Strong P and T Noninvariances in a Superweak Theory

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We show, by explicit construction of a highly convergent superweak theory of CP nonconservation, that the problem of natural suppression of strong P and T noninvariances —in the presence of instantons—may be resolved without invoking symmetries which imply massless quarks or nearly massless bosons.

Recent proposals for securing natural suppression of strong P and T noninvariances[1-4]—which necessarily appear, in the presence of instantons, if neither P nor T is an exact symmetry of the entire Lagrangian—through the introduction of a global U(1) symmetry appear to have run into grave difficulties. If the symmetry is realized by setting the mass of one of the quarks equal to zero, one loses all the elegant features of chiral perturbation theory[5]; if the symmetry is realized in Nambu-Goldstone fashion, the concomitant pseudo Goldstone boson,[6] the axion, appears to be ruled out by experiment.[7] These difficulties underline the need for new theories[1] of CP nonconservation which can, without any additional symmetry input, ensure naturally the smallness of P and T noninvariances in strong interactions. Our purpose in this Letter is to provide an example of such a theory. The theory is of the superweak[8] type, and is such that the P and T noninvariance effects, which are transferable from the quantum flavordynamics (QFD) sector to the quantum chromodynamics (QCD) sector, are finite and first show up at the two-loop level.

To have a precise statement of the problem to be solved, let us assume that CP noninvariance arises in the QFD sector via preferably spontaneously[9] generated phases in the quark mass matrix. Diagonalization of the mass matrix then augments the QCD Lagrangian with a term of the form

$$\mathcal{L} = \frac{\theta_{\rm QFD}}{64\pi^2}\,\mathrm{Tr}\,\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}, \tag{1}$$

where G is the covariant curl of the color-gluon field matrix and

$$\theta_{\rm QFD} = i\ln(\det M/\det M^\dagger), \tag{2}$$

M being the $R \to L$ mass matrix.

If the phases in M lie in the range $\frac{1}{2}\pi - 10^{-3}$, and this covers a wide spectrum of available theories, the P- and T-nonconserving effects induced in the QCD sector have an absurdly large magnitude. The presence of instantons prevents us from discarding the interaction in Eq. (1), even though it is a total divergence, and thereby renders these effects observable. To control these effects we may add to the QCD Lagrangian a counter term of the same form as Eq. (1) and chosen such that

$$\theta_{\rm eff} = \theta_{\rm QCD} + \theta_{\rm QFD} = 0. \tag{3}$$

However, radiative corrections in the QFD sector will generate a

$$\delta\theta_{\rm eff} = i\,\mathrm{Tr}[\ln(1 + M_D^{-1}\delta M) - \ln(1 + M_D^{-1}\delta M^\dagger)], \tag{4}$$

δM being the radiative correction to the diagonalized mass matrix M_D. In theories of T nonconservation which are milliweak at the tree level and convergent enough to yield a finite $\delta\theta_{\rm eff}$, one typically obtains a value $\sim 10^{-3}\alpha/\pi$ which is still too large to be physically acceptable. One is therefore obliged to carry $\theta_{\rm QCD}$ as a free parameter—as one has to in theories in which $\delta\theta_{\rm eff}$ is infinite—and adjust it once again.

Our criteria for a natural resolution of the strong P and T problem are that

$$\theta_{\rm QFD} = \theta_{\rm QCD} = 0 \tag{5}$$

(naturally), and that $\delta\theta_{\rm eff}$ be a finite, calculable number of an acceptable order of magnitude ($\lesssim 10^{-10}$).

We proceed to describe a model in which the above criteria are met.

