packet parameters, we calculate analytically the threshold and time for direct collapse. We show that direct collapse can occur for bump-on-tail instabilities, when the bump is fairly

broad, low, and at high velocity. An important example is furnished in connection with type-III solar radio bursts.<sup>12</sup>

Consider the cubic nonlinear Schrödinger equation for the envelope,  $\vec{\mathcal{E}}$ , of a Langmuir wave  $\vec{E} = \text{Re}[\vec{\mathcal{E}} \exp(-i\omega_p t)]$ :

$$i\partial_t \vec{\mathcal{E}} + \frac{1}{2} \nabla(\nabla \cdot \vec{\mathcal{E}}) + |\vec{\mathcal{E}}|^2 \vec{\mathcal{E}} = 0. \quad (1)$$

This equation assumes adiabatic ions<sup>7</sup> and ignores a  $c^2 \nabla \times \nabla \times \vec{\mathcal{E}}$  term<sup>13</sup> associated with coupling to nonelectrostatic field polarizations [see Eqs. (11) and (12)]. Time is measured in units of inverse plasma frequency,  $\omega_p^{-1}$ , and length in units of  $\sqrt{3}$  times the Debye length,  $\sqrt{3}k_D^{-1}$ , and  $|\vec{\mathcal{E}}|^2$  has the units of  $32\pi m(\theta_e + \theta_i)$ , where  $\theta_e$  and  $\theta_i$  are the electron and ion temperatures.

From the gauge and translational invariance of the Lagrangian density<sup>14,15</sup> associated with Eq. (1), a number of continuity equations and associated conservation laws follow. For our purposes, the important ones are

$$\partial_t |\vec{\mathcal{E}}|^2 + \nabla \cdot \vec{s} = 0, \quad (2a)$$

$$\partial_t \vec{p} + \nabla \cdot \vec{T} = 0. \quad (2b)$$

Here,  $\vec{s}$  is the current density, and  $\vec{p}$  is the momentum density of the field. For Eq. (1), they are both equal:

$$\vec{s} = \vec{p} = \frac{1}{2i} [\vec{\mathcal{E}}^* \nabla \cdot \vec{\mathcal{E}} - \vec{\mathcal{E}} \nabla \cdot \vec{\mathcal{E}}^*]. \quad (3)$$

The stress tensor  $\vec{T}$  is given by<sup>16</sup>

$$T_{ij} = \text{Re}[(\nabla \cdot \vec{\mathcal{E}}) \nabla_i \mathcal{E}_j^*] - \frac{1}{2} \delta_{ij} [|\vec{\mathcal{E}}|^4 + \nabla \cdot (\text{Re} \vec{\mathcal{E}}^* \nabla \cdot \vec{\mathcal{E}})]. \quad (4)$$

For localized fields, Eqs. (2a) and (2b) may be integrated to give the two conserved quantities  $N \equiv \int d^3r |\vec{\mathcal{E}}|^2$  and  $\vec{S} \equiv \int d^3r \vec{s}$ . In addition, the field energy,  $\mathcal{H}$ , is conserved:

$$\mathcal{H} = \frac{1}{2} \int d^3r [|\nabla \cdot \vec{\mathcal{E}}|^2 - |\vec{\mathcal{E}}|^4]. \quad (5)$$

In order to discuss particlelike behavior of a wave packet, it is useful to introduce the spatial average of any function  $f(r)$ , using  $|\vec{\mathcal{E}}|^2/N$  as a probability weighting factor:  $\langle f \rangle \equiv \int d^3r f(\vec{r}) |\vec{\mathcal{E}}|^2/N$ . Thus, we define for a wave packet its centroid coordinate  $\langle \vec{r} \rangle$ , its rms spatial width  $\langle \delta r^2 \rangle^{1/2}$  [where  $\delta r^2 \equiv |\vec{r} - \langle \vec{r} \rangle|^2$ ], and its mean intensity  $\langle |\vec{\mathcal{E}}|^2 \rangle$ . From (2a) we then obtain the Ehrenfest theorem,  $\partial_t \langle \vec{r} \rangle = \vec{S}/N = \text{const}$ . From Eqs. (2a) and (2b) and the fact that  $\vec{s} = \vec{p}$ , it is trivial to prove the following virial theorem:

$$\partial_t^2 \langle \delta r^2 \rangle = +2 \left[ \frac{1}{N} \int d^3r \text{Tr} \vec{T} - \left( \frac{\vec{S}}{N} \right)^2 \right]. \quad (6)$$

