Convection Currents and Spin Magnetization in $E2$ Transitions of ^{12}C

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The transverse electromagnetic form factors squared of the ^{12}C 2^+ levels at 4.439 MeV ($T=0$) and at 16.107 MeV ($T=1$) have been measured by means of 180° electron scattering over a momentum-transfer range from $q=0.51$ to $2.05\ \text{fm}^{-1}$. Evidence is presented for appreciable contributions of nuclear convection currents to the transverse 4.439-MeV form factor at low q, and spin magnetization contributions to the transverse 16.107-MeV form factor at higher q.

Transverse inelastic-electron-scattering form factors are sensitive to the spatial distribution of nuclear convection and magnetization currents. Yet for transitions of electric character, there have been few determinations of transverse form factors,[1] even though corresponding longitudinal form factors are often well known. For such transitions the transverse form factor squared at a momentum transfer q is given in the plane-wave Born approximation by[2]

$$|F_T(q)|^2=\frac{4\pi}{Z^2}\frac{|T_{EJ}(q)+T_{EM}(q)|^2}{2J_i+1},$$

where

$$T_{EJ}(q)=\langle J_f\|(1/q)\int d^3r\,\vec{j}(\vec{r})\cdot\vec{\nabla}\times j_L(qr)\vec{Y}_{LL}{}^M(\hat{r})\|J_i\rangle,$$
$$T_{EM}(q)=\langle J_f\|q\int d^3r\,\vec{\mu}(\vec{r})\cdot j_L(qr)\vec{Y}_{LL}{}^M(\hat{r})\|J_i\rangle. \tag{1}$$

In Eq. (1) $e\vec{j}(\vec{r})$ and $e\vec{\mu}(\vec{r})$ represent the nuclear convection current and magnetization density operators. The convection current and magnetization amplitudes $T_{EJ}(q)$ and $T_{EM}(q)$ for a given multipolarity L are rich in nuclear-structure information and can facilitate the interpretation of data on pion electroproduction and photoproduction, processes which are determined mainly by the magnetization properties of the nucleus. However, few attempts, if any, have been made to understand how these two terms separately contribute to the measured transverse form factor of an electric transition. An inspection of the q dependence of Eq. (1) shows that whereas the magnetization amplitude will generally dominate at higher q, the convection-current components should be most clearly seen at low q. Although the continuity equation ensures the existence of a definite convection-current component at $q=\hbar\omega$, the photon point, direct evidence for convection currents in (e,e') data is essentially nonexistent.

In this Letter we compare the transverse form factors for two $E2$ transitions in ^{12}C. The transition to the $T=1$, 16.109-MeV level is known to be a good example of a spin-flip excitation, and hence the magnetization amplitude dominates strongly. For the $T=0$, 4.439-MeV state, however, the magnetization amplitude should be reduced approximately by a relative factor[3] of $(\mu_p-\mu_n)/(\mu_p+\mu_n)=5.3$. Therefore, especially at low q, the magnetization amplitude should not mask the convection-current amplitude.

The scarcity of transverse form-factor measurements for electric transitions is a consequence of the strong dominance of the longitudinal cross section at usual scattering angles. However, for scattering angles θ near 180°, the longitudinal scattering is heavily suppressed, according to the expression for the differential cross section[4]

$$\frac{d\sigma(\theta\simeq\pi)}{d\Omega}=\eta\frac{Z^2\alpha^2}{4E_i{}^2}\left[\left(\frac{(\pi-\theta)^2}{4}+\frac{m^2}{E_iE_f}\right)|F_L|^2+|F_T|^2\right], \tag{2}$$

where E_i and E_f are the incident- and scattered-electron energies, Ze is the nuclear charge, m is the electron rest mass, η is a kinematic recoil factor, and F_L is the longitudinal form factor. The use of the University of Massachusetts four-magnet 180° electron scattering apparatus,[5] recently installed to operate in conjunction with the high-resolution magnetic spectrometer at the Bates Linear Accelerator Center, permitted the measurement of transverse scattering without being overwhelmed by longitudinal scattering.

Electrons of incident energies from 57 to 215 MeV were scattered from natural carbon targets of thickness 137 mg/cm^2 in the case of the 4.439-MeV level, and 25.9 mg/cm^2 for the 16.107-MeV level. Those electrons scattered through 180° were deflected out of the incident beam path into a magnetic spectrometer operating in the energy-loss mode, and were detected by a drift chamber system[6] in the focal plane. Data were normalized to proton cross-section[7] measurements taken after each ^{12}C run. The deviation of the scattering angle from 180° due to multiple scattering and the finite spectrometer acceptance solid angle was established by measuring the carbon elastic cross section. For the 4.439-MeV level the longitudinal contribution at 180° was found to be less than 10% of the measured cross section for all beam energies. If instead the experiment was performed at 160°, then the longitudinal component would be 50 times larger than the transverse component.

