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Unified Gauge Theories and the Baryon Number of the Universe

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I suggest that the dominance of matter over antimatter in the present universe is a consequence of baryon-number–nonconserving reactions in the very early fireball. Unified guage theories of weak, electromagnetic, and strong interactions provide a basis for such a conjecture and a computation in specific SU(5) models gives a small ratio of baryon- to photon-number density in rough agreement with observation.

It is known that the present universe is predominantly made of matter, at least in the local region around our galaxy, and there has been no indication observed[1] that antimatter may exist even in the entire universe. I assume here that in our universe matter indeed dominates over antimatter, and I ask within the framework of the standard big-bang cosmology[2] how this evolved from an initially symmetric configuration, namely an equal mixture of baryons and antibaryons. Since the baryon number is not associated with any fundamental principle of physics,[3] such an initial value seems highly desirable. I find in this paper that generation of the required baryon number is provided by grand unified gauge theories[4] of weak, electromagnetic, and strong interactions, which predict simultaneous violation of baryon-number conservation and *CP* invariance. More interestingly, my mechanism can explain why the ratio of the baryon- to the photon-number density in the present universe is so small, roughly of the order[2] of 10^{-8}–10^{-10}.

The essential point of my observation is that in the very early, hot universe the reaction rate of baryon-number–nonconserving processes, if they exist, may be enhanced by extremely high temperature and high density. In gauge models discussed below, the relevant scale of temperature is given by the grand unification mass around 10^{16} GeV where fundamental constituents, leptons and quarks, begin to become indistinguishable. This mass is high enough to make futile virtually all attempts to observe proton decay in the present universe: proton lifetime $\gg 10^{30}$ year.[5] Instead, if my mechanism works, we may say that a fossil of early grand unification has remained in the form of the present composition of the universe.

The laws obeyed by the hot universe at temperatures much above a typical hadron mass (~1 GeV) might, at first sight, appear hopelessly complicated because of many unknown aspects of hadron dynamics. Recent developments of high-energy physics, however, tell that perhaps the opposite is the case. At such high temperatures and densities hadrons largely overlap and an appropriate description of the system is given in terms of pointlike objects—quarks, gluons, leptons, and any other fundamentals. The asymptotic freedom[6] of the strong interaction and weakness of the other interactions further assure[7] that this hot universe is essentially in a thermal equilibrium state made of almost freely moving objects. I shall assume that this simple picture of the universe is correct up to a temperature close to the Planck mass, $G_N^{-1/2} \sim 10^{19}$ GeV, except possibly around the two transitional regions where spontaneously broken weak-electromagnetic and grand-unified gauge symmetries become restored.[8]

In such a hot universe the time development of the baryon-number density $N_B(t)$ is given by

$$\frac{dN_B}{dt} = -3\frac{\dot{R}}{R}N_B + \sum_{a,b}(\Delta n_B)\langle\sigma v\rangle N_a N_b. \qquad (1)$$

Here R is the cosmic scale factor; $\langle\sigma v\rangle$ is the thermal average of the reaction cross section for $a+b\to$anything times the relative velocity of a and b; Δn_B is the difference of baryon number between the final and initial states of this elementary process; N_a is the number density of a in thermal equilibrium. At the temperatures I am considering here, the energy density is dominated by highly relativistic particles and the following relation holds[2]:

$$\dot{R}/R = -\dot{T}/T = (8\pi\rho G_N/3)^{1/2}, \qquad (2)$$

where ρ is the energy density

$$\rho = d_F \pi^2 T^4/15. \qquad (3)$$

I use units such that the Boltzmann constant $k=1$. The effective number of degrees of freedom d_F is counted as usual, $\frac{1}{2}$ and $\frac{7}{16}$, respectively, for each boson or fermion species and spin state. To solve Eq. (1) with (2) and (3) given, it is convenient to rewrite (1) in the form

$$\frac{dF_B}{dT} = -(8\pi^3 G_N d_F/45)^{-1/2}(\tfrac{3}{8})^2 F_\gamma^{\,2}\delta, \qquad (4)$$

where $\delta = \sum(\Delta n_B)\langle\sigma v\rangle$; $F_B = N_B/T^3$; $N_a = 3N_\gamma/8$; $F_\gamma = N_\gamma/T^3$ with N_γ the photon-number density. I used the fact, valid in the following example, that only massless fermions participate in baryon-number–nonconserving reactions.

To obtain a nonvanishing baryon number one must break the microscopic detailed balance (more precisely, reciprocity), because otherwise the inverse reaction would cancel the baryon number gained. This necessity of simultaneous violation of baryon-number conservation and CP or T invariance has a further consequence that the amount of generated baryon number may be severely limited. This is because the detailed balance is known[9] *not* to be broken by Born terms, and one must deal with higher-order diagrams.

