Real Part of the p-p, p-d, and p-n Forward Scattering Amplitudes from 50 to 400 GeV

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Proton-proton and proton-deuteron elastic scattering has been measured for incident laboratory energy from 50 to 400 GeV; minimum $|t|$ values were, for p-p, 0.0005 (GeV/c)^2, and for p-d, 0.0008 (GeV/c)^2. From the differential cross sections we have determined the ratios of the real to imaginary parts of the forward scattering amplitude, ρ_{pp} and ρ_{pd}, for p-p and p-d scattering. Using a Glauber approach and a sum-of-exponentials form factor we obtain ρ_{pn} for p-n scattering.

Previous experiments at Fermilab have yielded measurements of the ratio of the real to imaginary parts of the forward scattering amplitude, ρ_{pp}, for pp scattering[1,2] as well as the slope of the forward diffraction peak, b, of p-p[3] and p-d scattering.[4] Recent measurements have been reported from the intersecting storage rings at CERN on ρ_{pp} and the resultant dispersion-relation implications on σ_{pp} at ultrahigh energies.[5] Measurements at Serpukhov on p-d elastic scattering below 70 GeV have yielded a parametrization of the deuteron form factor, $|S(t)|$, and ρ_{pd}, for p-d scattering.[6,7] Further analysis using a Glauber approach has resulted in values reported from the Serpukhov data for ρ_{pn}, for p-n scattering.[8]

We have extended these measurements to higher energies and with lower momentum transfer, $|t|$, further into the Coulomb and Coulomb-nuclear interference regions. It is observed that ρ_{pp} rises and crosses through zero at approximately 335 GeV, a consequence and reflection of the rise in the total cross section. Isospin invariance in strong interactions makes the prediction that asymptotically at high energies we would expect to find $\rho_{pp} \simeq \rho_{pn}$. Our motivation was to test this prediction at high energies and in a single experimental setup.

The circulating beam in the Fermilab accelerator intercepted a gas-jet hydrogen (deuterium) target of thickness 2×10^{-8} g/cm^2 and width ±6 mm. Recoil particles were detected at a distance of 7.5 m by sets of totally depleted surface-barrier silicon detectors with typical dimensions 4×20 mm^2. The angular resolution was ±0.8 mrad, a large improvement over previous experiments.[1-4] The front detectors ranged from 15 to 250 μm thick and the back detectors from 200 to 1500 μm. Two permanently fixed stacks of detectors were used to monitor the jet-beam interaction rate. During readout of a stack the inputs to all other stacks were inhibited. Thus all channels had the same precentage of dead time ($\lesssim3\%$).

The $|t|$ values studied were $0.0005<|t|<0.03$ (GeV/c)^2 for hydrogen and $0.0008<|t|<0.08$ (GeV/c)^2 for deuterium. In our t range multiple scattering of the outgoing recoil particle was small.

The method used to separate proton and deuteron recoils is described elsewhere.[4] The recoil momentum spectra were fitted over the range $>\pm5.0\sigma$ by a formula which contained Gaussian plus exponential (background) terms. The number of elastic events was obtained after applying cuts at $\pm4\sigma$ and subtracting the background determined from the fit. The background was ~1% except for the lowest-$|t|$ deuteron data where background was ~3%. The correct normalization was obtained in the final fits from the optical theorem and is uncertain by $\simeq0.7\%$ for p-p and $\simeq0.5\%$ for p-d.

The detectors were calibrated with a $_{90}$Th^{234} α-particle source. The absolute angles determined from the elastic peak and α-particle energy calibrations when compared with survey measurements show an offset difference of $\lesssim0.15$ mrad. We estimate our angle uncertainty at $\lesssim0.05$ mrad. The magnetic field in our detector system was reduced by shielding to $\lesssim0.03$ G in order to minimize angular errors at very low $|t|$. The main contribution to the systematic error in ρ_{pp} (±0.009) and in ρ_{pd} (±0.008) is the angular uncertainty in the position of the detectors. Other systematic errors are the uncertainty in detector area ($\pm0.15\%$) and nuclear interactions in the detector. The latter are approximately proportional to the particle range and $\lesssim0.07\%$ for our $|t|$ range. Tables of pp and pd differential cross sections as well as more details on the analysis will be published elsewhere.[9]

