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Collective Resonances in Pion-Nucleus Scattering

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Pion-nucleus scattering is described by the excitation of bound nucleons into the Δ resonance leading to isobaric resonances of the whole nucleus. A few broad collective resonances of different multipolarity are shown to dominate elastic and inelastic π-^{12}C scattering. From the qualitatively good agreement with the data, we conclude that those giant isobaric resonances are a general feature of the nuclear excitation spectrum in the $\Delta(3,3)$ energy range.

It is well known that pion-nucleon scattering at intermediate energies is dominated by the $\Delta(3,3)$ resonance. From the large elementary cross section of $\sigma_{\pi^+p} \sim 200$ mb at the resonance energy, it is to be expected that a conventional multiple-scattering approach to pion-nucleus scattering converges slowly. A natural way of introducing many-body effects resulting from the strong pion-nucleon interaction is the explicit inclusion of the isobar degrees of freedom of bound nucleons. For such a description various related models have been developed during the last few years: the isobar doorway model of Kisslinger and Wang,[1] the collective model of Dillig and Huber,[2] and the multiple-scattering approach of Lenz and co-workers[3]; related aspects have been discussed by Brown and Weise.[4]

In this paper we adopt the approach of Ref. 2 to describe elastic and inelastic pion-^{12}C scattering: We assume that the incoming pion excites a bound nucleon into the $\Delta(3,3)$ resonance, thereby creating an isobaric particle-hole $(\Delta\bar{N})$ configuration. Since the Δ interacts strongly with the surrounding nucleons the various $(\Delta\bar{N})$ configurations of the same quantum numbers are coupled, thus leading to new eigenmodes of the whole nucleus. Those nuclear excitations, $|A_\nu^{*J^\pi}\rangle$, can decay emitting a pion thereby leaving the target nucleus in its ground or one of its excited states, respectively. The corresponding T matrix for pion-nucleus scattering then is given by

$$T(\vec{k},\vec{k}',\omega) = \sum_{J^\pi,M,\nu} \frac{\langle \vec{k}',f|\mathcal{L}^\dagger|A_\nu{}^{*J^\pi,M}\rangle\langle A_\nu{}^{*J^\pi,M}|\mathcal{L}|\vec{k},i\rangle}{\omega - \mathcal{E}_\nu{}^{J^\pi}}. \tag{1}$$

Here $|i,\vec{k}\rangle$ and $|f,\vec{k}'\rangle$ denote the initial and final states, $\mathcal{E}_\nu{}^{J^\pi}$ is the complex eigenenergy of the A^* resonances, $\mathcal{L}$ is the usual $\pi N\Delta$ transition operator,[4] and ω is the pion energy in the π-nucleus c.m. system.

For the actual calculations the Δ particle and the nucleons are assumed bound in a common harmonic oscillator potential, $h_{\rm ext}$ $(\hbar\omega = 41A^{-1/3})$. Accordingly, the resonance energies $\mathcal{E}_\nu{}^{J^\pi}$ of Eq. (1) are obtained as the (complex) eigenvalues of the many-body Hamiltonian

$$H = \sum_{i=1}^{A} [h_{\rm ext}(i) + h_{\rm int}(i)] + \frac{1}{2}\sum_{i\neq k} V_{\Delta N}(i,k). \tag{2}$$

Here $h_{\rm int}$ describes the internal degrees of freedom of the baryons. For $V_{N\Delta}$ we used a one-boson-exchange (OBE) interaction taking into account explicitly π and ρ exchange.[5-7] The π-exchange part is dominating and describes the coupling to the pion-nucleus elastic channel. Therefore in the OBE description the exchanged pion can be a real pion, propagating on its mass shell and thereby leading to a complex and energy-dependent ΔN interaction. The usual coupling constants[8] were used: $f_{\pi NN}^2/4\pi = 0.08$, $f_{\rho NN}^2/4\pi = 3.32$, $f_{\pi N\Delta}^2/4\pi = 0.3$, and SU(6) relations for $f_{\rho N\Delta}$, $f_{\pi\Delta\Delta}$, and $f_{\rho\Delta\Delta}$ ($f_{N\Delta}^2 = \frac{72}{25} f_{NN}^2$ and $f_{\Delta\Delta}^2 = \frac{1}{25} f_{NN}^2$). Furthermore, we introduced a monopole form factor in the potential ($\Lambda_\pi = 3.5 m_\pi$, $\Lambda_\rho = 7 m_\pi$).[9,10] This interaction has been diagonalized in a finite basis of isobaric p-h (particle-hole) configurations up to $5\hbar\omega$. The resulting A^* resonances then are given as a coherent superposition of p-h configurations:

