Fluctuations Due to Zero-Point Motion of Surface Vibrations in Deep-Inelastic Rections

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The quantal fluctuations in energy loss, scattering angle, and angular momentum transfer in heavy-ion collisions due to the excitation of surface modes is studied in a classical model by utilizing a distribution of shapes, representing the zero-point fluctuations as initial conditions. For grazing impact parameters this leads to a clear separation between deep-inelastic and quasielastic events.

The classical treatment of heavy-ion reactions discussed by Broglia and co-workers[1-3] leads to deformations and to trajectories of relative motion, which at each instant of time are equal to the expectation values of the corresponding quantal variables insofar as the fluctuations in these quantities are small.

On the basis of the classical results one may calculate the expected spread in the energy and angular momentum dissipated in the two nuclei from the number of quanta absorbed in each vibrational mode. One thus finds

$$[\langle(E_{n\lambda\mu}-\langle E_{n\lambda\mu}\rangle)^2\rangle]^{1/2}=[\hbar\omega_{n\lambda\mu}\langle E_{n\lambda\mu}\rangle]^{1/2}, \qquad (1)$$

where $\langle E_{n\lambda\mu}\rangle$ is the expectation value of the energy $E_{n\lambda\mu}$ absorbed by the mode of frequency $\omega_{n\lambda}$. The mode is specified by the quantum numbers n, λ, and μ, where λ is the multipole order and μ is its component perpendicular to the scattering plane.

This estimate[3] of the fluctuation breaks down in situations where the energy loss or scattering angle varies rapidly with impact parameter, or even shows a discontinuity (cf. Hill and Wheeler[4]). This is typically the case for heavy-ion collisions, where the interaction between the two nuclei is a rapidly changing function of the distance between the surfaces and where discontinuities appear at the onset of fusion.

In order to give a more realistic estimate of the fluctuations, we utilize the fact that the statistical aspects of the excitation of harmonic vibrational states can be reproduced by starting the classical calculation with a statistical ensemble of initial conditions describing the zero-point fluctuations in the amplitude α and the conjugate momentum Π. Thus, with the Hamiltonian

$$H=\Pi^2/2D+\tfrac{1}{2}D\omega^2\alpha^2-F(t)\alpha, \qquad (2)$$

the solution $a(\Pi,\alpha,t)$ to Liouville's equation of motion for the phase-space distribution is related to the corresponding density matrix $\rho(\alpha,\alpha',t)$ by the Wigner transformation[5]

$$a(\Pi,\alpha,t)=(2\pi\hbar)^{-1}\int d\xi e^{i\xi\Pi/\hbar}\rho(\alpha+\tfrac{1}{2}\xi,\alpha-\tfrac{1}{2}\xi,t)$$

or

$$\rho(\alpha,\alpha',t)=\int d\Pi\, e^{i(\alpha'-\alpha)\Pi/\hbar}a(\Pi,\tfrac{1}{2}(\alpha+\alpha'),t). \qquad (3)$$

Here the momentum Π is a classical quantity. Utilizing (3) and the ground-state wave function of the oscillator, one finds that the ground state is to be described by a Gaussian phase-space distribution

$$a_0(\Pi,\alpha)=\frac{1}{\pi\hbar}\exp\left\{-\frac{\Pi^2/(2D)+\frac{1}{2}D\omega^2\alpha^2}{\frac{1}{2}\hbar\omega}\right\}. \qquad (4)$$

Classically we thus have an (unphysical) spread in the oscillator energy. However, the increase in the energy spread as well as the average energy calculated from the solution to Liouville's equation with a Hamiltonian given by Eq. (2) and with the initial distribution (4) at $t=-\infty$ are identical to the quantal results.

Moreover, since the angular momentum $L_{n\lambda}$ carried by a surface vibration of multipole order λ is related to the absorbed energy $E_{n\lambda\mu}$ through the relation

$$L_{n\lambda}=\sum_{\mu=-\lambda}^{\lambda}\hbar\mu\frac{E_{n\lambda\mu}}{\hbar\omega_{n\lambda}}, \qquad (5)$$

the increase in the average and in the spread of the angular momentum is the same in classical and quantal treatments, when the perturbation is linear in the vibrational amplitudes.