As an illustration of the ideas presented above, I shall work in grand unified models based on the group SU(5), which are direct generalizations of the original Georgi-Glashow model[4] to allow more than six flavors of quarks and heavy leptons sequentially in accord with recent experimental observations.[10] Fundamental fermions are thus classified as follows:

$$\underline{5},\quad (\varphi)_{iR} = \begin{pmatrix} l_1 & q_3{}^a \\ l_2 & \end{pmatrix}_{iR} \qquad (5a)$$

$$\underline{10},\quad (\psi)_{iL} = \begin{pmatrix} & q_1{}^a & \\ l_3 & & C\bar{q}_4{}^a \\ & q_2{}^a & \end{pmatrix}_{iL}, \qquad (5b)$$

with several sequences ($i=1-n_S$). Here $(l_1 l_2)$ and $(q_1 q_2)$ form $\mathrm{SU}(2)_W$ doublets and the others singlets; $a=1,2,3$ are three colors. The baryon number is assigned as $\frac{1}{3}$ to any $\mathrm{SU}(3)_C$ triplet and $-\frac{1}{3}$ to any antitriplet.

There are two characteristic mass scales, $m(W)$ and $m(\tilde{W})$, in this class of models with W a representative of ordinary weak bosons and with $\tilde{W}$ that of colored weak bosons. Existence of the two extremely different mass scales reflects two Higgs systems, $\tilde{H}(\underline{24})$ and (presumably several of) $\tilde{H}(\underline{5})$, being responsible for the breaking of SU(5) and $[\mathrm{SU}(2)\otimes\mathrm{U}(1)]_W$,[11] respectively. At the temperatures that most concern us, $m(W) \ll T \lesssim m(\tilde{W})$, the universe is effectively $[\mathrm{SU}(2)\otimes\mathrm{U}(1)]_W \otimes \mathrm{SU}(3)_C$ symmetric and all fermions remain massless, which makes subsequent computations easier.

The baryon nonconservation is caused by exchange of $\tilde{W}$ coupled to fermions,

$$(g/2\sqrt{2})\sum_i[(\bar{l}_1\bar{l}_2)\gamma_\alpha(q_3{}^c)_R + \bar{l}_3\gamma_\alpha(q_2{}^c, -q_1{}^c)_L + \epsilon_{abc}(\bar{q}_1{}^a\bar{q}_2{}^a)\gamma_\alpha(C\bar{q}_4{}^b)_L]_i\begin{pmatrix}\tilde{W}_1{}^c\\ \tilde{W}_2{}^c\end{pmatrix}_\alpha + (\mathrm{H.c.}), \qquad (6)$$

and by exchange of colored Higgs $H_i(\underline{5})$ contained in the full Yukawa coupling,

$$L_Y = \tfrac{1}{2}f_{ij}\bar{\psi}_i{}^{\alpha\beta}[H_1{}^\alpha(\varphi_j{}^\beta)_R - H_1{}^\beta(\varphi_j{}^\alpha)_R] + \tfrac{1}{4}h_{ij}\epsilon_{\alpha\beta\gamma\delta\eta}\psi_i{}^{\alpha\beta}C(\psi_j{}^{\gamma\delta})_L H_2{}^\eta + (\mathrm{H.c.}), \qquad (7)$$

where the two Higgs bosons of $\underline{5}$, H_1 and H_2, may or may not coincide. The Yukawa coupling constants in (7) are related to fermion masses when $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ is broken,

$$(f_{ij}) = (2\sqrt{2}G_\mathrm{F})^{1/2}\cos\zeta M_1, \qquad (8a)$$

$$(h_{ij}) = (2\sqrt{2}G_\mathrm{F})^{1/2}\sin\zeta U^T M_2 U, \qquad (8b)$$

where G_F is the Fermi constant, $G_F m_N{}^2 \simeq 10^{-5}$; M_1 and M_2 are diagonalized mass matrices for quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$, respectively [masses of charged leptons are equal to those of $q(-\frac{1}{3})$ except for renormalization effects[12]]; U is a unitary matrix that generalizes the Cabibbo rotation for more than three flavors.[13] To allow CP nonconservation also in the Higgs sector as in the Weinberg model[14] of CP nonconservation, I introduce the complex dimensionless parameters, α and β, in propagators by

$$i\langle T(H_1{}^{\dagger a} H_2{}^a)\rangle_{q=0} = \begin{cases} \alpha/m^2(W) \text{ for } a=1,2 \\ \beta/m^2(W) \text{ for } a=3-5, \end{cases} \tag{9}$$

where q is the momentum involved in propagation. This is possible only if more than three Higgs bosons $H_i(\underline{5})$ exist with complex, trilinear and quartic couplings to $\tilde{H}(\underline{24})$.

