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# Charged-Particle Emission Following Muon Capture in Complex Nuclei

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A model combining pre-equilibrium and compound-nucleus emission is used to explain the observed emission of charged particles following $\mu$ capture from nuclei with $A \gtrsim 20$. The good agreement obtained constitutes additional evidence for a recently proposed distribution of the nucleon momenta in nuclei.

There is good understanding[1] of the main features of the well-studied neutron emission process following $\mu$ capture in nuclei. In contrast, the infrequent emission of charged particles has been less explored experimentally and a theoretical accounting for its strength and spectrum has been lacking.[1] In this paper, we report a new approach to this problem. Our results agree remarkably well with the recently measured[2] rates of charged-particle emission following $\mu$ capture in a wide range of nuclei, as well as with older data on $p$ and $\alpha$ spectra and rates from emulsion experiments.[3-5]

The fractional rate of charged-particle emission ranges from ~15% in light nuclei[6] to 1–3% in intermediate and heavy ones.[2-5,7] Earlier theoretical attempts failed to account for the rate by orders of magnitude[8,9] or dealt with a limited part of the emission process.[10-12]

The model we consider has the following main ingredients: (1) We include both pre-equilibrium and compound-nucleus emission from the excited nucleus. (2) The excitation function is calculated by using the Amado distribution for the nucleon momenta of the capturing nucleus,

$$n(p)=N/\cosh^2\gamma p\,, \tag{1}$$

where $N$ is a normalization constant and $\gamma$ a momentum scale, $\gamma=0.8$ fm. This distribution was derived[13] as the Hartree solution of a one-dimensional many-body problem with $\delta$-function forces and has been used[14] to explain the energetic protons observed by Frankel *et al.*[15] at 180° in proton-nucleus collisions. These experiments, which are sensitive to the high-energy components of $n(p)$, appear to confirm[16] that the nucleon momentum distribution decreases exponentially in $p$ rather than Gaussian, and give roughly the above value of $\gamma$. We conjecture now that $n(p)$ can be used for the full nucleon momentum distribution. Indeed, in our problem a wide range of momenta is involved, though it is largely because of the high-energy part of $n(p)$ that we succeed in accounting so well for the observed charged-particle emission. The main steps of the calculation are as follows:

*(a) Capture and excitation function.*—Along lines previously used,[1] we express the muon cap-

 

ture rate in terms of an effective coupling constant $G_{\rm eff}$,

$$\Lambda = N G_{\rm eff} |\psi_\mu|_{\rm av}^2 \int d^3k_\nu d^3p\, d^3q\, g(p)[1-h(q)]\,\delta(\vec{p}-\vec{k}_\nu-\vec{q})\,\delta(E_0-k_\nu c-E), \tag{2}$$

where $g(p)$ and $1-h(q)$ are the capturing proton and neutron-hole momentum distributions, the latter accounting for the Pauli exclusion principle. $E$ is the excitation energy of the nucleus ${}_{Z-1}X^A$ and $E_0 = M(A,Z)c^2 - M(A,Z-1)c^2 + M_\mu c^2 - B_\mu$. For $g(p)$ and $h(q)$ we use Eq. (1) and after numerical integrations one arrives at the nuclear excitation function

$$\Lambda = N' \int_0^{E_0} I(E)\, dE\,. \tag{3}$$

$E$ is related to the particle variables by $E=(2M^*)^{-1} \times (q^2-p^2)$, where for the effective nucleon mass we use an average value[1] $M^* = 0.68M$.