As an illustration of this approach we study the collision of Xe on Pb at 1130 MeV. We include vibrations of multipolarities 2^+, 3^-, 4^+, and 5^- in both nuclei with one low-frequency and one high-frequency mode (giant resonance) for each multipolarity. The calculations are performed as described in Ref. 2 except that both nuclei in a random way are given a deformation (in $\alpha_{n\lambda\mu}$ and $\Pi_{n\lambda\mu}$) according to (4), and that the damping of the surface modes is suppressed until the time of contact. For each multipole order λ we still only consider the components of vibration μ that fulfill the selection rule $(-1)^{\lambda+\mu}=+1$, which ensures that the trajectories remain in the scattering plane. The zero-point vibrations with $(-1)^{\lambda+\mu}=-1$ would, in fact, generate trajectories going out of the scattering plane.

The main features of the influence of zero-point vibrations are revealed in Figs. 1(a)–1(c), which show histograms of the final total kinetic energy E_f of relative motion in the center-of-mass system at different impact parameters near grazing. Each distribution can be separated into two different distributions, a quasielastic one for $E_f>660$ MeV and a deep-inelastic distribution for $E_f<660$ MeV. For decreasing impact parameters ρ, the total quasielastic component decreases and the deep-inelastic component becomes more symmetric with a decreasing mean value. For large values of ρ, the deep-inelastic component degenerates into a long tail. In Fig. 1(d) we compare our results for the average and the spread of E_f for quasielastic and deep-inelastic events with the results obtained in our previous calculations without zero-point vibrations.

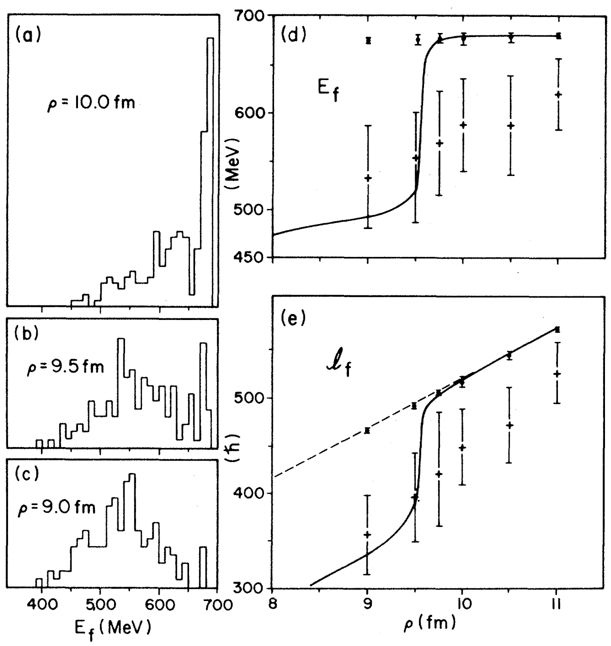

FIG. 1. Final kinetic energy and final angular momentum for collisions of Xe on Pb at a bombarding energy of 1130 MeV (683 MeV in the center-of-mass system) and with impact parameters close to grazing. In (a)–(c) are shown histograms of final kinetic energy for events with impact parameter ρ = 9, 9.5, and 10 fm. A rather clear separation is seen between a quasielastic peak and a broad deep-inelastic energy distribution. The average energies and the widths of these two distributions are shown in (d) as a function of the impact parameter together with the "average" curve obtained in Ref. 2 without zero-point fluctuations. In (e) is shown a similar plot of the final relative angular momentum.

A qualitative understanding of the separation of the grazing events into two groups can be obtained by interpreting the zero-point fluctuation as giving rise to a distribution in the effective impact parameter. By folding the average energy-loss curve $E_f(\rho')$ with a Gaussian distribution $G(\rho-\rho')$, one obtains two peaks which have about equal weight at the impact parameter $\rho=\rho_0$ corresponding to the inflection point in $E(\rho)$.

In Fig. 1(e) is shown the final angular momentum of relative motion for the quasielastic and deep-inelastic components and the result of the calculations without zero-point vibrations. This figure further supports the interpretation of Figs. 1(a)–1(d) already given.

