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New, Generalized Cabibbo Fit and Application to Quark Mixing Angles in the Sequential Weinberg-Salam Model

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We carry out a new, generalized analysis of nuclear β decay and semileptonic hyperon and K_{e3} decays. The results are used to determine two quark mixing angles in the sequential Weinberg-Salam model.

The Weinberg-Salam (WS) $SU(2)_L \otimes U(1)$ gauge theory[1] has been quite successful in accounting for a wide variety of weak-interaction data. The four-quark version of the model, due to Glashow, Iliopoulos, and Maiani (GIM),[2] gained strong support with the discovery of charmed hadrons. Recently, the version of the model with three doublets of leptons and quarks, first discussed by Kobayashi and Maskawa (KM),[3] has gained prominence, since it incorporates the τ lepton[4] and the ($Q=-\frac{1}{3}$) Υ-constituent quark,[5] b. We assume that any additional fermions will transform under $SU(2)_L \otimes U(1)$ in the same way as the known ones, and we denote the corresponding model as the sequential WS model.

The mixing angles which describe how the weak-gauge-group eigenstates of the fermions are composed of mass eigenstates are of fundamental significance. With more than two quark doublets the old Cabibbo theory must be generalized. In this Letter we report the results of a new analysis of nuclear β decay and semileptonic hyperon and K_{e3} decays which makes use of the most up-to-date data to determine two quark mixing parameters and corresponding angles in the sequential WS model. Previous bounds on mixing angles[6,7] have been based on the standard 1974 Cabibbo fit by Roos.[8] Since several of the experimental inputs have changed since that time, it is clearly important to carry out a new analysis.

Consider the sequential WS model with $n_f = 2n$ flavors of quarks. Denoting the vectors of $Q=-\frac{1}{3}$ left-handed components of quark mass and gauge-group eigenstates, respectively, as $\psi_L = (d, s, b, \ldots)^T$ and $\xi_L = (d', s', b', \ldots)^T$, we have $\xi_L = V\psi_L$, where V is the unitary $n \times n$ quark mixing matrix. With no loss in generality one can define V_{11} and V_{12} to depend, respectively, on just one and two rotation angles. In a KM-type parametrization $V_{11} = c_1$ and $V_{12} = s_1 c_3$, where $c_1 \equiv \cos\theta_1$, $s_1 \equiv \sin\theta_1$, etc. If the Cabibbo-GIM structure of the charged current were exact, the submatrix consisting of V_{ij}, $i, j = 1, 2$ would be unitary by itself. This would require that a number of new mixing angles must vanish and would imply the existence of at least one new stable quark[9]; for the WS-KM model this would be the b quark. However, two recent experiments[10] have established that, given a reasonable assumption about the hadronic production cross section for b-flavored hadrons, such particles are unstable, with lifetimes $\tau_b < 5 \times 10^{-8}$ sec. This implies that there must be finite deviations from Cabibbo-GIM universality and yields, via a quark model estimate, a correlated lower bound on $|s_2|$ and $|s_3|$ of order $\sim 10^{-3}-10^{-4}$.

Certain general conditions which constrain fermion mixing in the WS-KM model have been discussed previously by Lee and Shrock.[7] We shall assume here that all of the neutrinos are massless, so that one can define leptonic gauge-group eigenstates as simultaneous mass eigenstates. For this case the above constraints on quark mixing reduce to hadron-lepton weak universality and the degree of success of the Cabibbo-GIM theory of charged currents[6]; these will be used for our analysis below.

Summarizing our results, we find $|V_{11}|=|c_\perp| = 0.9737 \pm 0.0025$ and $|V_{12}|=0.219\pm 0.002$, which implies $|s_3|=0.28^{+0.13}_{-0.28}$ where the errors do not include any theoretical uncertainties. With the more realistic assumption of a ~5% fractional uncertainty in $|V_{12}|$ due to SU(3) symmetry breaking (SB) in the hyperon fit, and radiative corrections, we get $|V_{12}|=0.219\pm 0.011$ and $|s_3| = 0.28^{+0.21}_{-0.28}$. To bound other mixing angles would require precise knowledge of the charmed lifetime and b- and t-decay data, which are not available, and very model-dependent use of the K_L-K_S mass difference and associated CP-invariance violation. We proceed with our analysis; a detailed report will be published elsewhere.[11]

