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Classical Many-Body Calculation of Relativistic Nuclear Collisions

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A zero-parameter, classical, many-body model of relativistic heavy-ion collisions is proposed. Inclusive proton cross sections from 250- and 400-MeV/nucleon $^{20}\text{Ne} + \text{U}$, 400-MeV/nucleon $^4\text{He} + \text{U}$, and 800-MeV/nucleon $^{20}\text{Ne} + \text{NaF}$ collisions are in good agreement with the model.

Proton emission in relativistic $^{20}\text{Ne} + \text{U}$ collisions has been attributed by Westfall *et al.*¹ to evaporation from a nuclear "fireball" with temperature $\tau \approx 50$ MeV and recoil velocity $\beta = V/c \approx 0.25$. Light composite nucleus formation has been explained in terms of a final-state interaction among nucleons² or alternatively as thermal emission^{3,4} from a fireball. The idea that thermal equilibrium can be achieved within collision times of $\approx 10^{-22}$ sec is difficult to believe. It is, therefore, important to see if this is indeed a necessary assumption to obtain agreement with observations. For this reason microscopic descriptions of relativistic heavy-ion collisions, which follow the time evolution of the collision, have been tried.⁵⁻¹⁰ Microscopic descriptions have generally only been able to reproduce the gross features of the proton spectra, often differing at points by a factor of 10. A detailed microscopic model of heavy-ion collisions would be valuable in providing a baseline of what is to be expected in the absence of any exotic phenomena. In this Letter I will describe a classical many-body calculation of heavy-ion collisions I have developed that may fill this role.

The central assumption of this calculation is that relativistic nucleus-nucleus collisions may be treated as a succession of free two-body nucleon-nucleon collisions. The calculation proceeds as follows. At the beginning of each collision all nucleons are assigned randomly chosen positions in the projectile and target nuclei, which are assumed to be spherical with diffuse surfaces. Similarly the momentum, in the target or projectile frame, of each nucleon is chosen out of a Fermi distribution with $P_F = 265$ MeV/c.

Nucleons are assumed to follow straight-line trajectories and to interact at the point of closest approach if their separation d satisfies $\pi d^2 \leq \sigma(E_{c.m.})$, where σ is the appropriate experimental nucleon-nucleon total cross section, which depends on the center-of-mass energy $E_{c.m.}$ of the pair. If this condition is satisfied the scattering angle is randomly chosen from experimental elastic-scattering angular distributions, tabulated by Chen.¹¹ Finally, both nucleons must have momenta satisfying the exclusion principle, $P > P_F$ in the laboratory frame, or the collision is forbidden. Scattering is assumed to take place in a potential well of depth $V_0 = 45$ MeV. The effects of refraction and reflection are ignored. This simplification might be expected to distort the low-energy proton spectrum; however, there is no apparent systematic departure of the calculation from the data at energies down to 30 MeV. Roughly 2000 nucleus-nucleus collisions must be simulated to provide meaningful statistics. This requires about 1.5 h of CDC 7600 time for $^{20}\text{Ne} + \text{U}$ collisions.

One difficulty with this calculation is that there is no way to account simply for formation of light composite particles, which account for much of the emitted matter.² If these particles are formed by final-state interactions then the observed proton spectrum will be modified from its pre-final-state-interaction or "primordial" form.⁶ The primordial proton spectrum is given by

$$\left(\frac{d^2\sigma}{d\Omega dE} \right)_{\text{primordial}} = \sum_{\text{all isotopes}} Z \frac{d^2\sigma(Z, A)}{d\Omega dE}, \quad (1)$$