Using Eqs. (4) and (5) and integrating twice in time, this yields

$$\langle \delta r^2 \rangle = At^2 + Bt + C + (2-D) \int_0^t dt' \int_0^{t'} dt'' \langle |\mathcal{E}|^2 \rangle. \quad (7)$$

Here,  $D$  is the number of spatial dimensions,  $B$  and  $C$  are integration constants, and  $A$  is the following invariant:

$$A = 2\mathcal{K}/N - \vec{S}^2/N^2. \quad (8)$$

Since  $\langle \delta r^2 \rangle$  and  $\langle |\mathcal{E}|^2 \rangle$  are positive definite, it follows that collapse of  $\langle \delta r^2 \rangle$  to zero occurs in a finite time whenever  $A < 0$  and  $D \geq 2$ . The field  $\vec{\mathcal{E}}$  must then become singular,<sup>17</sup> since  $N$  is conserved. Physically, however, the collapse will stop when  $\langle \delta r^2 \rangle^{1/2}$  is a few Debye lengths, because of Landau damping,<sup>18</sup> which has been omitted from Eq. (1).

Let us now apply the virial theorem to study the stability of an initial wave packet in two dimensions. We assume that initially

$$\vec{\mathcal{E}}(\vec{r}, t=0) = -\varphi_0 \nabla \exp[\mathbf{\hat{a}}_{k_0} \cdot \vec{r} - \frac{1}{2} r^2 (\Delta k)^2], \quad (9)$$

where  $\varphi_0$  is a constant amplitude, related to the mean intensity by  $\langle |\mathcal{E}|^2 \rangle_0 = \frac{1}{2} k_0^2 \varphi_0^2$ . Assuming  $\Delta k \ll k_0$ , for this wave packet we find  $B=0$ ,  $C = \langle \delta r^2 \rangle_0 \approx (\Delta k)^{-2}$ , and  $A = 4[(\Delta k)^2 - \langle |\mathcal{E}|^2 \rangle_0]$ . The threshold and time for direct collapse are therefore given (in physical units) by

$$\frac{\langle |\mathcal{E}|^2 \rangle_{\text{th}}}{2\pi n(\theta_e + \theta_i)} = \frac{48(\Delta k)^2}{k_D^2}, \quad (10a)$$

$$\frac{1}{\omega_p t_c} = \frac{1}{2} \frac{\Delta k}{k_D} \left[ \frac{\langle |\mathcal{E}|^2 \rangle_0 - \langle |\mathcal{E}|^2 \rangle_{\text{th}}}{2\pi n(\theta_e + \theta_i)} \right]^{1/2}. \quad (10b)$$

Note that the threshold and collapse time are independent of  $k_0^2$ . An approximate calculation in three dimensions shows that these values are not greatly changed.<sup>16</sup>

In order to justify the adiabatic ion approximation which underlies Eq. (1), we must verify that the inertial term in the ion hydrodynamic equation is much smaller than the pressure term. Since the ion density in this approximation is proportional to  $|\mathcal{E}|^2$ , and the inertial term to  $\partial_i^2 |\mathcal{E}|^2 = \nabla \nabla : \vec{\mathbb{T}}$ , the comparison is easily made for the assumed initial wave packet. We find<sup>16</sup>

$$\frac{9}{2} \frac{k_0^2}{k_D^2}, \frac{\langle |\mathcal{E}|^2 \rangle_0}{32\pi n(\theta_e + \theta_i)} \ll \frac{m}{M}, \quad (11)$$

which is the adiabatic ion approximation.

In order to justify the electrostatic approximation, it is necessary<sup>13</sup> to have  $mc^2 |\nabla \times \nabla \times \vec{\mathcal{E}}| \ll 30_e |\nabla \nabla \cdot \vec{\mathcal{E}}|$ . This is automatically satisfied

at time  $t=0$ , since we begin with the pure electrostatic field given by Eq. (9). However, even with a pure electrostatic field,  $|\vec{\mathcal{E}}|^2 \vec{\mathcal{E}}$  will have a transverse component, which acts as a source for  $\nabla \times \nabla \times \vec{\mathcal{E}}$ . An approximate condition for the validity of the electrostatic approximation is

$$\Delta k \ll k_0, \quad (12)$$

which is the electrostatic approximation. In the late stages of collapse, both the adiabatic and electrostatic approximations usually break down.