The differential cross sections are fitted by the

Bethe interference formula[10]

$$d\sigma/dt = \pi |f_n + f_c|^2, \quad (1)$$

where the nuclear and Coulomb scattering amplitudes have the forms

$$f_n = (\sigma_{tot}/4\pi\hbar)(\rho_{pp} + i)e^{bt/2}, \quad (2)$$

$$f_c = (2\alpha\hbar/t)G_p^2(t)e^{i\alpha\varphi}. \quad (3)$$

The free parameters in the fit are ρ_{pp} and the overall normalization. We have assumed that the real and imaginary parts have the same t dependence and that spin effects may be neglected. For σ_{tot} and b we use empirical fits to the data of Carroll *et al.*,[11] $\sigma_{tot} = (50.866 - 5.2302 \ln s_{pp} + 0.5437 \ln^2 s_{pp})$ mb and Bartenev *et al.*,[3] $b(s) = b_{pp} = 8.27 + 0.556 \ln s_{pp}$.[12] α is the fine-structure constant, $G_p(t) = (1 + |t|/0.71)^{-2}$ the proton electromagnetic form factor, and $\alpha\varphi = 2\alpha \ln(1.06\hbar/R\sqrt{|t|})$ the Coulomb phase with $R = \sqrt{10}$ mb$^{1/2}$.

TABLE I. The ratio of the real to the imaginary part of the forward scattering amplitude as a function of energy.

E_{lab} (GeV)	$\rho = \mathrm{Re}A/\mathrm{Im}A$	$\Delta\rho_{stat}$	$\Delta\rho/\Delta b$ [(GeV/c)^2]	$\Delta\rho/\Delta\sigma$ (mb^{-1})
Reaction $pp \to pp$ (systematic error in ρ ± 0.009)				
52	-0.153	0.012	0.038	-0.033
80	-0.096	0.010	0.036	-0.035
199	-0.034	0.009	0.025	-0.039
261	-0.009	0.009	0.023	-0.032
303	-0.011	0.008	0.024	-0.083
398	$+0.012$	0.009	0.022	-0.028
Reaction $pp \to pd$ (systematic error in ρ ± 0.008)				
49	-0.110	0.013	0.059	-0.011
82	-0.113	0.010	0.065	-0.011
182	-0.037	0.009	0.058	-0.008
281	-0.031	0.011	0.060	-0.007
379	$+0.003$	0.010	0.046	-0.007
397	$+0.034$	0.010	0.057	-0.008

The results for ρ_{pp} are listed in Table I. Typical χ^2 are 1.2–1.3 per degree of freedom. In Fig. 1(a) we show these values together with previous published results.[5,13] Results from Refs. 1 and 2 are not plotted but agree with the present experiment. An empirical expression valid within our energy range is

$$\rho_{pp}(s) = (-0.490 \pm 0.034) + (0.076 \pm 0.006) \ln s_{pp}. \quad (4)$$

The solid curve shown in Fig. 1(a) is taken from a dispersion relation.[5] The normalization of the curve is arbitrary using our data and that of Amaldi *et al.*[5] Our pp data show good agreement with the dispersion relation calculation. By comparing our pp data with the prediction obtained from a simple vacuum exchange model [dashed curve in Fig. 1(a)] we can estimate the importance of other exchanges.

The p-d elastic cross sections have also been fitted with Eq. (3), where we treat the deuteron as a single particle. In this case the nuclear amplitude is parametrized as a sum of exponentials,[4,14]

$$f_n = (\sigma_{tot}/4\pi\hbar)(\rho_{pd} + i)e^{bt/2}[0.34e^{141.5t/4} + 0.58e^{26.1t/4} + 0.08e^{15.5t/4}], \quad (5)$$

where we assume that[4] $b(s) = 8.46 + 0.94 \ln s_{pp}$ and use an empirical fit to the published pd total cross section[11,15] $\sigma_{tot} = (99.73 - 9.40 \ln s_{pd} + 0.829 \times \ln^2 s_{pd})$ mb. The Coulomb amplitude is written as

$$f_c = (2\alpha\hbar/t)G_p(t)G_d(t)e^{i\alpha\varphi} \quad (6)$$

with $G_d(t) = \exp[(25.95t + 60t^2)/2$ and $\alpha\varphi = 2\alpha \ln(1.06\hbar/R\sqrt{|t|})$ with $R = 2.7\sqrt{10}$ mb$^{1/2}$.