where E is the energy per nucleon and the sum is

over all isotopes. In practice only hydrogen and helium isotopes contribute significantly. Figures 1 and 2 compare the model proton spectrum with the experimental primordial spectrum from Eq. (1) for 250- and 400-MeV/nucleon $^{20}\text{Ne} + \text{U}$ and 400-MeV/nucleon $^4\text{He} + \text{U}$.^{1,2} In all cases the calculations reproduce the shape of the primordial proton spectrum with rms fractional errors of about 25%. Roughly half of this error is due to counting statistics of the calculation at small cross sections. Note that the data have all been lowered by a factor of 3 in Figs. 1 and 2 from the originally published values. Recently the authors of Ref. 1 have made new measurements¹² which show that their spectra for $^{20}\text{Ne} + \text{U} \rightarrow p + x$ at 250 and 400 MeV/nucleon should be lowered by a factor of 2 to 2.5. Although they have not yet checked all their hydrogen and helium isotope data, or data for $^4\text{He} + \text{U}$ collisions, these will probably be lowered by similar factors. This essentially eliminates any discrepancy between this calculation and the data.

It is of some interest to know if the onset of

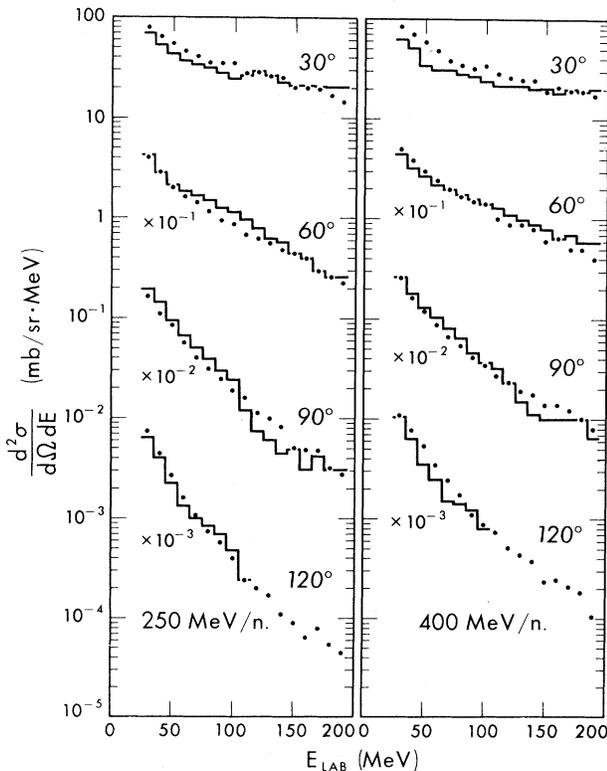


FIG. 1. Single-particle inclusive cross section for production of protons in $^{20}\text{Ne} + \text{U}$ collisions. The solid line is based on the calculations presented in this Letter. See text regarding the normalization of the data.

pion production radically alters the nucleus-nucleus collision process. Figure 3 compares the calculation with data¹³ for 800-MeV-nucleon $^{20}\text{Ne} + \text{NaF} \rightarrow p + x$. The calculation yields relatively good agreement over a wide dynamic range despite the fact that it does not include pion production. The calculation does, however, systematically overestimate the data at high momenta.

Figure 4 shows the relative frequency of multiple collisions for nucleons emitted in 250-MeV-nucleon $^{20}\text{Ne} + \text{U}$ collisions. Koonin¹⁴ suggested that a major portion of the inclusive proton cross section for this reaction might be explained by single scattering of nucleons. He suggested that two-proton azimuthal-angle correlations would be a sensitive probe of this process. My calculation indicates that only 13% of the emitted nucleons scatter only once, and that azimuthal correlations due to nucleon-nucleon scattering should be quite small and would require an enormous

