the surface and therefore reduces the surface viscous damping. This helps explain the reduction in resistivity with increasing surface magnetic fields.

Within the present range of parameters the current channel does not pinch because the plasma pressure gradient (Fig. 2) is just sufficient to counteract the gradient of the self-pinching magnetic field, the pinching condition being $3nKT/(B_z^2/8\pi) < L_P/L_B$, where B_z is the magnetic field produced by the induced current and L_P and L_B are the gradient scale lengths of the plasma pressure and magnetic field, respectively. According to our data in Fig. 4 both nKT and B_z (or J_φ) scale linearly with the surface field. We expect current pinching to take place when the surface field and the induced current are further increased beyond the present marginal balancing situation.

This research was supported by the U. S. Energy Research and Development Administration.

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Stability of Bound Eigenmode Solutions for the Collisionless Universal Instability

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(Received 18 April 1978)

The collisionless universal instability in slab geometry with a sheared magnetic field is considered. It is shown from the differential eigenvalue equation that no bound growing eigensolutions exist. The method of solution consists of first assuming that exponentially growing eigenmodes exist and then forming quadratic quantities from which a contradiction is obtained. The relation between the eigenmodes and convective modes is discussed.

Recently, several authors have considered the stability of drift-wave eigenmodes in a sheared magnetic field.[1,2] By employing numerical and approximate analytical procedures, the conclusion was drawn that the discrete eigenmodes were stable. The proof of this conclusion is the subject of this Letter.

The stability problem will be approached by assuming that unstable, bound modes exist. Then, quadratic forms will be derived from the governing equations from which it will be seen that the initial assumption leads to a contradiction, hence proving that there are no bound eigenmodes.

The starting point is the well-known eigenvalue equation for the perturbed drift-wave potential in a sheared magnetic field,[1,2]

$$\frac{\partial^2\varphi}{\partial\xi^2}+\left\{\xi^2\frac{\omega_*^2 r_n^2}{\omega^2 L_s^2}-\left(1-\frac{\omega_*}{\omega}\right)\left[1+\frac{\omega}{|k_\parallel v_e|}Z\left(\frac{\omega}{|k_\parallel v_e|}\right)\right]-b^2\right\}\varphi=0, \tag{1}$$

where $\xi=x/\rho_s$, x is the direction of inhomogeneity, $\rho_s^2=cT_e/eB\Omega_i$, $\omega_*=k_y cT/eBr_n$, $r_n^{-1}=-n^{-1}(x)\times dn/dx$, $\Omega_i=eB_0/m_i c$, $\vec{B}=B_0(\hat{a}_z+x\hat{a}_y/L_s)$, ω is the mode frequency, $b^2=k_y^2\rho_s^2$, $k_\parallel=k_y x/L_s$, and Z is the plasma dispersion function. Equation (1) is derived by treating the ions as a cold fluid subject to the $\vec{E}\times\vec{B}$ and polarization drifts and retaining the effects of inertia when motion along the field is considered [here $\vec{E}=-\nabla\varphi(x)\exp(ik_y y-i\omega t)$ is assumed]. The electrons are governed by the drift kinetic equation with only $\vec{E}\times\vec{B}$ drift. Equation (1) is obtained by demanding the electron and ion density perturbations be quasineutral. It is assumed in equilibrium that only a density gradient is present.

Equation (1) is derived for real ξ (real x), but it can be considered to apply in the complex ξ plane. In particular, Eq. (1) will be considered as it applies on the line $\xi=-i\omega\eta/|\omega_*|$ where η is real and positive. On this line the electron contribution, $1+(\omega/|k_\parallel v_e|)Z(\omega/|k_\parallel v_e|)$, is found to be purely real and positive. Thus, quadratic forms integrated along this line (henceforth called the η line) will be particularly simple.

Before making the transformation $\xi=-i\omega\eta/$

$|\omega_*|$ in Eq. (1) the asymptotic behavior of $\varphi(\zeta)$ must be considered. It must be shown that if $\varphi(\zeta)$ is bound on the real ζ axis then it is bound on the η line as well. From Eq. (1) it is seen that as $\zeta \to \infty$

$$\varphi \sim \exp\left(-i\frac{|\omega_*|}{\omega}\left|\frac{r_n}{L_s}\right|\frac{\zeta^2}{2}\right) \tag{2}$$

is the appropriate asymptotic behavior when $\omega = \omega_r + i\gamma$, $\gamma > 0$. To determine the asymptotic behavior of $\varphi(\zeta)$ on the η line the location of the Stokes lines implied by Eq. (2) must be considered. Figure 1 shows the location of the η line and the Stokes lines in the complex ζ plane for the separate cases $\omega_r > 0$ and $\omega_r < 0$. In both cases a Stokes line lies between the η line and the real ζ axis. However, the asymptotic solution given by Eq. (2) is subdominant on these Stokes lines so that the transformation $\zeta = -i\omega\eta/|\omega_*|$ can be inserted in Eq. (2) to obtain the asymptotic behavior on the η line,

$$\varphi(\eta) \approx \exp\left(i\frac{\omega}{|\omega_*|}\left|\frac{r_n}{L_s}\right|\frac{\eta^2}{2}\right).$$