In Table I we list our results for ρ_{pd}. In Fig. 1(b) we show these values together with previous published results.[6] The line shown is taken from our $\rho_{pp}(s)$ parametrization, Eq. (4). An empirical expression valid in our energy range is

$$\rho_{pd} = (-0.450 \pm 0.035) + (0.070 \pm 0.006) \ln s_{pp}. \quad (7)$$

ρ_{pp} and ρ_{pd} both cross zero at about 335 GeV. The deuteron forward scattering amplitude can be written as $f_d = f_p + f_n + i\hbar \langle r^{-2}\rangle_d f_p f_n$, where the deuteron inverse radius squared is $\langle r^{-2}\rangle_d \simeq 0.033$ mb^{-1} (approximately equal to I_G discussed in the next paragraph). We define the Glauber screening parameter $\delta \equiv \langle r^{-2}\rangle_d \sigma_{tot}/4\pi \simeq 0.111$. If we assume that the forward proton and neutron amplitudes are equal, $f_n \simeq f_p = \sigma_{tot}(\rho_{pp} + i)/4\pi\hbar$, then, omitting the small ρ_{pp}^2 term, $f_d \simeq \sigma_{tot}[\rho_{pp}(1-\delta) + i(1-\delta/2)]/2\pi\hbar$. The result is that we would expect $\rho_{pd} = \rho_{pp}(1-\delta)/(1-\delta/2) \simeq 0.941\rho_{pp}$. Both ρ_{pp} and ρ_{pd} change sign at the same energy as predicted. Using the empirical fits to our data, $\rho = \rho_0 + \rho_1 \ln s$ as expressed in Eqs. (4) and (8), we find $\rho_{1pd}/\rho_{1pp} = 0.92 \pm 0.11$, also in agreement with the prediction. Since the derivation is based upon the equivalence of the proton and neutron, our results support isospin invariance in proton-neutron collisions at high energies.

In the Glauber approach[13,16-18] elastic p-d scattering is described as a coherent sum of Coulomb, single-nucleon, and double-nucleon scattering. Assuming only s-wave contribution and only elas-

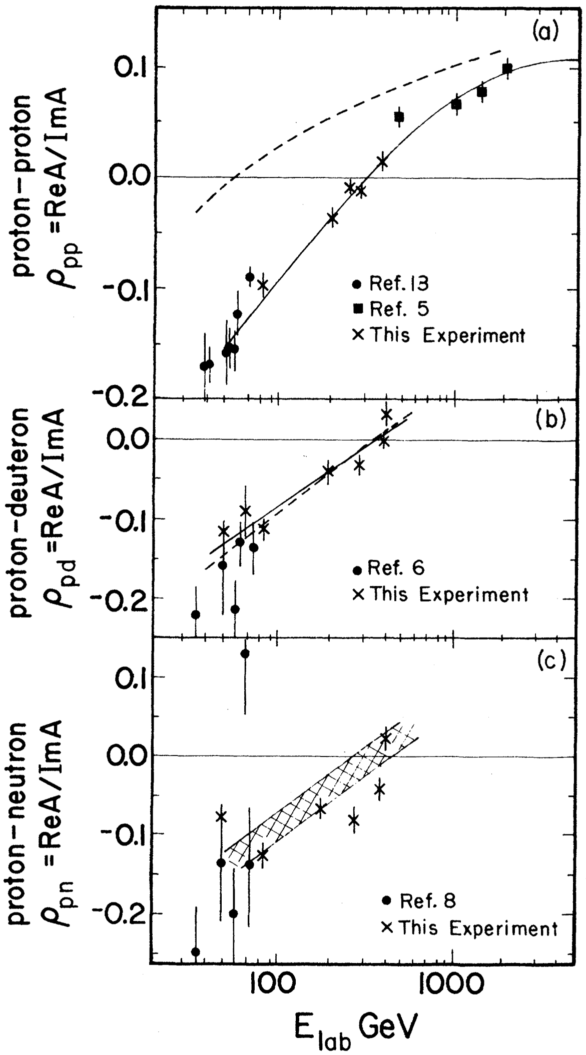

FIG. 1. (a) The ratio of the real to the imagniary part of the forward p-p nuclear amplitude. The curves are (dashed) one-Pomeron formula, $\mathrm{Re}A = \frac{1}{2}\pi(\partial\,\mathrm{Im}A/\partial\ln s)$, (solid) a dispersion-relation calculation discussed in text. (b) The ratio of the real to the imaginary part of the forward p-d nuclear amplitude. The solid line is the best fit, $\rho(s) = -0.450 + 0.070\ln s_{pd}$, to the proton-deuteron results. The dashed line is our fit $\rho(s) = -0.490 + 0.076\ln s$ to the proton-proton results. (c) The ratio of the real to the imaginary part of the forward p-n amplitude. The error corridor shown is from the empirical fit $\rho(s) = (-0.490 \pm 0.034) + (0.076 \pm 0.006)\ln s$ to the proton-proton results including systematic errors.