*(b) Pre-equilibrium emission.*—There is ample evidence[17,18] that the spectra of particles emitted in nuclear reactions of 15–100 MeV excitation energy have components which are not accounted for by direct one-step mechanisms or by emission from a compound nucleus. In dealing with this problem, Griffin has introduced[19] the exciton model to describe the equilibration process, by which the final state is reached through a series of two-body interactions, with some probability for emission at each step. In muon capture, the excited nucleus is left with an average of nearly 20 MeV excitation energy, its distribution extending to several tens of MeV. Since the excitation function extends over the energy range in which pre-equilibrium emission was shown[17,18] to play an important role in nuclear reactions, it should obviously be taken into account also in our problem. We employ the hybrid model approach to pre-equilibrium emission,[20,21] the emitted spectrum of particle $x$ given by

$$P_x(\epsilon)d\epsilon = \int_0^{E_0} I(E)\, dE \sum_{\substack{n=n_0 \\ (\Delta n=2)}}^{\bar{n}} {}_nf_x \left[\frac{\rho_{n-1}(U,\epsilon)}{\rho_n(E)}\, g\right]\left[\frac{\lambda_c(\epsilon)}{\lambda_c(\epsilon)+\lambda_{n+2}(\epsilon)}\right] D_n\, d\epsilon\,. \tag{4}$$

${}_nf_x$ is the number of nucleons of type $x$ in an $n$-exciton state, the expression in the first bracket gives the fraction of the $n$-exciton state population which has one particle in an unbound level with energy $\epsilon$ and the second bracket gives the fraction of particles that are emitted rather than undergoing a transition to an $n+2$ state. $D_n$ is the depletion factor, expressing the probability of survival from states $m<n$. In $\mu$ capture, the initial state has $n_0=2$ with one proton hole and one neutron, the proton emission occurring from states with $n \geq 4$. We performed our calculations for $x=p, n$, using for (4) the explicit formula of Blann and Mignerey.[21]

*(c) Compound-nucleus emission.*—Statistical emission of particle $x$ (we considered $x=p, n, d, \alpha$) from the equilibrium stage was calculated from the well-known expression

$$P_x(\epsilon) = \frac{(2S_x+1)M_x\epsilon_x\sigma_x\rho(U)}{\pi^2\hbar^3\rho_c(E)}. \tag{5}$$

We did not use specific information for each nucleus; rather we employed the well-tested parametrization for the inverse-reaction cross section $\sigma_x$ of Dostrovsky *et al.*[22] For the level densities we take

$$\rho(U) \propto \frac{a^{1/2}\exp[2(aU)^{1/2}]}{A^{5/2}(U+t)^2} \tag{6}$$

with $a=A/10$, $U=at^2-t$, and $U=E-B_x-\epsilon_x-\delta$, $\delta$ being the pairing energy. Equation (6) has been used successfully[23] for a wide range of nuclear reactions at comparable energies. The calculation of the statistical emission is done only for nuclei which reach the equilibrium state without prior emission, which we find to happen in 70%–80% of the cases. The small amount of equilibrium emission from nuclei which emitted during equilibration is thus neglected.

In Table I we present our results for the reaction $(\mu,\alpha)$. This was calculated as compound-nucleus emission only, after considering the pre-equilibrium emission of protons and neutrons. There is no good treatment yet[18] for cluster emission from the pre-equilibrium stage, and moreover, for nuclei with $A<80$ for which experimental data exist,[2] one expects compound-nucleus emission to dominate.[23] Our results confirm this assumption.

Table II contains our results for single $(\mu,p)$

TABLE I. $10^3$ times the calculated probabilities per muon captured for the reaction ${}_ZX^A(\mu,\alpha)_{Z-3}X^{A-4}$, compared to $10^3$ times the experimental data of Ref. 2.