$$|A_\nu^{*J^\pi}\rangle = \sum_{\alpha,\beta} C_{\alpha\beta}^{J^\pi,\nu} |[\Delta_\beta, \bar{N}_\alpha]^{J^\pi}\rangle, \tag{3}$$

where α and β denote the corresponding single-particle quantum numbers. As a general result of such a diagonalization it turns out that, for each partial wave, there are one or at most two resonances which gain a large elastic width and therefore dominate the scattering processes. A measure for their importance is the mesonic transition strength $S_\nu^{J^\pi}$ from the target ground state into the various A^* resonances, defined by

$$S_\nu^{J^\pi} = \sum_{\alpha,\beta} [C_{\alpha\beta}^{J^\pi,\nu}]^* \langle\!\langle \Delta_\beta \bar{N}_\alpha)^{J^\pi} | \mathfrak{L} | 0, \vec{k}\rangle. \tag{4}$$

In Table I this connection between $\Gamma_{J_\nu^\pi}^{\mathrm{el}}$ and $S_\nu^{J^\pi}$ is explicitly demonstrated for the 1^+ partial wave. It is obvious that also the integrated elastic cross section $\sigma_\nu^{J^\pi}$ of only one intermediate state $|A_\nu^{*J^\pi}\rangle$ has to reflect this dramatic dependence on $\Gamma_{J_\nu^\pi}^{\mathrm{el}}$. As a typical feature it turns out that for each multipolarity there exist a few (generally one or two) collective resonances with a large elastic width and transition strength. Similar results have also been found for ^{4}He and ^{16}O in Ref. 3.

TABLE I. Comparison of elastic width $\Gamma_{J_\nu^\pi}^{\mathrm{el}}$, mesonic transition strength $S_\nu^{J^\pi}$, and contribution $\sigma_\nu^{J^\pi}$ to the elastic cross section of the individual 1^+ resonances (up to $3\hbar\omega$ with $\delta\Gamma = 0$) in π-^{12}C scattering at $\omega = 290$ MeV.

ν	$S_\nu^{1^+}$	$\Gamma_{1^+,\nu}^{\mathrm{el}}$(MeV)	$\sigma_\nu^{1^+}$(mb)
1	$(-0.139 - 0.312i)\times 10^{-1}$	0.014	2.817×10^{-6}
2	$(-0.511 + 0.742i)\times 10^{-2}$	0.001	1.374×10^{-8}
3	$(0.155 - 0.172i)\times 10^{-1}$	0.005	6.026×10^{-7}
4	$(0.635 - 1.280i)\times 10^{-2}$	0.002	8.474×10^{-8}
5	$-0.107 + 0.120i$	0.217	1.489×10^{-3}
6	$(0.112 - 0.998i)\times 10^{-1}$	0.104	1.927×10^{-4}
7	$(0.766 - 4.832i)\times 10^{-1}$	2.116	1.297×10^{-1}
8	$(2.457 - 0.124i)\times 10^{-1}$	0.010	9.751×10^{-3}
9	$(0.358 + 0.130i)\times 10^{-1}$	0.014	6.727×10^{-6}
10	$(-1.970 - 0.750i)\times 10^{-2}$	0.002	6.405×10^{-7}
11	$-1.221 + 0.986i$	19.398	10.660
12	$1.588 + 2.512i$	85.130	44.962

This collective phenomenon is well known from low-energy nuclear physics; its physical background is best exhibited in the schematic model of Brown and Bolsterli.[11] Recently this model has been extended in Ref. 3 to pion-nucleus scattering. Because of this similarity in the description, the occurrence of those collective A^* resonances can be considered as giant (3, 3) resonances or giant isobaric resonances (GIR).[2]

Before comparing our results with the experimental data we add here comments on two technical details. Firstly, the OBE interaction $V_{N\Delta}$ of Eq. (2) generates the elastic width only. An additional damping has been introduced by an energy-dependent though state-independent width parameter $\Gamma(E)$:

$$\Gamma(E) = \Gamma_{\mathrm{free}}(E) + \delta\Gamma(E). \tag{5}$$

Here $\delta\Gamma$ supposedly incorporates in a rough approximation all the various medium corrections

