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Ambiguities in Pion-Nucleus Optical Potentials and the Determination of Neutron Radii

Morton M. Sternheim and Kwang-Bock Yoo
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
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The $\pi^{\pm}$-^{48}Ca elastic scattering data are shown to be reproduced with a wide range of rms neutron radii if the optical-model parameters are suitably adjusted. The corresponding total cross sections vary significantly.

It has long been expected that any difference between the proton and neutron distributions in nuclei will lead to corresponding differences in $\pi^{\pm}$-nucleus differential[1] and total[2] cross sections in the energy region dominated by the (3, 3) πN resonance. This is because at these energies $\sigma(\pi^+ p) \simeq 3\sigma(\pi^+ n)$, so that the π^+ mainly "sees" protons and, by symmetry, the π^- mainly sees neutrons. With the aid of recent data[3,4] for ^{40,48}Ca, we have now found that if both differential- and total-cross-section data are fully analyzed, the ambiguities are reduced in extracting radial parameters.

Electron and proton scattering experiments[5] indicate that in ^{40}Ca the rms proton and neutron radii, r_p and r_n, are nearly equal; also, $r_p(^{48}\mathrm{Ca}) \simeq r_n(^{40}\mathrm{Ca})$. Hartree-Fock calculations[6] give $\Delta r_n = r_n(^{48}\mathrm{Ca}) - r_n(^{40}\mathrm{Ca}) = 0.31$ fm; however, scattering of 79-MeV α particles[7] indicates $\Delta r_n = 0.08 \pm 0.08$ fm, and 1-GeV proton scattering[8] gives 0.16 $\pm$ 0.02 fm. Generally, in neutron-rich nuclei, measured neutron radii are smaller than the Hartree-Fock predictions.[9]

Two $\pi^{\pm}$ experiments have been done on ^{40,48}Ca, yielding somewhat different conclusions about Δr_n. Jakobson *et al.*[3] performed measurements at the Clinton P. Anderson Meson Physics Facility (LAMPF) at several energies of the differences in the total cross sections, $\Delta\sigma^{\pm} = \sigma_T^{\pm}(^{48}\mathrm{Ca}) - \sigma_T^{\pm}(^{40}\mathrm{Ca})$. They adjusted the radial parameters in an optical-model calculation to obtain a fit and concluded that the rms neutron radii differ by $\Delta r_n = 0.14 \pm 0.05$ fm. At the Swiss Institute for Nuclear Research, Egger *et al.*[4] measured elastic scattering over a range of angles at 130 MeV. They associated the minima with nuclear radii via a black-disk model, and found that the π^- radius exceeded the π^+ radius by 0.21 fm for ^{40}Ca, and by 0.51 fm for ^{48}Ca. They ascribed the ^{40}Ca difference to Coulomb effects, and assumed that the additional radius difference of 0.3 fm in ^{48}Ca is due to a larger neutron radius. They state that these results are consistent with the Hartree-Fock predictions[6] but not those of Jackson *et al.*[3]

One problem in interpreting both of these experiments is that some knowledge of the optical potential is required. Jakobson *et al.*[3] used optical-model parameters based on free-πN data; these parameters reproduce the general features of pion scattering in this energy region, and, as they noted, their analysis is relatively insensitive to the values of the parameters as long as they are the *same* for both ^{40}Ca and ^{48}Ca. In the elastic scattering experiment,[4] the "blackness" of the nucleus is needed to associate the disk radius R with a neutron radius. If the medium is made more absorptive, the absorptive region will move further out into the nuclear surface; since the black-disk diffraction minimum occurs at $2kR\sin(\theta/2) = 3.83$, θ decreases as the imaginary part of the optical potential increases. The same effect occurs if the potential is made more attractive, so that the pion wave number k is increased.[10] *By themselves, the positions of the minima yield little information about the nuclear radii.*

Since we know the complete elastic-scattering distribution at 130 MeV, we can in fact check the optical-model assumptions at that energy. Using the electron-scattering density parameters[11] for both the protons and the neutrons and the free-πN phase shifts, we found that the differential cross sections calculated with the ABACUS-M code[12] agree qualitatively with the data, although they do not fit in detail. The minima are located rather well, suggesting that $r_n = r_p$ is consistent with the elastic data. However, the calculated minima are too deep; the predicted absorption is too large. Varying the off-shell behavior (Kisslinger[13] versus Laplacian models[14]) and including the Lorentz-Lorenz effect[15] or the c.m.-system to laboratory correction[16] does not change this general pattern (Fig. 1).