Thus, for a growing mode ($\omega = \omega_r + i\gamma$) $\varphi(\eta) \to 0$ for real η as $\eta \to \infty$. That $\varphi(\eta)$ decays in this manner quarantees that the continuation of the integration path has not crossed an anti-Stokes line.

Attention is now turned to the analytic continuation of the electron contribution from the real ζ axis to the η line. The electron term is given by

$$\frac{\omega}{|k_\parallel v_e|} Z\left(\frac{\omega}{|k_\parallel v_e|}\right) = \frac{1}{\pi^{1/2}}\int_{-\infty}^{\infty} \frac{\omega dt \exp(-t^2)}{t|\omega_*|a\zeta - \omega}, \tag{3}$$

where $a = k_y v_e \rho_s/(|\omega_*| L_s)$. Without loss of generality we take k_y and hence a to be positive. The causality condition ($\gamma > 0$) properly defines the path of integration in the t plane around the singularity in Eq. (3) for ζ real and positive. As ζ is rotated to the η line the singularity rotates in the opposite sense. The locations of the singularity in the t plane for ζ real and positive and ζ on the η line are shown in Fig. 2 again for the separate cases $\omega_r > 0$ and $\omega_r < 0$. In both cases the singularity rotates away from the integration contour

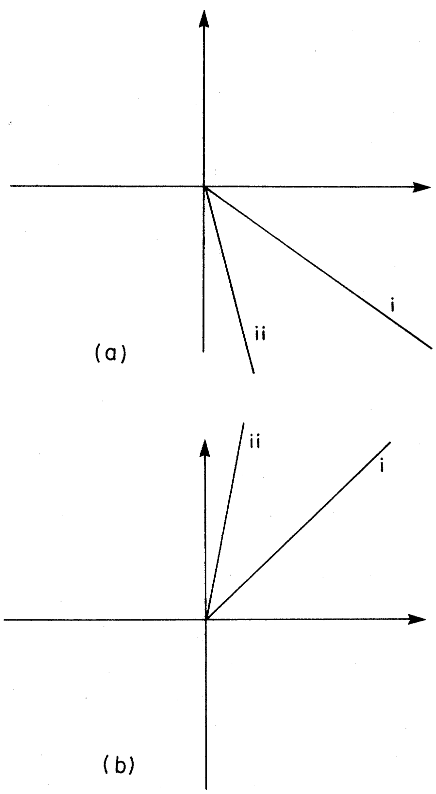

FIG. 1. The location of (i) the Stokes line and (ii) the η line in the ζ plane for (a) $\omega_r > 0$ and (b) $\omega_r < 0$.

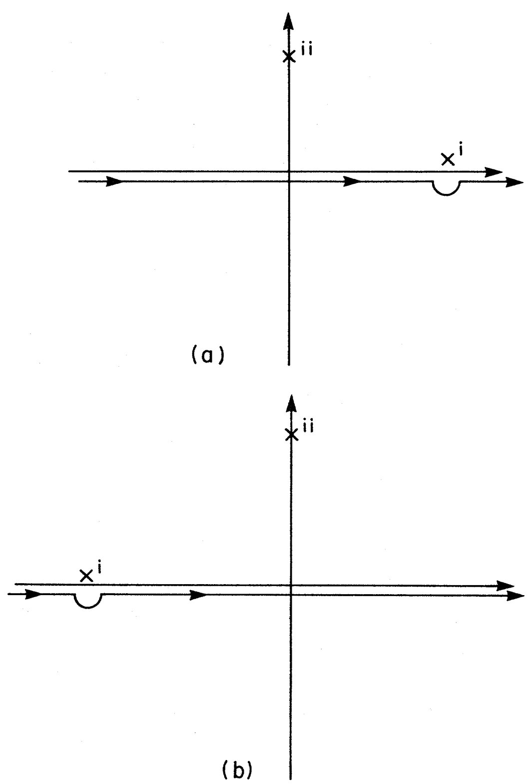

FIG. 2. The contour of integration and the location of the singularity of Eq. (3) in the t plane for (i) ζ real and (ii) ζ on the η line. Shown separately are the cases (a) $\omega_r > 0$ and (b) $\omega_r < 0$.