tic rescattering we can write

$$d\sigma/dt = \pi|S(t/4)(A_C + A_p + A_n) + A_G|^2, \qquad (8)$$

where $S(t/4)$ is the deuteron form factor and the amplitudes A_C, A_p, A_n, and A_G are functions of the slope parameters, real-to-imaginary forward nuclear scattering-amplitude ratios, and total cross sections for pp and pn scattering as well as relative phases between the amplitudes and the Glauber integral defined as

$$I_G = \tfrac{1}{2}\int_{-\infty}^{0} S(t)\exp[\tfrac{1}{2}b_{pp}t]\exp[\tfrac{1}{2}b_{pn}t]\,dt. \qquad (9)$$

We assume t independence for ρ_{pp} and ρ_{pn}, as no experimental information exists on this point. We calculate phases using the formulas in Ref. 14. For the p-p total cross section and b_{pp} we use the empirical formulas given previously. We assume that $\sigma_{pn} = \sigma_{pp}$ and $b_{pn} = b_{pp}$. For ρ_{pp} we use the empirical fit, Eq. (4). The spherical deuteron form factor, $S_0(t/4)$, is the same as used in Eq. (5). A quadrupole form factor is significant only at large $|t|$ values and introduces a maximum contribution of $\simeq 1.6\%$ at our highest $|t|$.

TABLE II. The Glauber-analysis results, ρ_{pn} and I_G, for the reaction $pd \to pd$.

E_{lab} (GeV)	ρ_{pn}	$\Delta\rho_{pn}$	I_G (mb^{-1})	ΔI_G (mb^{-1})
49	−0.081	0.018	0.0281	0.0011
82	−0.127	0.013	0.0298	0.0006
182	−0.065	0.014	0.0333	0.0005
281	−0.084	0.020	0.0348	0.0006
379	−0.045	0.017	0.0362	0.0005
397	+0.021	0.017	0.0346	0.0007

The results of the Glauber analysis are given in Table II and shown in Fig. 1(c). The free parameters are ρ_{pn} and I_G. One notes a larger statistical error on ρ_{pn} in comparison with the ρ_{pd} statistical error. The systematic errors in our ρ_{pn} values introduced by uncertainty in the deuteron form factor $S(t/4)$ [not shown in Fig. 1(c)] are of the order of our statistical errors. In Fig. 1(c) the error corridor on our ρ_{pp} results is used to compare ρ_{pn} values with ρ_{pp}. We find no significant difference between ρ_{pp} and ρ_{pn}. The shadow correction increases with energy but remains small, $\lesssim 0.03\ \mathrm{mb}^{-1}$.

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Remark on Factorization of *S* Matrices

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I point out that the structural property of two-dimensional perturbation theory, which allows the existence of nontrivial, factorizing *S* matrices, has an analogy in higher dimensions.

The computation of a relativistic, exact *S* matrix for some nontrivial model theory in four dimensions is of course one of the most desirable topics in today's theoretical physics. Such a *S* matrix is built up by infinitely many scattering functions which are defined by the connected kernels. In principle there are two possibilities:
(1) There exist infinitely many independent scattering functions and the calculation of an exact *S* matrix seems to be a hopeless undertaking for all times. (2) The other possibility is that the scattering functions are built up (algebraically) from finitely many independent functions. In this note I present some arguments in support of the second possibility.

Recently a variety of nontrivial exact *S* matrices, which factorize in terms of two-particle scattering amplitudes, have been computed in $1+1$ dimensions.[1-5] Nontrivial in this context means $S\neq 1$. The concept of factorization seems at first glance very limited because in dimensions $d>2$, $S\neq 1$ implies pair creation and annihilation[6] and a factorization in terms of two-particle elastic scattering amplitudes is therefore impossible. Furthermore it is in $d>2$ a consequence of the Coleman-Mandula theorem[7] that infinitely many local charges which govern the dynamics of the two-dimensional systems are not allowed. Nevertheless the second look is less pessimistic. The nonlinear σ model is one of the models[3] whose exact *S* matrix is now known. This model previously received a lot of attention because of its analogies [O(3) case] to the pure Yang-Mills theory in $d=4$. For this model (other models can be treated similarly) it has been shown by Lüscher[8] that nonlocal charges which do *not* commute with the *S* matrix (and are therefore not symmetries of the free theory) govern the dynamics of the model. There are no objections against the possibility of such charges in higher dimensions.

Finally there are arguments coming from the general structure of perturbation theory. If perturbation theory makes sense for the *S* matrix and possibility (2) is realized for the *S* matrix, then it has to be realized in each order of perturbation theory. The point of the present note