In Fig. 2(a) is shown the correlation between the final energy and angular momentum at an impact parameter of 10 fm. The result shows a remarkable linear correlation,

$$L_f=c_0E_f+(\langle L_f\rangle-c_0\langle E_f\rangle)\,, \tag{6}$$

with $c_0=0.78\hbar/\text{MeV}$. Neglecting the effect of particle transfer, one can calculate the correlation factor on the basis of the model without zero-point vibrations. Since the correlation factor c_0 is given by

$$c_0=\frac{\langle EL\rangle-\langle E\rangle\langle L\rangle}{\langle(E-\langle E\rangle)^2\rangle}\,, \tag{7}$$

we obtain from Eqs. (5) and (1)

$$c_0=\sum_{n\lambda\mu}\langle E_{n\lambda\mu}\rangle\hbar\mu\,\Big(\sum_{n\lambda\mu}\langle E_{n\lambda\mu}\rangle\hbar\omega_{n\lambda}\Big)^{-1}. \tag{8}$$

With the values of $\langle E_{n\lambda\mu}\rangle$ calculated in the model without zero-point vibrations we obtain $c_0\simeq 0.5\hbar/$MeV for $\rho=10$ fm. The difference between this number and the empirical value quoted above is partly due to the effect of particle transfer.

In Fig. 2(b) is shown the correlation in final energy E_f and scattering angle $\theta_{\text{c.m.}}$ at the impact

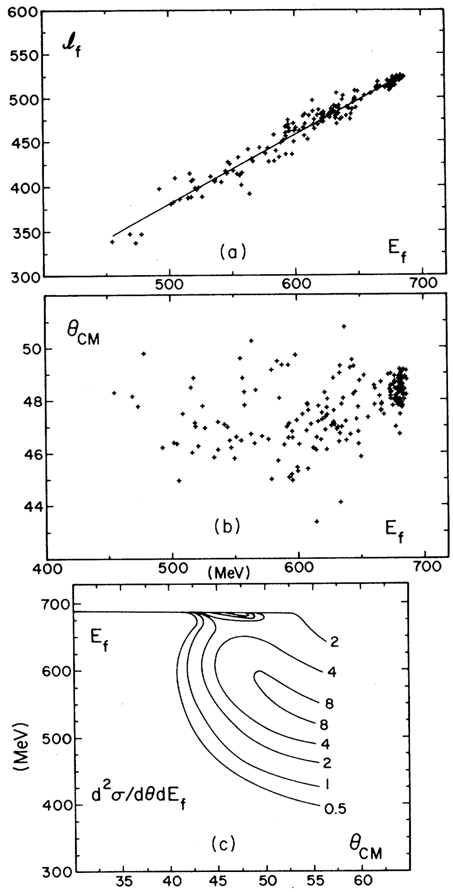

FIG. 2. Correlations between different observables in the collision between Xe and Pb at a bombarding energy of 1130 MeV. In (a) is shown the correlation between the final kinetic energy and the final relative angular momentum for an impact parameter of 10 fm. Each cross indicates an event. The straight-line fit has a slope of $c_0 = 0.78\hbar/\mathrm{MeV}$. For other impact parameters in the range 9–11 fm the slope changes between (0.74 and 0.80)$\hbar$/MeV. In (b) is similarly shown the correlation between scattering angle (in the center-of-mass system) and final kinetic energy E_f for an impact parameter of 10 fm. The spread in angle, though larger for the deep-inelastic than for the quasielastic component, is rather small. In (c) we indicate the relative cross section $d^2\sigma/d\theta\, dE$ as a function of θ and E_f. The curves indicate contours of equal cross sections.

parameter $\rho = 10$ fm. Also here there is a clear separation into quasielastic and deep-inelastic components. The rather small fluctuation in scattering angle is to be expected in the present case where the average deflection function does not show any rapid variation. In Fig. 2(c) we have constructed the inelastic cross section $d^2\sigma/dE\, d\theta$ from a statistical weighting of the results for different impact parameters. A comparison with the experimental results of Schröder and Huizenga[6] shows that the events fall inside the bulk part of the experimental inelastic cross section. We expect, furthermore, that a comparable fluctuation in scattering angle will result from a proper statistical treatment of particle transfer as well as from particle evaporation.[7]

From the calculations presented above we conclude that the fluctuations due to the zero-point motion of the surface are of crucial importance in heavy-ion collisions. It is expected that they would be of similar importance for time-dependent Hartree-Fock calculations. The quantitative value of the fluctuations is, however, somewhat uncertain because of a series of unsolved problems associated with the fact that the surface-surface interaction used in the calculations is empirically adjusted to reproduce the properties of the nuclear surface which is already influenced by the zero-point fluctuations.

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