(I) *Determination of* $|V_{11}|$.—From the muon lifetime,[12] and the equation

$$\tau_\mu^{-1} = \left(\frac{G_\mu^2}{192\pi^3}\right) f\left(\frac{m_e^2}{m_\mu^2}\right)\left[1-\frac{\alpha}{2\pi}\left(\frac{\pi^2-25}{4}\right)\right],$$

where $f(x)=1-8x+8x^3-x^4+12x^2\ln(1/x)$, we find

$$G_\mu = (1.166\,32 \pm 0.000\,04)\times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$= (1.435\,82 \pm 0.000\,04)\times 10^{-49}\ \mathrm{erg\ cm}^3.$$

The nuclear β-decay constant G_V' is determined from $0^+\to 0^+$ superallowed Fermi β transitions. Extracting the nuclear-dependent radiative correction δ_R and the Coulomb correction δ_C one obtains a nuclear-independent $\mathfrak{F}t$ value: $\mathfrak{F}t = ft(1+\delta_R)\times(1-\delta_C)$.[13,14] Then $G_V'^2 = \pi^3 \ln 2/(\mathfrak{F}t m_e^5)$. Two recent values[13,14] for G_V' are, in units of 10^{-49} erg cm^3, 1.4128 ± 0.0005 and $1.412\,48\pm 0.000\,44$; we use the weighted mean, $G_V' = 1.412\,46\times 10^{-49}$ erg cm^3. From a comparison of the different values of $1-\delta_C$ and G_V' obtained in Refs. 13 and 14 we assign as our own estimates of the fractional errors in these quantities 2×10^{-3}, and 1×10^{-3}, respectively.

The mixing coefficient $|V_{11}|$ is then given by $|V_{11}| = [G_V'^2/G_\mu^2(1+\delta_W)]^{1/2}$, where δ_W denotes the weak-interaction correction to the ratio of the β and μ decay coupling constants squared. The correction to the ratio $\Gamma(d\to ue\bar\nu_e)/\Gamma(\mu\to\nu_\mu e\bar\nu_e)$ of free-quark to μ decay rates was calculated by Sirlin,[15] who used current-algebra arguments to estimate the corresponding ratio for physical hadrons. This calculation was performed in the WS model with four quarks; for our work it is necessary to generalize it to the case of $2n$ quarks. We have done this by explicitly checking the graphs which contribute to δ_W, and we find that, to leading order, δ_W is not modified by the existence of heavy quarks,[16] as long as certain obvious criteria are satisfied.[17] It is difficult to ascertain the theoretical uncertainty inherent in this method of computing δ_W starting with free quarks; we estimate the fractional error in $1+\delta_W$ to be $\sim\alpha/2\pi = 1\times 10^{-3}$. We find, finally, that $|V_{11}| =$ (a) 0.9736 ± 0.0025, (b) 0.9738 ± 0.0025 for $\sin^2\theta_W =$ (a) 0.20, (b) 0.25. Thus, to within the errors, the present uncertainty in $\sin^2\theta_W$ does not affect our determination of mixing coefficients. For the subsequent analysis we shall take the average value $|V_{11}|=0.9737\pm 0.0025$.

(II) *Determination of* $|V_{12}|$; (A) *hyperon decays*.—In order to find $|V_{12}|$ we have carried out a new two-parameter fit to $\Delta S=1$ semileptonic hyperon decays. The two parameters are $|V_{12}|$ and $\alpha_D = D/(D+F)$, where D and F are the symmetric and antisymmetric SU(3) reduced matrix elements for the axial-vector current. Although this fit is sufficient for our purposes, we have also performed one[11] which includes the two $\Delta S = 0$ decays $\Sigma^\pm \to \Lambda e^\pm \overset{(-)}{\nu}_e$.