One condition for the adiabatic ion approximation (11) is that  $k_0/k_D < (m/M)^{1/2}$ . For a beam instability,  $k_0$  is roughly determined to be  $v_e/v_b$ , the ratio of electron thermal to beam speed. Hence, the adiabatic ion condition can only be satisfied for very high-velocity beams, moving through relatively cold plasmas. A good example is provided by the electron beams in the solar corona, which are associated with type-III solar radio bursts.<sup>12,19</sup> Here  $v_b$  can be as large as  $c/2$ , and  $k_0/k_D$  can be on the order of  $10^{-2}$ . The half-width,  $\Delta k$ , is controlled by the beam width,  $\Delta v_b$ , ( $\approx v_b/3$ ), and can be of order  $k_0 \Delta v_b / 6v_b$ . Using this, the threshold value of  $W \equiv \langle |\mathcal{E}|^2 \rangle / 4\pi n \theta$  for direct collapse is found from Eq. (10a) to be  $10^{-5}$  assuming  $\Delta k = k_0/18$ . Values of  $W$  this high have been measured at about 0.5 a.u. by spacecraft<sup>20</sup> during type-III bursts. All of the inequalities of (11) and (12) can be satisfied initially for these parameters, and one could expect all but the final stages of the time history of direct collapse to be properly described by our virial theory, provided one can ignore the continuous input of energy into the beam modes by the beam, and provided secondary instabilities driven by the beam mode are slower than collapse. We have studied such effects<sup>21</sup> by numerical solutions<sup>12</sup> of a more general set of equations than (1).

In the numerical work,<sup>12</sup> dynamic ions are introduced through the use of a hydrodynamic equation for the ion density. Heavy linear damping of the ion-acoustic (quasi-) modes is included. In addition, in the equation for the envelope field,  $\vec{\mathcal{E}}$ , constant (quasilinear theory) growth rate for the beam modes is retained. In this work the real and envelope fields are related by  $E = 2 \text{Re}[\mathcal{E} \times \exp(-i\omega_p t)]$ , leading to a definition of  $W$  which is one-quarter that of the present Letter.

These equations were solved numerically, starting with initial white noise. The beam modes were observed to grow slowly, until they rose about a factor of 3 above the threshold for direct collapse, to a value  $W \equiv |\mathcal{E}|^2 / 4\pi n \theta = 4 \times 10^{-4}$ . At

this point a very rapid direct collapse sets in. The beam modes have no time to grow during this collapse, and, in fact, these waves are quickly depleted, as energy flows to regions of  $k$  space out of phase with the beam. Effectively, the plasma decouples from the beam; further growth of the beam modes becomes irrelevant, and the system evolves like an initial-value problem. In fact, a numerical solution with no beam, but with an initial random wave packet having  $W = 4 \times 10^{-4}$  and  $\Delta k = k_0/6$ , gives the identical subsequent evolution, which is illustrated in Fig. 1. The time for collapse obtained numerically is about  $9 \times 10^4 \omega_p^{-1}$ , compared with the prediction of Eq. (10b), which is  $7 \times 10^4 \omega_p^{-1}$ .

The quasilinear growth rate of the beam modes,  $\gamma_B \approx (n_b/3n_e)(v_b/\Delta v_b)^2 \omega_p$ , can be ignored during collapse if  $\gamma_B t_c \ll 1$ . Quasilinear beam-plateau formation cannot occur if the resonant modes never acquire an energy density comparable to that of the beam, or, equivalently, if  $W \ll 4(n_b/n_e)(v_b/v_e)^2$ . The energy density,  $W$ , in these two inequalities is effectively set by the collapse threshold to be several times  $48(\Delta k)^2$ , where  $\Delta k$  is determined by the beam width,  $\Delta v_b$ . We thus arrive at the following necessary ordering of beam parameters for the virial theory of direct collapse:  $9(\Delta v_b/v_b)^2(v_e/v_b)^4 \ll n_b/n_e \ll \frac{3}{2}(\Delta v_b/v_b)^4 \times (v_e/v_b)^2$ . For the type-III burst problem,  $n_b/n_e \approx 10^{-6}$ , and these inequalities are well satisfied.

It is also necessary that the collapse time,  $t_c$ , be short compared to the time for several  $e$  foldings of the secondary-wave instabilities driven by the beam modes, if the collapse is to be direct, rather than indirect. Typical of the competing secondary instabilities are induced (forward) scattering off ions and the forward-cone modulational instability.<sup>3,12</sup> The main parameter which determines whether collapse is faster than secondary instability is  $\Delta k$ , the width of the beam-mode wave packet. The collapse time  $t_c$  decreases with  $\Delta k$ , while the secondary instabilities take *longer* to develop as  $\Delta k$  increases. This is because the growth rate is reduced by so-called broadband pump effects.<sup>9,22</sup> Let  $\gamma_g^r$  be the (reduced) growth rate for one of these instabilities. The two conditions for direct collapse are therefore that the threshold (10a) be exceeded, *and* that  $\gamma_g^r t_c / \ln A \ll 1$ , where  $A$  is the noise amplification factor. Both are well satisfied for the parameters of the type-III burst problem.