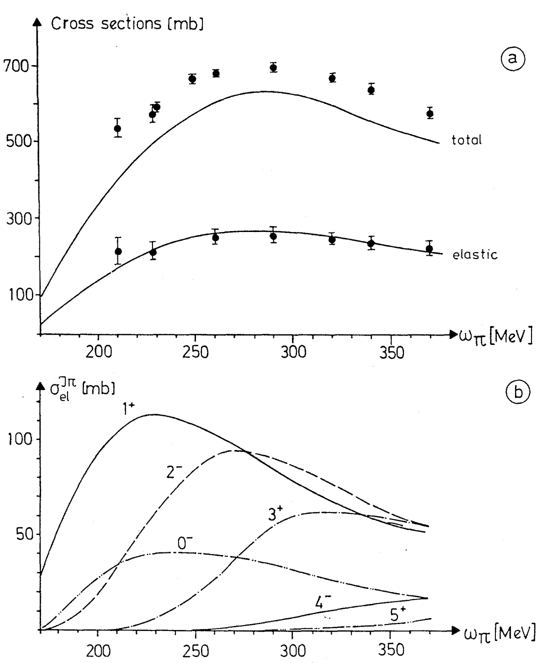

FIG. 1. (a) Total and total elastic cross sections for π-^{12}C scattering; experimental data from Ref. 12 throughout this paper. (b) Contributions of different partial waves to the total cross section.

to Γ_{free}, such as Pauli blocking,[3] binding effects,[3] true absorption, and spreading; $\delta\Gamma$ was adjusted to the total elastic cross sections: $\delta\Gamma/2$ turns out to vary smoothly between 26 and 16 MeV in the energy range 200 MeV $\leq \omega \leq$ 360 MeV. In Fig. 1(a) the results of our model are shown for σ_{el} and σ_{tot}. Obviously the effects mentioned above can be accounted for approximately by this simple parametrization.

Secondly, the kinematical transformation from the π-nucleus (ACM) to the π-nucleon c.m. system (NCM) has been performed similarly to Ref. 3. It yields for the π momentum

$$\vec{k}_{NCM} = \beta(\vec{k} - \alpha\vec{K}), \tag{6}$$

with $\beta = M/(M+\omega)$ and $\alpha = \omega/M$. Here, $\vec{k}$ and $\vec{K}$ denote the momenta of the pion and the bound nucleon in the ACM, respectively.

In Fig. 2 we present our results for the elastic π-^{12}C differential cross sections. The structure of the experimental data of Binon *et al.*[12] is reproduced rather satisfactorily indicating that our basic ideas about the excitation of isobaric nuclear resonances are realistic. Within the present model inelastic and elastic pion-nucleus scatter-

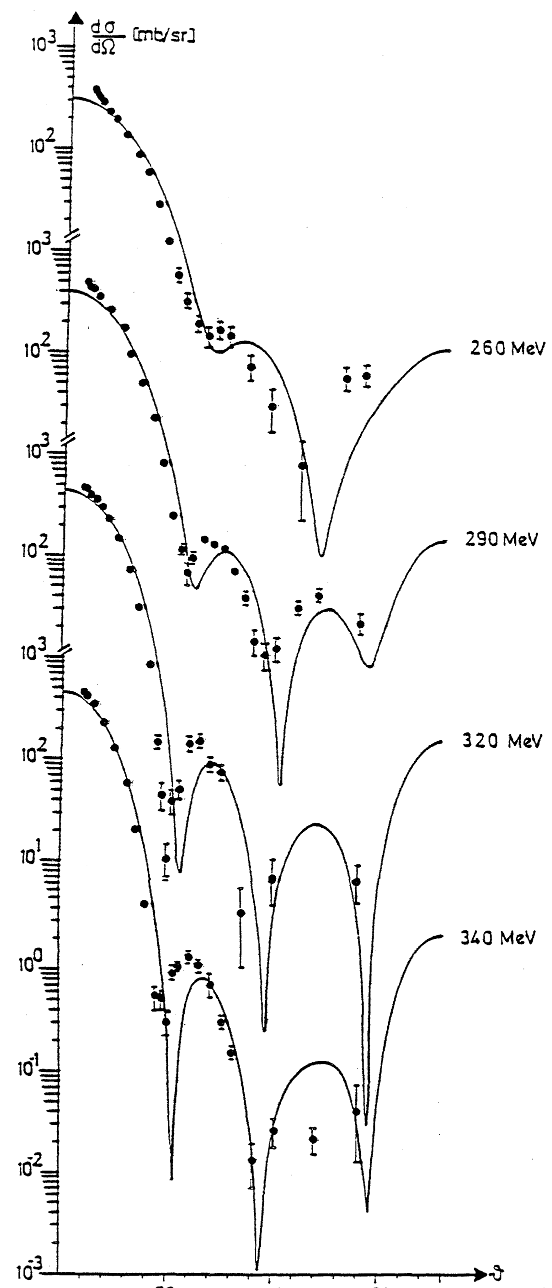

FIG. 2. π-^{12}C elastic scattering for different pion energies ω.