Any theory in which P or T is broken spontaneously will yield $\theta_{\rm QCD} = 0$. To secure $\theta_{\rm QFD} = 0$ we assume that the theory is characterized by the attribute of *manifest* left-right (L-R) symmetry[10]; in such theories $M = M^\dagger$ is a natural[11] relationship and the desired objective is painlessly attained. Now the smallest gauge group which accommodates manifest L-R symmetry, in a physically acceptable way, is $\mathcal{G} \equiv \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$; however, to have CP-nonconserving phases in fermion–gauge-field couplings, which survive in the presence of possible symmetries that prevent the mixing of the four "low-lying" (u,d,c,s) quark flavors with higher flavors $(t,b,\text{etc.})$, it is necessary to work with a larger gauge group (To have phases in the couplings of fermions to physical Higgs fields, the group enlargement is not necessary.[12] However, we require that all essential physics stem from the fermion gauge-field sector and assume that the plethora of parameters in the Higgs sector may be adjusted to meet this requirement.) The simplest possibility is $U(1)_N \otimes \mathcal{G}$; our considerations, in this paper, are based upon this group, with the current corresponding to the extra U(1)—before the onset of the Higgs mechanism—taken to be

$$J_\mu^N = (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) - (\bar{c}\gamma_\mu c + \bar{s}\gamma_\mu s). \tag{6}$$

To trigger the Higgs mechanism,[13] we take over the usual assortment of Higgs fields needed to make a go of $\mathcal{G}$-based models,[14] modified and extended as follows:

Fields transforming as $(\frac{1}{2},\frac{1}{2})_{Y=\pm 2}$, $(1,0)_{Y=0}$, and $(0,1)_{Y=0}$ are assigned $N=0$. Fields transforming as $(\frac{1}{2},\frac{1}{2})_{Y=0}$ are bracketed into three classes: Φ_a with $N=0$, Φ_b with $N=+2$, and Φ_c with $N=-2$, with the stipulation that under the operation of left-right conjugation $\Phi_a \to \Phi_a^\dagger$, $\Phi_b \to \Phi_c^{\dagger *}$ and $\Phi_c \to \Phi_b^\dagger$. For a wide range of parameters in the invariant Higgs potential we find[11]

$$\langle \Phi_a \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}, \quad \langle \Phi_b \rangle = \langle \Phi_c^\dagger \rangle = \begin{pmatrix} v_3 & 0 \\ 0 & v_4 e^{i\delta} \end{pmatrix}, \tag{7}$$

with all v_i real. The overall pattern, of vacuum expectation values, is such that parity nonconservation stems entirely from the unequal expectation values of the (1,0) and (0,1) fields—which, operationally, means unequal masses for the charged gauge fields W_L and W_R.

Equation (7) guarantees independent Hermitian mass matrices for the u-c and the d-s quarks, respectively, and also ensures, at the tree level, the absence of mixing between V, the $U(1)_N$ gauge field, and other neutral gauge fields. Diagonalization of the quark mass matrices leads to the following form, for the interactions relevant to the present context:

$$\begin{aligned}\mathcal{L}_I = (g/\sqrt{8})[\bar{\psi}_1\gamma_\mu(1-\gamma_5)U_C\psi_2\cdot W_L^\mu + \bar{\psi}_1\gamma_\mu(1+\gamma_5)U_C\psi_2\cdot W_R^\mu + \text{H.c.}] \\ + f[\bar{\psi}_1\gamma_\mu U_1^\dagger\tau_3 U_1\psi_1\cdot V^\mu + \bar{\psi}_2\gamma_\mu U_2^\dagger\tau_3 U_2\psi_2\cdot V^\mu] \\ + \text{other neutral-current–gauge-field couplings.}\end{aligned} \tag{8}$$

Here $\bar{\psi}_1 \equiv (\bar{u},\bar{c})$, $\bar{\psi}_2 \equiv (\bar{d},\bar{s})$, and the U_j $(j=1,2)$ are unitary matrices which may be parametrized in the Euler form:

$$U_j = \exp(i\tau_3\chi_j')\exp(i\tau_2\theta_j)\exp(-i\tau_3\chi_j). \tag{9}$$

Furthermore $U_C \equiv U_1^\dagger U_2$, the Cabibbo matrix, may, without loss of generality, be chosen to be orthogonal; this choice eliminates the phases χ_j' from Eq. (8) and implies the relationships

$$\begin{aligned}\tan\theta_C &= \tan(\theta_2-\theta_1)\,\frac{\cos(\chi_1-\chi_2)}{\cos(\chi_1+\chi_2)}, \\ &= \tan(\theta_2+\theta_1)\,\frac{\sin(\chi_1-\chi_2)}{\sin(\chi_1+\chi_2)},\end{aligned} \tag{10}$$

θ_C being the Cabibbo angle.