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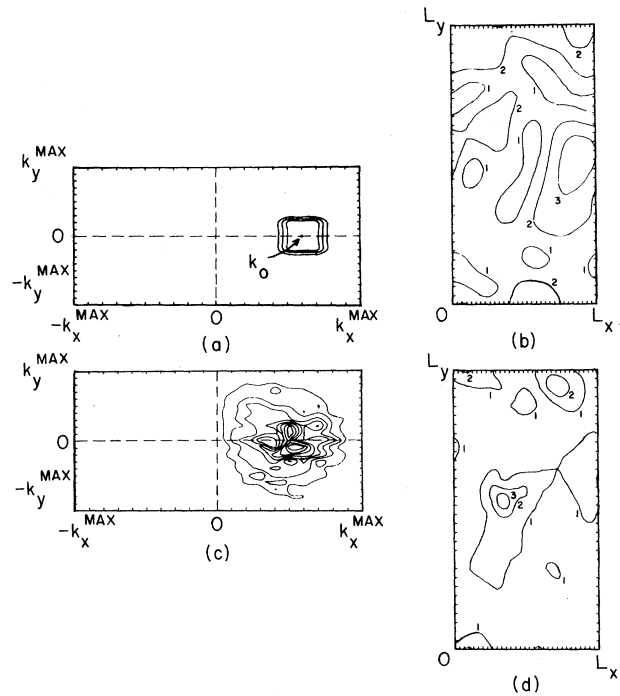


FIG. 1. Evolution of a set of waves, as found numerically by solving a set of equations with ion hydrodynamics included, but reducing to (1) in the present parameter regime (see Ref. 9). The initial spatially averaged electric field energy density is  $W = \langle |E|^2 \rangle / 4\pi n \theta_e = 4 \times 10^{-4}$ . The central wave number is  $k_0 \lambda_e = 0.011$ , while the full width in either wave-number direction of the initial spectrum is  $k_0/3$ . Numerically, the initial wave-number distribution consists of nine modes, each with a random phase. (a) Initial electric field amplitude in wave-number space. (b) Initial electric field amplitude in real space. Contours indicate relative absolute value of electric field, with contour 1 indicating  $W = 1.2 \times 10^{-4}$  and contour 3 indicating  $W = 10^{-3}$ . (c) Electric field amplitude in wave-number space at time  $9 \times 10^4 \omega_p^{-1}$ . (d) Electric field amplitude in real space at time  $9 \times 10^4 \omega_p^{-1}$ . Contours indicate relative absolute value of electric field, with contour 1 indicating  $W = 4.8 \times 10^{-4}$  and contour 3 indicating  $W = 4 \times 10^{-3}$ .

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## Discovery of an Internal-Friction Peak in the Metallic Glass Nb<sub>3</sub>Ge

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A well-defined internal-friction peak has been observed near 260 K in amorphous rf-sputtered films of Nb<sub>3</sub>Ge, studied at audio frequencies by a vibrating-reed technique. The characteristics of the peak are consistent with a stress-induced ordering mechanism involving a presently unidentified center which undergoes reorientation by an atomic jump with a sharply defined activation energy of 0.52 eV. The peak appears to be the first example of its type found in a metallic glass.

Metallic glasses are presently under intensive investigation from many points of view, and the eventual use of selected alloys in some applications now seems highly probable. However, much remains to be learned about the detailed structure of these metastable amorphous alloys, and particularly about the mechanism and kinetics of the internal atomic movements that ultimately control their stability. As is well known from much work on crystalline materials, internal-friction peaks associated with the stress-induced directional ordering of point defects or local structural groupings can provide valuable information in both of these areas.<sup>1</sup> The purpose of this Letter is to de-

scribe results on amorphous films of Nb<sub>3</sub>Ge which are thought to provide the first known example of such a peak in a metallic glass.

The measurements were performed on unsupported thin films by a vibrating-reed technique,<sup>2</sup> following the observation that amorphous films of Nb<sub>3</sub>Ge are mechanically much superior to their crystalline counterparts.<sup>3</sup> Amorphous films of about 6 μm thickness have been detached from their substrates and found to be strong, flexible, and easily handled, in marked contrast to the extreme brittleness and fragility produced by a crystallization anneal. Although amorphous Nb<sub>3</sub>Ge is a superconductor, the transition tem-