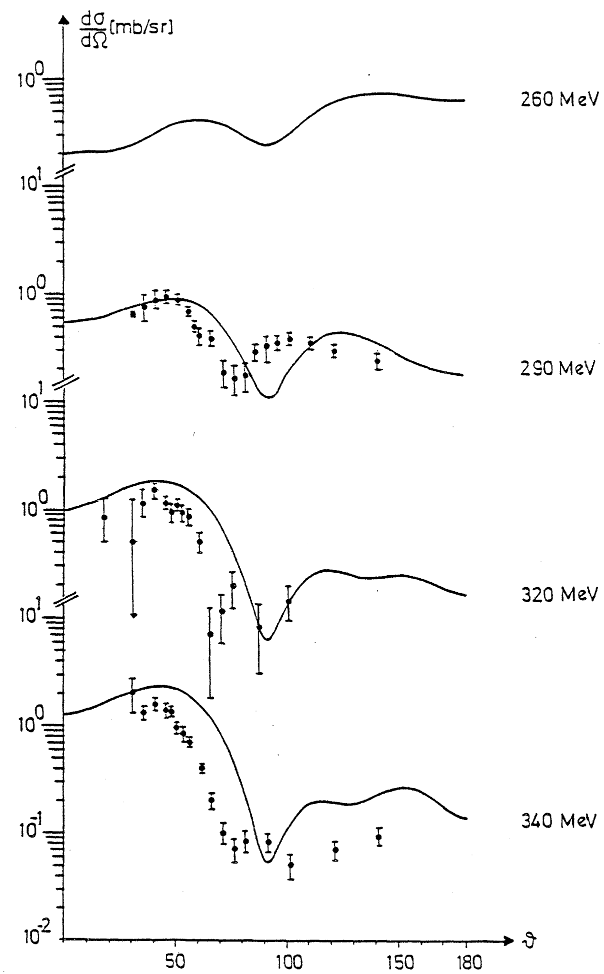

FIG. 3. π-^{12}C inelastic scattering into the 3^- state for different pion energies ω.

ing can be treated consistently on the same footing providing a further stringent test of our model. In Fig. 3 we present as an example the results for the excitation of the 3^- state in ^{12}C at 9.64 MeV [the wave function for this 3^- state has been obtained from a conventional 1p-1h random-phase approximation (RPA) calculation[13]]. As in the elastic case, both the magnitude and the structure of the experimental cross sections can be reproduced nicely. [In the calculation of the inelastic cross sections the kinematical transformation from Eq. (6) is not yet included for technical reasons.] This consistent overall description of elastic as well as inelastic pion-nucleus scattering indicates that the nuclear dynamics at intermediate energies are well described in the present model.

To obtain more insight into the dynamics of the scattering process we show in Fig. 1(b) the individual contributions of the various resonances to the elastic cross section. Evidently we are faced with the fact that the excitation spectrum of a complex nucleus in this energy region is characterized by a number of broad, strongly overlapping resonances of different multipolarities. The complex energies and the excitation strength distributions of those excitation modes reflect both the elementary πN interaction and the response of the nuclear many-body system to an incoming pion wave of definite angular momentum.

The strong dominance of a few broad collective resonances also explains the success of the phenomenological approach to the isobar doorway model (IDM) of Kisslinger and Wang[1] using the closure approximation to Eq. (1). However, the characteristic difference from the results of Ref. 1 is that in the present calculation all the differential cross sections for various energies show an increase for backward angles and the appearence of a third maximum for $\omega_\pi > 260$ MeV. This additional structure at large angles seems to be rather independent with respect to a moderate variation of the model parameters, whereas the magnitude of the cross section is more sensitive to finer details. Therefore large-angle scattering seems to provide particularly useful information about the internal dynamics of the intermediate doorway resonances.

Summarizing the results of our present investigation we find that the cross section for π-^{12}C scattering is built up by contributions from broad, strongly overlapping resonances of definite multipolarity and parity. For each partial wave only a few (one or at most two) collective A^* resonances dominate π-^{12}C scattering. Evidently those coherent nuclear excitations are a characteristic feature of the resonant π-nucleus scattering system. From their origin they reflect the pionic quantum numbers and can be visualized as pion currents interacting resonantly with the whole target nucleus. The collectivity of those giant isobaric resonances can be characterized by each of the quantities $\Gamma_{J_\nu}{}^{\pi\,\mathrm{el}}$, $S_\nu{}^{J\pi}$, or $\sigma_\nu{}^{J\pi}$.

In conclusion we remark that the present model allows a natural extension to other elementary (πN) or ($K^- N$) resonances: Explicit inclusion of the internal nucleonic degrees of freedom of a nucleus in such a particle-hole expansion could be the first step towards a unified description of medium-energy reactions.

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