Calculations[8] of the isoscalar and isovector 2^+ transverse form factors in a particle-hole ($1p_{1/2}$, $1p_{3/2}^{-1}$) model using harmonic-oscillator wave functions were found not to be in good agreement with the data. The calculated isovector transverse form factor agrees with the data in q dependence but not magnitude, and the calculated isoscalar transverse form factor disagrees with both the q dependence and magnitude of the data. Therefore, we consider the intermediate-coupling model of Cohen and Kurath,[9] which gives a good description of the energy levels and transition rates for p-shell nuclei. Curves A and B of Fig. 1 are the transverse form factors squared generated using the Cohen and Kurath (8-16) POT p-shell configurations with harmonic-oscillator wave functions for the isovector and isoscalar 2^+ transitions, respectively, and curve C is the longitudinal form factor squared for the isoscalar 2^+ transition. No convection-current contributions are present in the calculated transverse form factors since the initial and final states are within the same p shell and have the same radial wave functions in this harmonic-oscillator basis. This calculation agrees with transverse 16.107-MeV form-factor data without normalization for an oscillator length parameter $b = 1.64$ fm, and with longitudinal 4.439-MeV form-factor data,[10,11] with an upward normalization of 2.0, for $b = 1.76$ fm. However, curve B upward normalized by a factor of 3.0 fails to agree in either shape or magnitude with the transverse 4.439-MeV form-factor data.

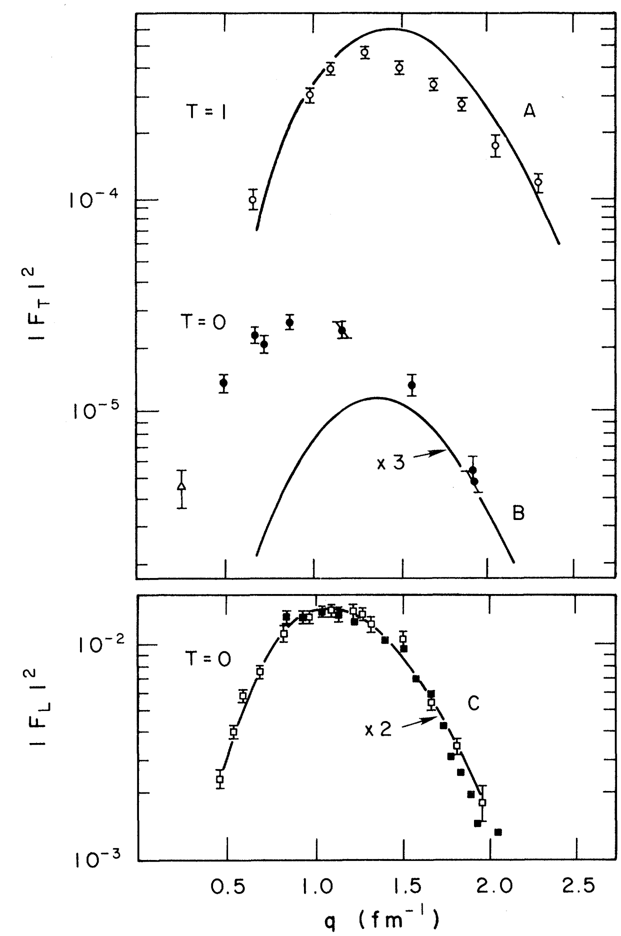

FIG. 1. Squared form factors for the $T=1$, 16.107 MeV and the $T=0$, 4.439-MeV 2^+ states in ^{12}C. Curves A and B are the transverse form factors squared of the $T=1$ and $T=0$ states calculated with $b=1.64$ and 1.76 fm, respectively. Curve C is the longitudinal form factor squared for the $T=0$ state calculated with $b=1.76$ fm. (Triangles, data of Ref. 15; unfilled squares, data of Ref. 10; and filled squares, data of Ref. 11.)

The moderate success in correctly predicting the transverse form factor of the 16.107-MeV isovector excitation by using the Cohen and Kurath configurations encourages us to believe that the magnetization part of the 4.439-MeV transverse isoscalar form factor is correctly given by this model. It is therefore conjectured that the remaining strength in the 4.439-MeV transverse form factor is largely due to convection currents not accounted for in the model. The fact that the additional strength is concentrated at low q supports this hypothesis.