To see the effect of using phenomenological parameters in extracting neutron density information, we began with ^{40}Ca. We used a Kisslinger model with a full Lorentz-Lorenz correction. Initially, we set the proton and neutron half-density (c_p and c_n) and thickness parameters (a_p and a_n) equal to the electron-scattering values[11]; we held the small s-wave coefficient b_0 in the potential constant and varied the p-wave coefficient b_1, which was taken[17] to be pure isospin $\frac{3}{2}$. When we fitted to the elastic $\pi^{\pm}$ data simultaneously, we obtained an excellent fit; as anticipated, the fitted value of Imb_1 was significantly smaller than the free value (see Fig. 2 and Set I in Table I). Alternatively, we fitted the π^+ data alone, since these cross sections are rather insensitive to the neutron distribution. We then used the resulting b_1 parameter in π^- calculations, varying the neutron parameters c_n and a_n to optimize the fit; this gave $r_n < r_p$, with $c_n > c_p$ but $a_n < a_p$ (Set II in Table I). However, this procedure was not unique; when we changed the distribution used in fitting the π^+ data, the neutron parameters found from the π^- data also changed somewhat.

Turning to ^{48}Ca, we found that using the ^{40}Ca Set I or Set II best-fit b_1 values and the electron-scattering ^{48}Ca density parameters[11] for both the protons and the neutrons gave a better agreement than found with the free-πN parameters. (Figure 2 shows the Set II calculations; results with Set I are similar.) Again the minima are located fairly well, so that there is no necessity to introduce

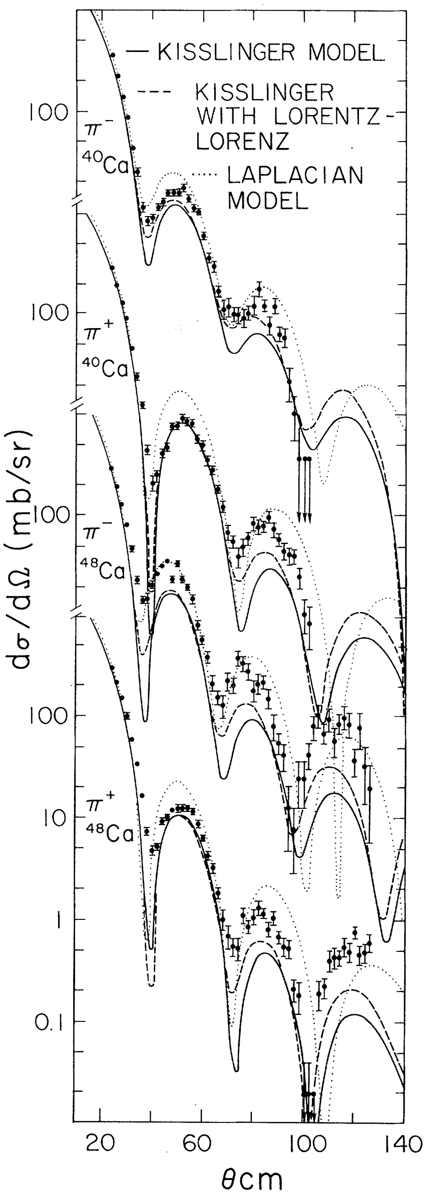

FIG. 1. Differential cross sections calculated with parameters obtained from free-πN phase shifts (Table I). Poorer results are obtained if the angle transformation is included. Data are from Ref. 4.

a larger neutron radius.