It is evident from Eq. (8) that CP nonconservation can arise if, and only if, none of the angles which occur in Eq. (10) vanish: In other words, if CP is good in the D_0-$\bar{D}_0$ system it will also be good in the K_0-$\bar{K}_0$ system. Note also, since $\chi_2 \to 0$ implies $\theta_1 \to 0$ and $\theta_2 \to \theta_C$, one cannot simultaneously choose χ_2 and θ_2 to be arbitrarily small. In the following we take χ_2 to be $O(1)$.

The experimentally known properties of the K_L-K_S system[15] are correctly reproduced if

$$(f^2/m_V^2)\sin 2\theta_2[\sin 2\theta_2 + O(G_F m_c^2 \sin\theta_C)] \cong 10^{-9} G_F, \tag{11}$$

where the second term in the square brackets includes loop corrections, involving W fields and concomitant Higgs fields, to the tree amplitude, m_c being the mass of the charm quark. Note that the correction is of $O(G_F)$ and not $O(\alpha)$, as it might be in a theory less convergent than ours; this feature permits us to choose θ_2 as small as 10^{-4} without running into stability problems (i.e., loops overwhelming trees). Thus if $f \sim e$, a natural identification if our gauge group is viewed as part of a larger group, we may satisfy Eq. (11) by choosing m_V as small as $\sim 4m_W$. The maximum value for m_V, allowed by Eq. (11), is $4\times 10^4 m_W$.

We may now calculate the loop corrections to the quark mass matrix. In the interest of simplicity, we ignore W_L-W_R mixing (which, fortunately, is phaseless at the tree level) and choose to work in the 't Hooft–Feynman gauge.[13]

Consider first the graphs involving only W and V fields. It is easy to see that one-loop graphs do not generate any phase-dependent terms in $\det(M_D+\delta M)$, and that the first such terms arise from the two-loop graphs of Figs. 1(a)–1(d). Less readily apparent, but not difficult to verify, is the following property of these graphs: They yield phases if, and only if, the helicity of the internal fermion line flips at least *three* times, with two flips occurring in the segment enclosed by the W propagator. Consequently, graphs contributing to $\delta M-\delta M^\dagger$ are individually convergent.

Next, let us consider graphs in which one gauge-field propagator is replaced by a linear combination of Higgs propagators. Again, only two-loop graphs are relevant and a finite value for $\delta M-\delta M^\dagger$ is obtained. However, an amusing feature is worth noting: The divergences cancel only if both unphysical and physical Higgs propagators are retained.

Very similar remarks may be made about the remaining graphs; without further ado, therefore, we display the final result:

$$\delta\theta_{\rm eff}\sim(m_s/m_d)(m_c/m_V)^2(\alpha/\pi)^2\sin 2\theta_2\sin 2\theta_C \cong 10^{-13}(m_V/m_{W_L})^{-1}. \quad (12)$$

Here we have omitted logarithms of mass ratios, factors of $O(1)$ and terms proportional to the squares of light-quark masses. Also Eq. (11), with $f\sim e$, has been used.

Equation (12) yields an acceptable value for $\delta\theta_{\rm eff}$ for any value for m_V in the allowed interval $4m_W \lesssim m_V < (4\times10^4)m_W$. The problem of natural suppression of strong P and T noninvariances may therefore be deemed to have been solved.

In conclusion we make the following remarks:

(i) Our discussion, geared to four quark flavors, can be modified without difficulty to accommodate higher flavors. Indeed, if there are symmetries which decouple the mass matrices of the higher quarks from those of the "low-lying" ones, the modification is trivial.

(ii) We have tacitly assumed that the parameters in the Higgs sector may be chosen such that contributions to CP-nonconserving amplitudes from physical-Higgs exchange tree graphs may be ignored. However, we found it necessary to make use of physical Higgs fields to cancel infinities at the loop level. Problems of consistency, which may conceivably arise in QFD if kinematical Higgs fields are required to solve all divergence difficulties but otherwise stay in the background, remain to be investigated.