| Capturing nucleus | Experiment | Present calculation |
|---|---|---|
| ${}_{11}Na^{23}$ | $11 \pm 1.5$ | 10 |
| ${}_{15}P^{31}$ | $13 \pm 2$ | 10 |
| ${}_{23}V^{51}$ | $1.5 \pm 0.2$ | 1.6 |
| ${}_{25}Mn^{55}$ | $1.6 \pm 0.2$ | 2.3 |
| ${}_{26}Fe^{56}$ | $4.6 \pm 0.7$ | 3.8 |
| ${}_{29}Cu^{65}$ | $0.7 \pm 0.2$ | 0.36 |
| ${}_{33}As^{75}$ | $> 0.28 \pm 0.04$ | 0.39 |

and inclusive $(\mu, \tilde{p})$ proton emission. Here $(\mu, \tilde{p}) \equiv (\mu, p) + (\mu, pn) + (\mu, p2n) + \ldots + (\mu, d) + (\mu, dn) + \ldots$, since in most experimental setups (activation methods) the rates for $(\mu, pn)$ and $(\mu, d)$ are indistinguishable. The calculated deuteron emission is from the equilibrium stage only, for reasons given above.

Of special interest is the calculation of charged-particle emission from the emulsion nuclei (Ag, Br), where spectra are also available.[3-5] We find $(\mu, \tilde{p})_{Ag,Br} = 1.6\%$, versus the experimental $(2.2 \pm 0.4)\%$, the spectrum being exhibited in Fig. 1(a). For $(\mu, \tilde{\alpha})_{Ag,Br}$, calculated as emission from compound nucleus only, we obtain 0.23% versus the experimental 0.4–0.5%. The spectrum is shown in Fig. 1(b). This result is reasonable, since we know from the proton calculation that approximately 50% of the emission around $A = 100$ is of precompound nature. We make the following remarks: (1) The pre-equilibrium emission increases from a few percent in light nuclei to about 10% around $A = 50$ and becomes the major process for heavy nuclei, where the Coulomb barrier becomes prohibiting for evaporation. This agrees with the general trend[17] in nuclear reactions at similar energies. (2) The calculated energy spectra of the two contributing mechanisms [Fig. 1(a)] are different, though not dramatically so. Measurements of spectra in light and heavy nuclei would be of great value in checking our model. (3) We have calculated the charged-particle emission for all the nuclei measured by Wyttenbach *et al.*[2] and the agreement is of the same quality as for the selected sample presented in Table II. Moreover, we calculated neutron emissions for several nuclei and we find improved agreement with experiment, compared to previous calculations[1] which used a Gaussian nucleon momentum distribution. These results will be presented in a detailed paper.

In concluding we emphasize that the overall agreement obtained for a wide range of nuclei using a unified picture is a strong indication of its adequacy and our results lend support to the nucleon momentum distribution suggested previously.[13-15]

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TABLE II. Calculated probabilities per muon capture for the reaction ${}_ZX^A(\mu,p)_{Z-2}X^{A-1}$ and for inclusive proton emission $(\mu,\tilde{p})$. The experimental data are from Ref. 2, except when otherwise referenced. For $(\mu,\tilde{p})$ the experimental figures are lower limits, determined from the actually measured channels. The figures in parentheses are based on observed (Ref. 2) regularities of ratios among various channels. PE stands for pre-equilibrium.