To see how much better agreement with the ^{48}Ca data is possible, we chose several sets of neutron density parameters and varied b_1 for each set (Table I). We found equally good fits for a range of density parameters, e.g., with $a_n = 0.52$ fm, for c_n from 3.4 to 4.4. In each case the minima are correctly located. However, as c_n is increased, the fitted values of Reb_1 and Imb_1 both decreases. In terms of a black-disk

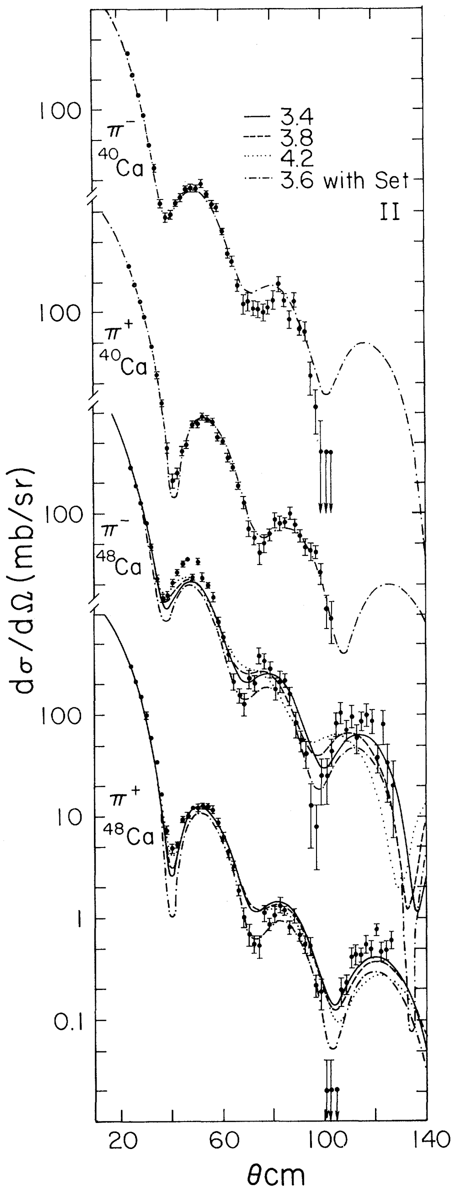

FIG. 2. ^{40}Ca best-fit calculations as discussed in the text. The ^{48}Ca curves are with fitted parameters for three r_n values plus the Set II parameters obtained by fitting ^{40}Ca. See Table I for parameter values.

model, the wave number k and the opaqueness of the surface are both decreasing so as to offset the change in c_n.

Clearly the elastic scattering data cannot be used by themselves at this point to find a neutron radius, since we have no empirical or theoretical basis for preferring any one of the fitted sets of optical-model parameters. However, even though they all reproduce the elastic data equally well, they do predict rather different results for the

TABLE I. Optical-model parameters. Electron-scattering density parameters (Ref. 11) for ^{40}Ca are $c = 3.31$ fm, $a = 0.60$ fm; for ^{48}Ca, $c = 3.60$ fm, $a = 0.52$ fm. The last column gives the χ^2 per degree of freedom (d.f.).

	c_n (fm)	a_n (fm)	r_n (fm)	Reb_1	Imb_1	χ^2/d.f.
Free πN				11.53	9.71	...
^{40}Ca Set I	3.31	0.60	3.40	11.32	6.86	3.8
Set II	3.60	0.46	3.27	11.46	6.61	3.3
^{48}Ca	3.4	0.52	3.27	12.36	4.81	11.1
	3.6	0.52	3.39	11.41	4.65	10.4
	3.8	0.52	3.52	10.46	4.54	10.3
	4.0	0.52	3.65	9.63	4.38	10.4
	4.2	0.52	3.78	9.03	4.26	10.6
	4.4	0.52	3.92	8.51	4.08	11.0

total cross sections. This means that these nuclei are not opaque enough for the total cross sections and minima to vary with the same equivalent black-disk radius.