I now calculate the quantity δ in Eq. (4) by keeping only two-body reactions. Here it is reasonable to suppose that masses of Higgs bosons are of the order of $m(W)$ for uncolored and of $m(\tilde{W})$ for colored ones and fermion masses are $\ll m(W)$, and hence α, $\beta = O(1)$. Remarkably, I found after summing over all sequences that δ vanishes if α and β are real. I can actually prove that this is true to any order of perturbation. The result of this computation is particularly simple when $m(W) \ll T \ll m(\tilde{W})$. The dominant contribution to δ of (4) comes from interference of the diagrams of Figs. 1(a) and 1(b) and, after the thermal average is taken, leads to

$$\frac{\delta}{T^2} \simeq \frac{3g^2}{8\pi^2 m^4(\tilde{W})} \operatorname{Im}\beta\alpha^* [\operatorname{tr}hh^\dagger \operatorname{tr}f^2 - \operatorname{tr}hh^\dagger f^2]. \tag{10}$$

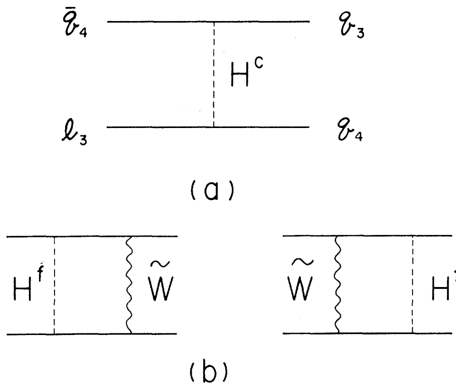

FIG. 1. Diagrams leading to generation of the baryon number. H^c (H^f) means a colored (uncolored) Higgs boson.

In this computation I ignored the Pauli exclusion effect in the final state caused by occupied thermal fermion states. Correct inclusion of this effect would not affect the result (10) drastically. Under the reasonable assumption $F_B \ll F_\gamma$ or $N_B \ll N_\gamma$, F_γ on the right-hand side of (4) may be approximated by the value at an initial temperature T_i where $N_B(T_i)=0$, and the rate equation (4) is integrated to

$$\frac{N_B(T)}{N_\gamma(T)} \simeq -(8\pi^3 G_N d_F/5)^{-1/2}(\tfrac{3}{8})^2 \frac{\delta}{T^2} N_\gamma(T_i). \tag{11}$$

During the evolution of universe from this high temperature T down to the recombination temperature (~4000°K) all fundamental constituents in the initial universe annihilate each other or go out of thermal contact, and the ratio (11) is roughly conserved to give finally the present value $(N_B/N_\gamma)_0$.

To obtain a rough quantitative idea of this ratio, I shall make a drastic extrapolation of formula (11) up to $T_i = m(\tilde{W})$. In the case of six flavors ($n_S = 3$) the best guess[12] for the parameters of the model is $g^2/4\pi = 0.022$, $m(\tilde{W}) = 2\times 10^{16}$ GeV. I also use the large difference of quark mass scales, $m(b) \gg m(s)$, $m(d)$ and $m(t) \gg m(c)$, $m(u)$, and the small mixing parameters[15] ($\lesssim 0.1$) in U of (8b). Combining (8), (10), and (11) and putting in some numerical factors, I find

$$\left(\frac{N_B}{N_\gamma}\right)_0 \approx 0.12 \frac{(d_F G_N)^{-1/2}}{m(\tilde{W})}\,\epsilon, \tag{12a}$$

$$\epsilon \simeq \frac{g^2}{\pi^2} G_F{}^2 m^2(t) m^2(b)(\sin\theta_2 + \sin\theta_3)^2 A, \tag{12b}$$

where $A = \operatorname{Im}\beta\alpha^* \sin^2\zeta \cos^2\zeta \approx O(1)$ in general. For definiteness, mixing angles are set by $\sin\theta_2 = \sin\theta_3 = 0.1$, which is consistent with present data.[15] Furthermore, $d_F = 63$ (69) with three H_i of $\underline{5}$ and real (complex) $\tilde{H}$ of $\underline{24}$, and hence

$$\left(\frac{N_B}{N_\gamma}\right)_0 \approx 2\times 10^{-9} \left[\frac{m(t)}{10\text{ GeV}}\right]^2 \left[\frac{m(b)}{5\text{ GeV}}\right]^2 A. \tag{13}$$

An estimate of this quantity[2] deduced from data ranges from 10^{-8} to 10^{-10}, which agrees with (13). The numerical value of (13) should not be taken too seriously because of the very crude approximations assumed, but it is hard to imagine that neglected corrections would alter (13) by more than 100.

Although my result (13) depends on the specific SU(5) model taken, the formula (12a) is presumably more general than this example provided that $\epsilon \approx$ (CP-invariance violation parameter)/137.

To this extent the small ratio of the number densities appears to be a general consequence of grand unification in the earliest history of our universe. The possibility that this fundamental parameter of cosmology is related to those of elementary-particle physics seems intriguing and deserves much investigation.

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