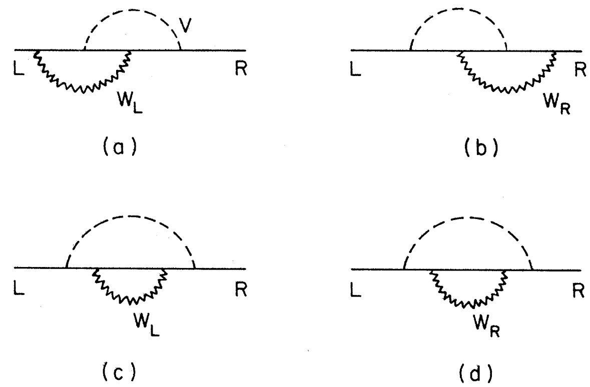

FIG. 1. Proper self-energy graphs which generate phases in the mass-matrix.

(iii) An interesting, and important, question is whether our model exemplifies a more general result. More precisely, does the requirement of natural suppression of strong P and T noninvariances—without benefit of axions and massless quarks—force CP nonconservation to be of the superweak type? This matter will be discussed elsewhere.

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[11]A discrete symmetry has been used to ensure that $\langle\Phi_b\rangle = \langle\Phi_c^\dagger\rangle$ is a natural relationship. The word "natural," as used here and as applied to the notion of manifest left-right symmetry, means that one has a solution, of the Higgs-potential minimization problem, which is stable under inclusion of loop corrections.

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Unified Gauge Theories and the Baryon Number of the Universe

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I suggest that the dominance of matter over antimatter in the present universe is a consequence of baryon-number–nonconserving reactions in the very early fireball. Unified guage theories of weak, electromagnetic, and strong interactions provide a basis for such a conjecture and a computation in specific SU(5) models gives a small ratio of baryon- to photon-number density in rough agreement with observation.

It is known that the present universe is predominantly made of matter, at least in the local region around our galaxy, and there has been no indication observed[1] that antimatter may exist even in the entire universe. I assume here that in our universe matter indeed dominates over antimatter, and I ask within the framework of the standard big-bang cosmology[2] how this evolved from an initially symmetric configuration, namely an equal mixture of baryons and antibaryons. Since the baryon number is not associated with any fundamental principle of physics,[3] such an initial value seems highly desirable. I find in this paper that generation of the required baryon number is provided by grand unified gauge theories[4] of weak, electromagnetic, and strong interactions, which predict simultaneous violation of baryon-number conservation and *CP* invariance. More interestingly, my mechanism can explain why the ratio of the baryon- to the photon-number density in the present universe is so small, roughly of the order[2] of 10^{-8}–10^{-10}.

The essential point of my observation is that in the very early, hot universe the reaction rate of baryon-number–nonconserving processes, if they exist, may be enhanced by extremely high temperature and high density. In gauge models discussed below, the relevant scale of temperature is given by the grand unification mass around 10^{16} GeV where fundamental constituents, leptons and quarks, begin to become indistinguishable. This mass is high enough to make futile virtually all attempts to observe proton decay in the present universe: proton lifetime $\gg 10^{30}$ year.[5] Instead, if my mechanism works, we may say that a fossil of early grand unification has remained in the form of the present composition of the universe.

The laws obeyed by the hot universe at temperatures much above a typical hadron mass (~1 GeV) might, at first sight, appear hopelessly complicated because of many unknown aspects of hadron dynamics. Recent developments of high-energy physics, however, tell that perhaps the opposite is the case. At such high temperatures and densities hadrons largely overlap and an appropriate description of the system is given in terms of pointlike objects—quarks, gluons, leptons, and any other fundamentals. The asymptotic freedom[6] of the strong interaction and weakness of the other interactions further assure[7] that this hot universe is essentially in a thermal equilibrium state made of almost freely moving objects. I shall assume that this simple picture of the universe is correct up to a temperature close to the Planck mass, $G_N^{-1/2} \sim 10^{19}$ GeV, except possibly around the two transitional regions where spontaneously broken weak-electromagnetic and grand-unified gauge symmetries become re-