The Set I and Set II parameters found for ^{40}Ca

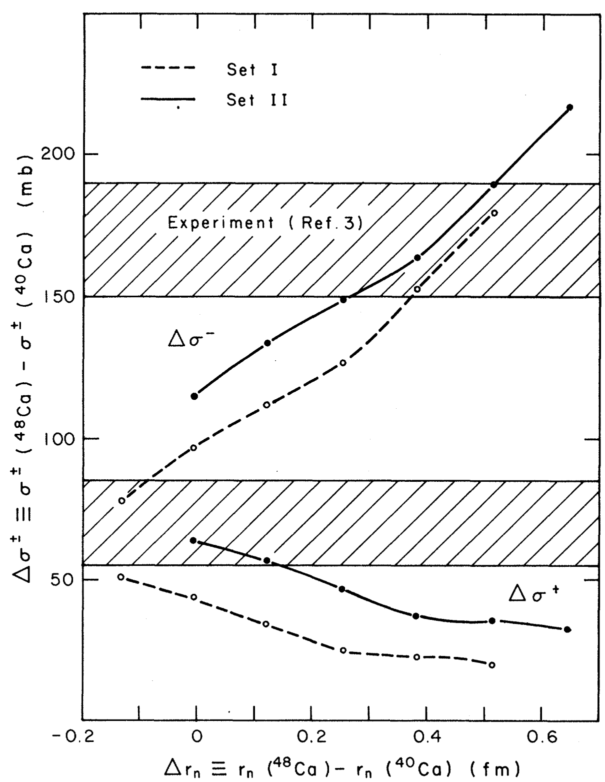

FIG. 3. Total-cross-section differences. The experimental data are taken from a smooth curve through the points at several energies in Ref. 3.

lead to different ^{40}Ca total cross sections; $\sigma^{\pm}$ are both smaller for Set II. These differences are reflected in the values of $\Delta\sigma^{\pm}$ shown in Fig. 3. In ^{48}Ca, as c_n increases, and the adjusted value of Imb_1 decreases, $\sigma^{+}(^{48}\text{Ca})$ decreases. This happens because the π^{+} interact mainly with the protons, whose radius is fixed, and the medium is becoming more transparent. However, at the same time $\sigma^{-}(^{48}\text{Ca})$ *increases*; the π^{-} mainly interact with the neutrons, and the effect of increasing the neutron radius apparently more than offsets the change in Imb_1.

It is clear from Fig. 3 that the variation in $\Delta\sigma^{\pm}$ corresponding to the ^{40}Ca Set I and II parameters obscure their dependence on Δr_n. Most likely other curves could have been obtained by varying the ^{40}Ca neutron distribution. The fact that the Set II curves are both more or less consistent with a common value, $\Delta r_n \simeq 0.2$ fm, while the Set I curves lead to inconsistent Δr_n values might be interpreted as an argument in favor of the former fit. Note that the results in Fig. 3 are insensitive to the *shape* of the neutron distribution.

To summarize, we have shown that the elastic scattering data alone cannot be used to determine neutron density parameters, since changes in the density can be offset by changes in the optical-model parameters. Since a single set of optical-model parameters does not describe the scattering from ^{40}Ca and ^{48}Ca very well, it is also impossible to extract such information from the total-cross-section differences alone. However, the two kinds of information taken together may permit more definitive statements to be made.

We might also note that preliminary data are now available[18] on the elastic scattering from ^{40,48}Ca at 291 MeV. The same kind of problems arise in extracting neutron parameters of this energy as at 130 MeV. It will be interesting to see whether the situation will be different at 180 MeV or so, where the nucleus is blackest.

One point we have not considered in this Letter is the possibility of an improved theoretical description of the pion-nucleus interaction. If sufficient progress is made so that the dependence on the particular nucleus is understood, and the adjustable parameters are thereby eliminated or at least reduced in number, reliable extraction of a neutron radius may become feasible.

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