mum $[1.75 \leq |t| \leq 2.00\ (\mathrm{GeV}/c)^2]$. The cross section here falls with increasing incident momentum until about $p_0 = 210$ GeV/c after which it appears to rise with further increase in momentum. This rise with increasing incident momentum is a general property of elastic-scattering models where the eikonal is a function of energy times a function of the impact parameter.[8]

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Gauge-Covariant Conformal Transformations

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The action of conformal transformations on gauge potentials can be defined in a gauge-covariant fashion. This unconventional procedure is used in several branches of theoretical physics, with the consequence that gauge potentials do not transform according to a representation of the conformal group; rather, they provide a representation for the "group of paths" of the conformal group. The nonintegrable phase factor is central to the construction.

Conformal transformations, which form a symmetry group for Yang-Mills theory, are usually realized on gauge potentials A^μ (taken as anti-Hermitian matrices in the space of infinitesimal group generators) by

$$A^\mu(x) \xrightarrow[\text{transformation}]{\text{conformal}} A'^\mu(x) = A^\alpha(x')\,\partial x_\alpha'/\partial x_\mu. \tag{1}$$

Here $x'(x,a)$ are the conformally transformed space-time coordinates, depending on the original coordinates x, and on the fifteen parameters $\{a\}$ of the conformal group. The above is equivalent to the infinitesimal formulas

$$x'^\mu = x^\mu + f^\mu,$$

$$A'^\mu(x) = A^\mu(x) + \delta_C A^\mu(x), \tag{2a}$$

$$\delta_C A^\mu = f^\alpha \partial_\alpha A^\mu + A^\alpha \partial^\mu f_\alpha \tag{2b}$$

with $f^\alpha(x)$ being the fifteen-parameter solution of Killing's equation:

$$\partial_\alpha f_\beta + \partial_\beta f_\alpha - \tfrac{1}{2} g_{\alpha\beta}\partial_\gamma f^\gamma = 0, \quad f^\alpha(x) = a^\alpha + b x^\alpha + \omega^{\alpha\beta} x_\beta + 2x^\alpha c^\beta x_\beta - c^\alpha x^2, \quad \omega^{\alpha\beta} = -\omega^{\beta\alpha}. \tag{3}$$

Expressions (1) and (2) reflect the fact that gauge potentials combine with derivatives to form gauge-covariant quantities. Covariance under coordinate transformations is maintained by transforming gauge potentials as derivatives, so that the differential form $A^\mu(x)dx_\mu$ is coordinate invariant.

In the usual way, the infinitesimal transformation determines the conserved currents by Noether's theorem.[1] These involve the energy-momentum tensor; however, it is the canonical, nonsymmetric tensor that appears. There are several disagreeable aspects to this procedure: The canonical tensor is not the one which couples to gravity; moreover, in a gauge theory it is not gauge invariant, and hence is obviously unphysical; the form of the conformal currents is rather inelegant. The remedy is well known[1]: By adding superpotentials to the Noetherian conformal currents, one can reexpress them simply in terms of the symmetric, Belinfante energy-momentum tensor, $\theta^{\mu\nu}$, which is gauge invariant and which does couple to gravity,

$$C^{\mu}=\theta^{\mu\nu}f_{\nu}. \tag{4}$$

Conservation of the currents C^{μ} follows from the zero trace of $\theta^{\mu\nu}$ when f^{μ} satisfies (3).

It is natural to inquire whether one can modify the transformation law (1) and (2), so that the Noether current is already given by (4), and no further improvement is needed. In this note it is shown that such a modified transformation law does indeed exist, that its infinitesimal form has already appeared in different contexts of theoretical physics, and that the corresponding finite construction is mathematically interesting with implications for supersymmetry transformations.

Let us observe that (2b) may also be written as

$$\begin{aligned}\delta_C A^{\mu}&=f_{\alpha}(\partial^{\alpha}A^{\mu}-\partial^{\mu}A^{\alpha}+[A^{\alpha},A^{\mu}])+\partial^{\mu}(f_{\alpha}A^{\alpha})+[A^{\mu},f_{\alpha}A^{\alpha}]=f_{\alpha}F^{\alpha\mu}+\mathfrak{D}^{\mu}f_{\alpha}A^{\alpha},\\ F^{\alpha\mu}&=\partial^{\alpha}A^{\mu}-\partial^{\mu}A^{\alpha}+[A^{\alpha},A^{\mu}],\qquad \mathfrak{D}^{\mu}=\partial^{\mu}+[A^{\mu},\quad.\end{aligned} \tag{5}$$

The second contribution to $\delta_C A^{\mu}$ is an infinitesimal (field-depedent) gauge transformation, which separately leaves the Yang-Mills theory invariant. Thus we see that by supplementing the infinitesimal conformal transformation by an infinitesimal gauge transformation, we arrive at a transformation law expressed in terms of the gauge field $F^{\mu\nu}$:

$$\delta A^{\mu}=f_{\alpha}F^{\alpha\mu}. \tag{6}$$

Because the response of the potential is gauge covariant (a pure gauge is left invariant) (6) may be called a gauge-covariant conformal transformation. It is clear that (6) is a symmetry transformation for Yang-Mills theory and that Noether's procedure yields the gauge-invariant current (4).