The most general form of the matrix element of the charged current between initial and final baryon states B_j and B_i is

$$\langle B_i(p_i)|J_\mu|B_j(p_j)\rangle = \bar u(p_i)[\gamma_\mu(F_1^{ij}+G_1^{ij}\gamma_5) - i\sigma_{\mu\nu}q^\nu(F_2^{ij}+G_2^{ij}\gamma_5) + q_\mu(F_3^{ij}+G_3^{ij}\gamma_5)]u(p_j),$$

where $q=p_i-p_j$. Given a choice for α_D, to obtain the actual values of D and F, we use as input the most accurately measured axial-vector coupling constant, viz., that for the decay $n\to pe\bar\nu_e$, $g_A(0)=D+F=1.253\pm 0.007$. We utilize the SU(3) octet property of the vector and axial-vector Cabibbo currents to calculate F_1^{ij} and F_2^{ij} in terms of the nucleon electromagnetic form factors, and to calculate G_1^{ij}

in terms of α_D and

$$G_1^{pn}(q^2) \equiv g_A(q^2) = 1.253/(1 - q^2/m_A^2)^2,$$

with $m_A = 0.9$ GeV. The partial conservation of axial-vector current and the generalized Goldberger-Treiman relation[18] are used to compute G_3. Further, we take $F_3 = G_2 = 0$ for the fits listed here. In order to incorporate radiative corrections we use the Fermi coupling

$$G_F = (1+\delta_W)^{1/2} G_\mu = 1.1785\times 10^{-5}\ \text{GeV}^{-2}.$$

The results of our hyperon fits are listed in Table I. The experimental values are taken from Ref. 12, except in the cases of the branching ratios $B(\Sigma^- \to \Lambda e\bar{\nu}_e)$ and $B(\Xi^- \to \Lambda e\bar{\nu}_e)$, for which we have used a weighted mean of the very recent results from Herbert *et al.*[19] and the previous world data. From the $\Delta S = 1$ fit we find $|V_{12}| = 0.220 \pm 0.003$ and $\alpha_D = 0.654 \pm 0.008$, with a satisfactory $\chi^2 = 6.9$ for 6 degrees of freedom (DF). The $\Delta S = 0, 1$ fit yields practically the same value of $|V_{12}| = 0.222 \pm 0.003$, but with a slightly smaller $\alpha_D = 0.645 \pm 0.008$ and only a fair $\chi^2 = 14.8/8$ DF. The single main contribution to this χ^2 is from the fit to $B(\Sigma^- \to \Lambda e\bar{\nu}_e)$, which is a pure D transition and, because q^2 is small, is consequently essentially purely axial vector. We regard the χ^2 as indicative of the magnitude of SU(3) SB in the axial-vector part of the matrix element. If we uniformly increase the experimental errors to render $\chi^2 = 8/8$ DF, then the resultant minimization error in $|V_{12}|$ becomes 0.004. The errors quoted for $|V_{12}|$ and α_D are defined as the changes in each of these parameters, with the other held fixed at the position of the minimum, which cause χ^2 to increase by one unit. This definition is the same as the one used in Ref. 8. These errors thus represent only the statistical uncertainty in the determination of these quantities by the minimization of χ^2. They do *not* include any part due to the uncertainty in the theoretical calculation of weak matrix elements and hence do *not* represent the full uncertainty in these parameters. From our various hyperon fits and their comparison with our K_{e3} results (see below) we estimate that the true fractional error in $|V_{12}|$ could easily be $\sim 5\%$.

(II) *Determination of* $|V_{12}|$; (B) K_{e3} *decays.*—$|V_{12}|$ can also be determined from K_{l3} decays, which have the advantage, relative to hyperon decays, that only the vector current contributes, so that the radiative correction δ_W is more reliable and SU(3) and chiral SU(3)⊗SU(3) SB[20] can be consistently incorporated. However, q^2 is much larger than for hyperon decays, and the SU(3) predictions apply strictly only at $q = 0$. We use only K_{e3}, and not $K_{\mu3}$, decays since for the former the theoretical calculation is more reliable, as it depends essentially only on the form factor $f_+(q^2)$ [although we do include $f_-(q^2)$ also].[21] From a comparison of our calculation (using G_F as given above) with the experimental branching ratios $B(K^+ \to \pi^0 e^+ \nu_e)$ and $B(K_L^0 \to \pi^\pm e^\mp \overset{(-)}{\nu}_e)$, we find $\sqrt{2} f_+(0)_{K^+} |V_{12}| = 0.216 \pm 0.003$ for the former and $f_+(0)_{K_L} |V_{12}| = 0.207 \pm 0.005$ for the latter decay. Correcting $f_+(0)$ for SU(3) and SU(3)⊗SU(3) SB,[20] we obtain $|V_{12}| = 0.221 \pm 0.003$ and 0.212 ± 0.005 from K_{e3}^+ and K_{e3}^0 decays, respectively. The weighted mean, $|V_{12}| = 0.219 \pm 0.003$, is slightly smaller than, but in reasonable agreement with, the value of $|V_{12}|$ found from hyperon decays. A final weighted mean of the hyperon and K_{e3} values