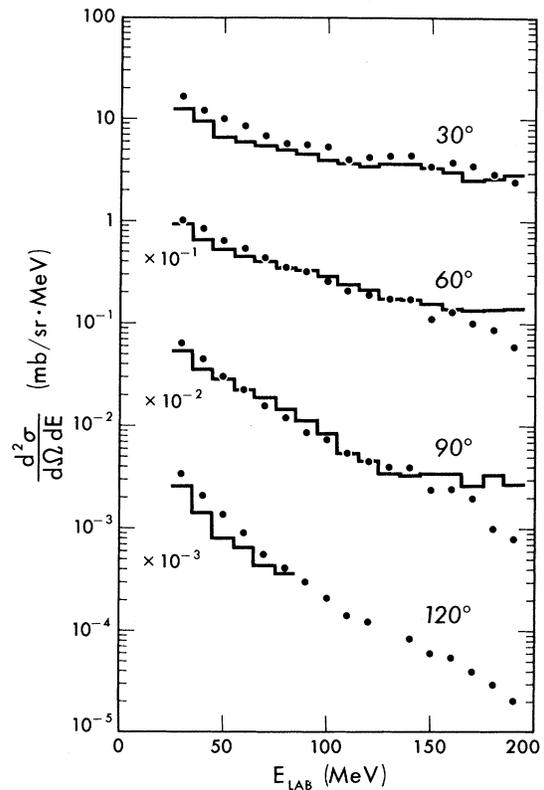


FIG. 2. Single-particle inclusive cross section for production of protons in 400-MeV/nucleon $^4\text{He} + \text{U}$ collisions. The solid line is based on calculations presented in the Letter. See text regarding the normalization of the data.

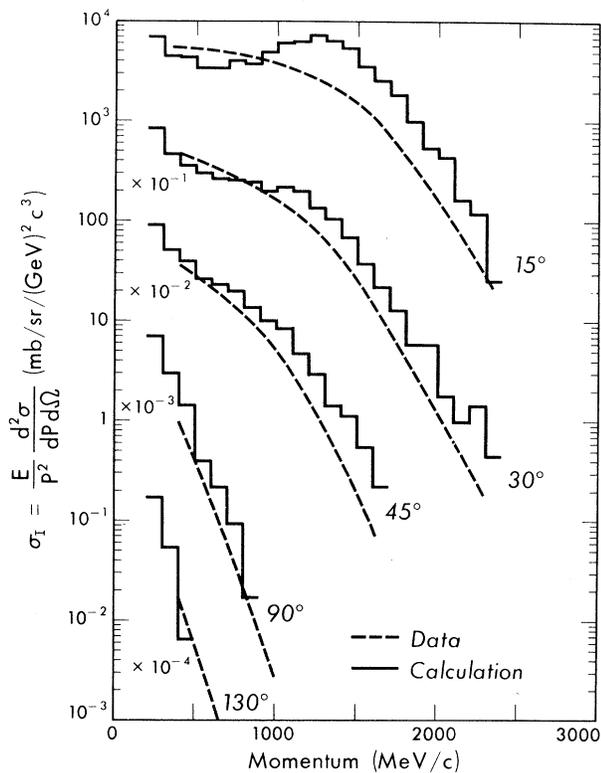


FIG. 3. Single-particle inclusive cross section for production of protons in 800-MeV/nucleon $^{20}\text{Ne} + \text{NaF}$ collisions.

amount of data to detect. The average number of scatterings, \bar{N} , is about five. This number is interesting for several reasons. A common approximation in previous cascade calculations was to neglect interactions of cascade nucleons with each other. The approximation limits the value that \bar{N} can assume to $\bar{N} < 2$. Clearly that approximation is not valid for nucleus-nucleus collisions. Studies¹⁵ of the approach of a hard-sphere gas to thermal equilibrium indicate that their energy spectrum can show some equilibrium features once \bar{N} reaches 4. Thus the assumption of thermal equilibrium of the fireball model may have some justification but should not be taken too literally.