In order to test the hypothesis further, we have estimated the convection-current contribu-

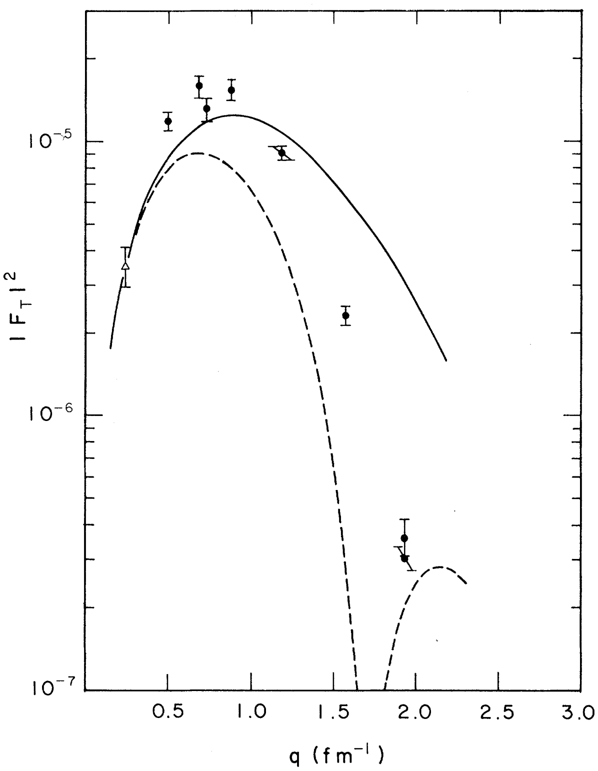

FIG. 2. Deduced convection-current part of the $T=0$, 4.439-MeV 2^+ transverse form factor squared. The dashed curve was obtained by using Cohen and Kurath configurations with Woods-Saxon wave functions normalized to the $B(E2)$ value at the photon point, and the solid curve by using a pure $(p_{1/2}, p_{3/2}^{-1})$ configuration. (Δ, datum point of Ref. 15.)

tion by repeating the calculations presented above with a Woods-Saxon potential[12] instead of a harmonic-oscillator potential. The $\vec{l}\cdot\vec{s}$ term in the Woods-Saxon potential splits the $1p_{1/2}$ and $1p_{3/2}$ levels and causes their radial dependences to differ. A transition between these levels therefore has associated convection currents. However, the computed convection-current amplitude $T_{EJ}(q)$ is insufficient to fit either the measured 4.439-MeV transverse form factor, or the observed reduced transition probability $B(E2)$. As in the case of the underestimation of the magnitude of the longitudinal form factor, we consider this failure to be due to the neglect of configurations outside the $1p$ shell. In fact, as a result of its higher excitation energy, even more configurations might enter into the 16.107-MeV transition, as evidenced by the disagreement of the calculated and experimental $B(C2)$ for this level.[13] Considering the agreement of the Cohen-Kurath calculation with the transverse form factor observed for this level in our q range, it would seem that although these neglected configurations contribute greatly to the transition charge- and convection-current densities, their influence on the transition magnetization involving spin flips is small.

In any event, for the 4.439-MeV level, we can renormalize the calculated convection-current amplitudes to give the observed $B(E2)$ value. At low momentum transfer, the shape of the form factor is insensitive to the exact form of the model, and if our hypothesis is reasonable, the renormalized convection-current calculations should provide plausible estimates of the form-factor strength unaccounted for by curve B.

The results are shown in Fig. 2. In order to make the comparisons more explicit, we have taken the assumed magnetization component out of the measured form factor squared by subtracting the square root of the unnormalized magnetization form factor squared from the square root of the measured form factor squared. In these models the signs of the current and magnetization components are the same. The agreement with respect to magnitude confirms our assumption that the 4.439-MeV transverse form factor contains large convection-current contributions. It is emphasized that the shapes of the computed form factors are probably unrealistic, especially in the high-q, model-dependent region, because essential configurations outside the $1p$ shell were neglected, as discussed previously.

In this Letter evidence has been presented for the observation of the convection-current part of a transverse form factor over a sizable q range for a $T=0$ state, and the magnetization part of a transverse form factor for a $T=1$ state. It is evident that a complete description of these states must involve configurations outside the $1p$ shell.[14] More detailed theoretical analyses are called for. The existence of extensive data on both the longitudinal and transverse form factors can probide stringent tests of theoretical wave functions. To simultaneously account for both of these form factors, more attention must be given to current conservation in nuclear models.

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Deep-Inelastic Electron Scattering from ^{12}C

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A systematic study of the deep-inelastic electron-scattering response function of ^{12}C has been carried out at scattering angles of 60° and 130° and electron energies between 160 and 520 MeV. A pronounced transverse strength, the origin of which is not understood, is found in the region between the quasielastic and the N peak.

Quasielastic electron scattering, which corresponds to the incoherent scattering of the electrons by individual nucleons within the nucleus, provides information on average kinetic and separation energies of nucleons. Previous experiments performed on several nuclei have shown that the quasielastic peak can be well fitted by a one-nucleon–knockout model.[1-3] However, it has