so that no deformation of the contour is necessary. The result is

$$1+\frac{\omega}{|k_{\parallel}v_e|}Z\left(\frac{\omega}{|k_{\parallel}v_e|}\right)\equiv g(\eta), \tag{4}$$

where

$$g(\eta)=\pi^{-1/2}\int dt\,\frac{\exp(-t^2)a^2\eta^2t^2}{1+a^2\eta^2t^2}\geqslant 0$$

[if only the resonant part of the Z function were considered $g(\eta)$ would be divergent].

Upon making the transformation $\zeta=-i\omega\eta/|\omega_*|$, Eq. (1) becomes

$$\frac{{\omega_*}^2}{\omega^2}\frac{\partial^2\varphi}{\partial\eta^2}+\left[\frac{{r_n}^2}{{L_s}^2}\eta^2+\left(1-\frac{\omega^*}{\omega}\right)g(\eta)+b^2\right]\varphi=0. \tag{5}$$

Multiplying Eq. (5) by φ^*, integrating from $\eta=0$ to $\eta=\infty$, taking into consideration that φ vanishes at $\eta=\infty$, and either φ or $\partial\varphi/\partial\eta$ vanishes at $\eta=0$, and separating real and imaginary parts yields the two quadratic forms,

$$\gamma\int_0^\infty d\eta\left(\frac{\omega_r{\omega_*}^2}{|\omega^2|^2}\left|\frac{\partial\varphi}{\partial\eta}\right|^2+|\varphi|^2\frac{g(\eta)\omega_*}{2|\omega|^2}\right)=0, \tag{6a}$$

and

$$\int_0^\infty d\eta\left\{\frac{{\omega_*}^2(\gamma^2-{\omega_r}^2)}{|\omega^2|^2}\left|\frac{\partial\varphi}{\partial\eta}\right|^2+|\varphi|^2\left[\eta^2\frac{{r_n}^2}{{L_s}^2}+b^2+\left(1-\frac{\omega_*\omega_r}{|\omega|^2}\right)g(\eta)\right]\right\}=0. \tag{6b}$$

If $\omega_r/\omega_*>0$, Eq. (6a) cannot be satisfied yielding a contradiction. Thus, there are no eigenmodes with $\omega_r/\omega_*>0$. If $\omega_r/\omega_*<0$, Eq. (6a) can be substituted into Eq. (6b) giving,

$$\int_0^\infty d\eta\left[\frac{{\omega_*}^2({\omega_r}^2+\gamma^2)}{|\omega^2|^2}\left|\frac{\partial\varphi}{\partial\eta}\right|^2+|\varphi|^2\left(\frac{{r_n}^2}{{L_s}^2}\eta^2+g(\eta)+b^2\right)\right]=0,$$

which cannot be satisfied. Thus, the assumption that bound, growing eigenmodes exist has led to a contradiction, thereby disproving the existence of such modes.

The preceding analysis has not shown that there is no universal instability, merely that there are no unstable eigenmodes. The stable eigenmodes which are bound in the sense of Pearlstein and Berk,[3] that is as $\zeta\to\infty$ ion Landau damping will cause the mode to decay, do not form a complete set. This can be seen by adopting the initial-value approach of Coppi *et al.*[4] If ω is treated as a Laplace variable and a function $f(\zeta,\omega)$ representing initial conditions is added to the right-hand side of Eq. (1) the general solution for $\varphi(\zeta,t)$ is given by

$$\varphi(\zeta,t)=\int_L^\infty\frac{d\omega}{2\pi}\int_{-\infty}^\infty d\zeta'\,\frac{\varphi_a(\zeta_>,\omega)\varphi_b(\zeta_<,\omega)}{W_{ab}(\omega)}f(\zeta',\omega)\exp(-i\omega t), \tag{7}$$

where φ_a and φ_b are homogeneous solutions of Eq. (1), $W_{ab}(\omega)$ is the Wronskian formed from φ_a and φ_b, $\zeta_>=\zeta$ if $\zeta>\zeta'$, $\zeta_>=\zeta'$ if $\zeta<\zeta'$, $\zeta_<=\zeta'$ if $\zeta>\zeta'$, $\zeta_<=\zeta$ if $\zeta<\zeta'$, and φ_a and φ_b satisfy the appropriate boundary condition [Eq. (2)] at $+\infty$ and $-\infty$, respectively. The contour L extends from $+\infty$ to $-\infty$ in the upper half of the ω plane. In order to determine the long-time behavior of $\varphi(\zeta,t)$ the contour L must be pushed into the lower-half ω plane. Clearly there will be contributions from the singularities $W_{ab}(\omega)=0$, (the eigenmodes) which have just been shown to occur for $\mathrm{Im}(\omega)<0$. There will also be contributions from the singularities of φ_a and φ_b (convective modes) which occur for $\omega=0$ [cf. Eq. (2)]. Thus, the long-time evolution of the initial disturbance is dominated by the convective modes. The stability condition in this case[4] is given by