There has been considerable interest in doing experiments that look selectively at central collisions of high-energy nuclei. In order to do this a criterion must be established for distinguishing central from noncentral events. Calculations of the type presented in this paper will provide a useful basis for choosing a best central-collision "trigger." For example, one central-collision trigger that has often been proposed is that there be no remaining projectile fragments, i.e., fast

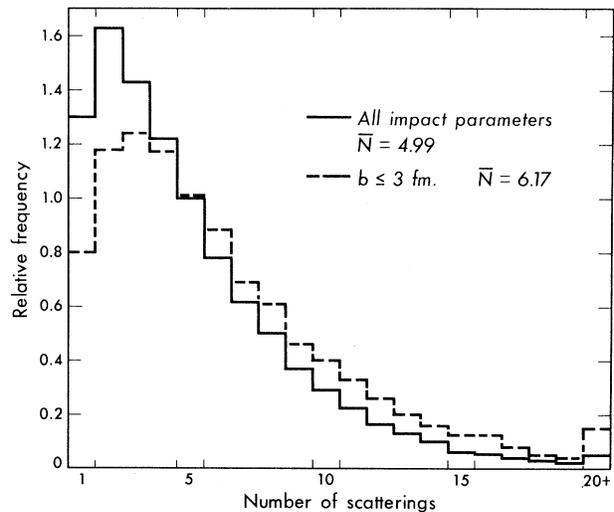


FIG. 4. Relative frequency of multiple scattering of nucleons emitted in 250-MeV/nucleon $^{20}\text{Ne} + \text{U}$ collisions.

particles at small laboratory angles. Consider the trigger requirement that no charged particles are within 5° of the beam axis from 400-MeV/nucleon $^{20}\text{Ne} + \text{U}$. For this case the calculation shows that 77% of the triggers come from the most central 30% of all the events, but the efficiency for triggering on the most central 30% of events is only 56%. Raising the trigger zone from 5° to 10° results in 97% of the triggers coming from the inner 30% of all the events, but only a 23% trigger efficiency. Clearly calculations of this type are valuable in designing and interpreting results of triggered experiments.

It is of some interest to understand which assumptions are responsible for the improved agreement of this calculation compared to previous microscopic approaches. There are four important features of this calculation, and no previous calculation contained all of them. These features are an exact treatment of multiple scattering, relativistic kinematics, use of experimental scattering cross sections, and treatment of Fermi motion in the target and projectile.

This calculation is in excellent agreement with single-particle inclusive proton data from relativistic heavy-ion collisions at beam energies of 250 and 400 MeV/nucleon. Although this calculation does not include pion production, it accounts reasonably well for the production of protons in nucleus-nucleus collisions at beam energies of 800 MeV/nucleon. This calculation shows that the radical assumption that a hot nuclear fireball^{1,16} is formed in nucleus-nucleus collisions is not

necessary to explain existing experimental results.

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Isoscalar Character of the $J^\pi = 6^+$, $E_x = 5.125$ MeV State in ^{58}Ni

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Transverse ($E6$) and longitudinal ($C6$) cross sections for the excitation of the $J^\pi = 6^+$, $E_x = 5.125$ MeV state in ^{58}Ni were measured by means of inelastic electron scattering. The deduced isoscalar component of the particle-hole wave function is found to be two orders of magnitude greater in amplitude than the isovector. This implies that the particle-hole isospin is relatively pure $\tau = 0$ and is not significantly mixed with $\tau = 1$.

Reported electron-scattering measurements of the transverse form factor for electric multipole transitions are nonexistent for $L > 2$. At forward scattering angles the longitudinal is generally much larger than the transverse cross section including both magnetization (spin) and convection (orbital) current contributions. This is not the case when electrons are observed at angles near or at 180° . Here it is possible to observe scattering due to transverse electric or magnetic form factors. It is the purpose of this Letter to

illustrate an important and unique aspect of transverse electric measurements. Specifically, it is shown how transverse measurements when combined with longitudinal ones for the same state of excitation can be used to deduce the isoscalar and isovector parts of the transverse form factor. Hence, this can then be used as a technique to study the isospin structure of particle-hole excitations and interactions in nuclei.

Recent electron-scattering experiments at $\theta = 160^\circ$ in nuclei ^{24}Mg (Ref. 1) and ^{58}Ni (Ref. 2)