TABLE I. Fits to semileptonic hyperon decays. Data are from Ref. 12 except as noted in the text.

Quantity	Experimental value	$\Delta S = 1$ fit	$\Delta S = 0, 1$ fit
$B(\Lambda \to pe\bar{\nu}_e)$	$(0.807 \pm 0.028)\times 10^{-3}$	0.801×10^{-3}	0.822×10^{-3}
$B(\Lambda \to p\mu\bar{\nu}_\mu)$	$(1.57 \pm 0.35)\times 10^{-4}$	1.36×10^{-4}	1.39×10^{-4}
$B(\Sigma^- \to ne\bar{\nu}_e)$	$(1.08 \pm 0.04)\times 10^{-3}$	1.06×10^{-3}	1.03×10^{-3}
$B(\Sigma^- \to n\mu\bar{\nu}_\mu)$	$(0.45 \pm 0.04)\times 10^{-3}$	0.506×10^{-3}	0.492×10^{-3}
$B(\Sigma^- \to \Lambda e\bar{\nu}_e)$	$(0.60 \pm 0.04)\times 10^{-4}$		0.705×10^{-4}
$B(\Sigma^+ \to \Lambda\bar{e}\nu_e)$	$(2.02 \pm 0.47)\times 10^{-5}$		2.29×10^{-5}
$B(\Xi^- \to \Lambda e\bar{\nu}_e)$	$(0.36 \pm 0.10)\times 10^{-3}$	0.453×10^{-3}	0.466×10^{-3}
$B(\Xi^- \to [\Lambda, \Sigma^0] e\bar{\nu}_e)$	$(0.68 \pm 0.22)\times 10^{-3}$	0.536×10^{-3}	0.549×10^{-3}
$G_1/F_1(n \to pe\bar{\nu}_e)$	1.253 ± 0.007	Input	Input
$G_1/F_1(\Lambda \to pe\bar{\nu}_e)$	0.62 ± 0.05	0.707	0.714
$G_1/F_1(\Sigma^- \to ne\bar{\nu}_e)$	-0.385 ± 0.07	-0.386	-0.363

of $|V_{12}|$ gives $|V_{12}|=0.219$, with an error of ± 0.002 or ± 0.011 if the theoretical uncertainty of ~5% due to SU(3) SB and radiative corrections is excluded or included, respectively. From these results we can determine the corresponding mixing angles: $|c_1|=0.9737\pm 0.0025$ and $|s_3|=0.28$, with errors of $^{+0.13}_{-0.28}$ or $^{+0.21}_{-0.28}$, defined as for $|V_{12}|$.

This determination of $|s_3|$ differs from the bound $|s_3|<0.25$ given in Ref. 6, which was based on the old Roos fit[8] and assumed that the *total* error in $|V_{12}|$ was just the statistical minimization error, ± 0.003. The fact that $|s_3|$ can attain reasonably large values makes it more difficult to bound the other quark mixing angles which, in the WS-KM model, consist of θ_2 and δ. In particular, even if one were to grant the validity of the free-quark–model assumptions and vacuum insertion technique underlying the use of the K_L-K_S mass difference to bound $|s_2|$, the approximation[6] of setting $|s_3|\simeq 0$ is not justified. Further, whatever the outcome of future experiments, from what is known at present, it is not at all clear that b should decay mainly to c rather than to u, since the relevant ratio of couplings,

$$\frac{|V_{ub}|}{|V_{cb}|}\equiv\frac{|V_{13}|}{|V_{23}|}=\frac{|s_1 s_3|}{|c_1 c_2 s_3 + s_2 c_3 e^{i\delta}|},$$

can easily be $\gg 1$ for allowed values of the mixing angles.

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