$$\frac{L_s}{r_n}<10n_0\left(\frac{m_i}{m_e}\right)^{1/2}\left[\ln\left(\frac{m_i/m_e}{\ln3(m_i/m_e)^{1/2}}\right)\right]^{-1}, \tag{8}$$

where n_0 is the number of e foldings of a convectively unstable wave that can be tolerated as it propagates from $\zeta=0$ to $\zeta=\infty$ where it will be damped by the ions. This stability condition also applies to the test-particle noise-amplification problem,[5,6] in which case n_0 is a measure of the amount of noise amplification that can be tolerated. Equation (8) was found by means of a WKB method. However, for the large values of L_s/r_n required to give significant amplification the WKB approximation should be valid.

In conclusion, it has been shown that the eigenmodes of the collisionless universal instability are stable and that the long-time behavior of an initial disturbance is dominated by the convective[4] modes.

The author acknowledges interesting discussions with Professor Bruno Coppi and Professor Y. Y. Lau. This work was supported by the U. S. Department of Energy.

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Pseudoharmonic Theory of Orientational Instabilities in Physisorbed Layers

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(Received 19 April 1978)

The rotation of physisorbed monolayers with respect to the substrate was first predicted by Novaco and McTague in the harmonic approximation. An anharmonic theory of this effect is given here. The rotation takes place at the commensurate-incommensurate transition, provided the longitudinal sound velocity of the free adsorbate is more than twice as large as the transverse sound velocity.

Physisorbed monolayers often show phases which are incommensurate with the substrate.[1-3] Similar discommensuration effects have been observed in charge-density waves,[4,5] liquid crystals under certain conditions,[6,7] ferroelectrics like $NaNO_2$, and many magnetic materials. However, adsorbed layers have particular properties due to the simultaneous existence of transverse and longitudinal modes. For instance, in certain cases, the adsorbate is tilted with respect to the substrate. This effect was first predicted by Novaco and McTague, using the harmonic approximation.[8]

This paper gives an anharmonic treatment of the effect. One advantage of the approach is that the anharmonic theory can be applied near the commensurate-incommensurate (C-I) transition, when the harmonic approximation cannot be used. Furthermore, our treatment is simpler and yields the energy as an extremely simple function of the tilt angle, whereas Novaco and McTague obtained the energy as an infinite sum from which the dominant term is difficult to extract.

As suggested by Venables,[2] the adsorbed layer is treated as a succession of domains separated by walls. [The terminology "dislocation"[2] or "soliton" is sometimes used. We prefer the word "wall" for a $(D-1)$-dimensional steady defect, "dislocation" for a $(D-2)$-dimensional defect, and "soliton" for a propagating defect.] Inside each domain the system is assumed to be harmonic and nearly in registry with the substrate. Thus all anharmonic features are contained in the walls. This approach is especially appropriate near the C-I transition, when the distance l between walls is large with respect to the wall thickness $1/\kappa$.

Pseudoharmonic theory: Basic equations.—Let the adatoms be labeled by a D-dimensional vector index $r=(x_1, x_2, \ldots, x_D)$ and let $\vec{R}(\vec{r})=(X_1, X_2, \ldots, X_D)$ denote their position. For physical applications $D=2$. More precisely, $\vec{r}$ can be chosen as the position that the adatom would occupy at zero temperature in the absence of interaction with the substrate, i.e., in the "free" adsorbate. Then the components of the strain tensor are[9]

$$U_{\alpha\gamma}(\vec{r})=\tfrac{1}{2}\partial_\alpha[X_\gamma(\vec{r})-x_\gamma]+\tfrac{1}{2}\partial_\gamma[X_\alpha(\vec{r})-x_\alpha], \qquad (1)$$

where $\alpha, \gamma=1, 2, \ldots, D$ and $\partial_\alpha=\partial/\partial x_\alpha$. The adsorbate is treated as an elastic continuum.

The registered state corresponds to $\vec{R}(\vec{r})=C\vec{r}+B$, where C and B are constants. The appropriate generalization for the incommensurate phase