| | $(\mu,p)$ Present calculation | | | $(\mu,\tilde{p})$ Present calculation | | |
|---|---|---|---|---|---|---|
| Capturing nucleus | % of PE emission | $10^3$ times total emission | $10^3$ times experiment | % of PE emission | $10^3$ times total emission | $10^3$ times experiment |
| ${}_{14}Si^{28}$ | 1.85% | 32 | $53 \pm 10$ [a] | 6.9% | 144 [b] | $150 \pm 30$ [b] |
| ${}_{23}V^{51}$ | 5.0% | 3.7 | $2.9 \pm 0.4$ | 12% | 25 | $> 20.1 \pm 1.3$ $(33 \pm 2)$ |
| ${}_{25}Mn^{55}$ | 7.4% | 2.4 | $2.8 \pm 0.4$ | 17% | 16 | $> 26 \pm 2$ $(37 \pm 3)$ |
| ${}_{29}Cu^{63}$ | 6.0% | 4.0 | $2.9 \pm 0.6$ | 13% | 25 | $> 17 \pm 3$ $(35 \pm 5)$ |
| ${}_{33}As^{75}$ | 11% | 1.5 | $1.4 \pm 0.2$ | 18% | 14 | $> 14 \pm 1$ $(19 \pm 2)$ |
| ${}_{51}Sb^{127}$ | 32% | 0.87 | $(> 0.37 \pm 0.05)$ | 37% | 11 | $> 8 \pm 1$ |
| ${}_{55}Cs^{133}$ | 79% | 0.47 | $0.48 \pm 0.07$ | 71% | 6.6 | $> 4.9 \pm 0.4$ $(6.8 \pm 0.6)$ |
| ${}_{67}Ho^{165}$ | 97% | 0.30 | $0.30 \pm 0.04$ | 91% | 4.3 | $> 3.4 \pm 0.2$ $(5.0 \pm 0.2)$ |
| ${}_{73}Ta^{181}$ | ~ 100% | 0.21 | $0.26 \pm 0.04$ | 93% | 3.4 | $> 0.7 \pm 0.1$ $(3.5 \pm 0.5)$ |
| ${}_{82}Pb^{208}$ | ~ 100% | 0.27 | $0.13 \pm 0.02$ | ~ 100% | 2.1 | $> 3.0 \pm 0.8$ $(4.9 \pm 1.2)$ |

[a]Ref. 24.

[b]These figures include all charged-particle emission (Ref. 6).

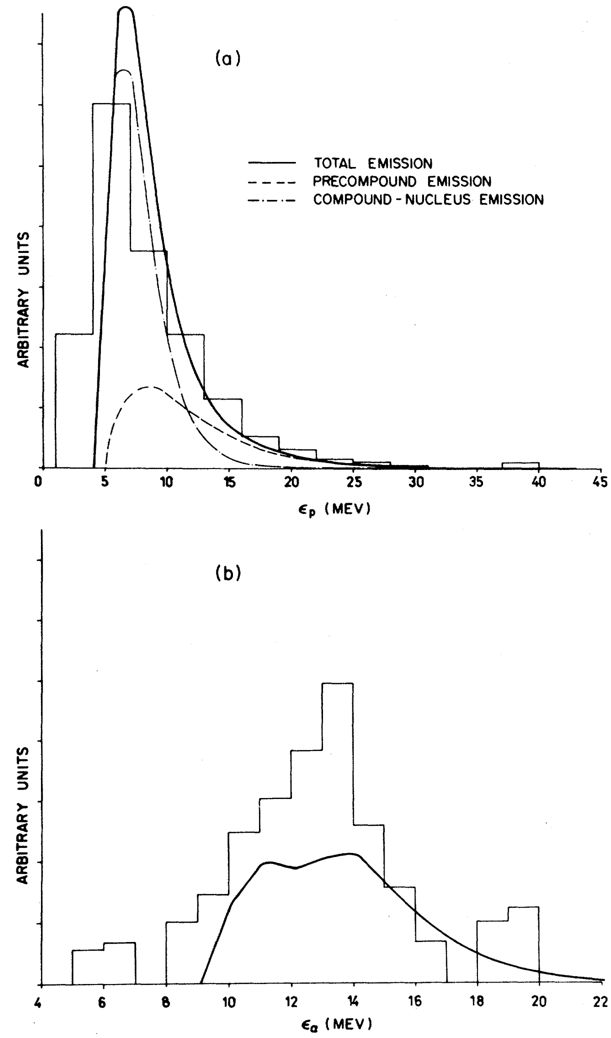


FIG. 1. (a) Calculated energy spectrum curves of protons emitted after $\mu$ capture in AgBr, versus the experimental histogram of Ref. 4. (b) Calculated energy spectrum of $\alpha$'s emitted after $\mu$ capture in AgBr, compared to the experimental histogram of Ref. 3. All curves and histograms are normalized to the calculated and observed rates, respectively.

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