The gauge-covariant transformation law arose previously in investigations of (extended) supersymmetry. The composition law for two infinitesimal supersymmetry transformations involves an infinitesimal (conformal) space-time transformation. It was noted that in this circumstance the conformal transformation appears in its gauge-covariant form.[2] Small deformations of pseudoparticle solutions to the Yang-Mills equation provide an apparently unrelated context for Eq. (6). Here one is also interested in deformations arising from infinitesimal conformal transformations. Again it proved useful to use the gauge-covariant expression.[3]

Given the infinitesimal form (6), we wish to determine the finite transformation law, for example, by integrating the Lie differential equations. So first we check the integrability condition. From (6) it follows that

$$(\delta^{(2)}\delta^{(1)}-\delta^{(1)}\delta^{(2)})A^{\mu}=\delta^{(12)}A^{\mu}+\mathfrak{D}^{\mu}(f_{\alpha}{}^{(1)}f_{\beta}{}^{(2)}F^{\alpha\beta}). \tag{7}$$

The infinitesimal coordinate transform for $\delta^{(1)}$ $(\delta^{(2)})$ is $f_{\alpha}{}^{(1)}$ $(f_{\alpha}{}^{(2)})$; for $\delta^{(12)}$ it is $f_{\alpha}{}^{(12)}$ given by

$$f_{\alpha}{}^{(12)}=f_{\mu}{}^{(1)}\partial^{\mu}f_{\alpha}{}^{(2)}-f_{\mu}{}^{(2)}\partial^{\mu}f_{\alpha}{}^{(1)}, \tag{8}$$

which is a Killing vector provided that $f_{\alpha}{}^{(1)}$ and $f_{\alpha}{}^{(2)}$ are. Thus we see that whereas the first term on the right-hand side of (7) has the correct form, the second term, which is a gauge transformation, spoils the integrability condition whenever it is nonvanishing. Consequently the infinitesimal gauge-covariant conformal transformations cannot be integrated to finite values; they do not form a group; and *a fortiori* they do not give a representation of the conformal group.

It is, of course, possible to provide a finite transformation law which reduces to (6) in the infinitesimal limit. I propose

$$A^{\mu}(x)\xrightarrow[\substack{\text{gauge-covariant}\\ \text{conformal}\\ \text{transformation}}]{}A'^{\mu}(x)=U^{-1}A^{\alpha}(x')\frac{\partial x_{\alpha}'}{\partial x_{\mu}}U+U^{-1}\frac{\partial}{\partial x_{\mu}}U=\left[U^{-1}A^{\alpha}(x')U+U^{-1}\frac{\partial}{\partial x_{\alpha}'}U\right]\frac{\partial x_{\alpha}'}{\partial x_{\mu}}. \tag{9a}$$

The new rule is a gauge transformation of the old one, with the gauge function U given by the noninte-grable phase factor,[4]

$$U=P\exp\left[-\int_x^{x'}A^\alpha(z)dz_\alpha\right] \tag{9b}$$

(P stands for path ordering). The substitution $x'=x+f$ and expansion to first order in f reproduces (6).

The finite gauge-covariant conformal transformation is seen to be path dependent, and to define it completely, we must specify the path for the integral in (9b). A useful prescription is obtained by recalling that for a one-parameter subset of transformations the integrability condition is satisfied [when $f_\alpha^{(1)}$ and $f_\alpha^{(2)}$ are proportional, the right-hand side of (7) vanishes] and (9) must reduce to a representation of the Abelian subgroup. It will be shown that the following path possesses this property.

To define the line integral, consider first the manifold of the conformal group where the "points" are the group elements. Any group element g can be reached by traversing a path on this manifold which proceeds from the point representing the identity e to g. Correspondingly in configuration space, a space-time path, which passes from x to x', is traced. It is this path that is used in the line integral (9b).

A one-parameter family of conformal transformations may be specified by λf, where f is fixed and λ varies from 0 to 1. The path is then given by $z(\lambda)=x'(x,\lambda a)$ with $z(0)=x'(x,0)=x$, $z(1)=x'(x,a)=x'$. The Lie equation is

$$\frac{\partial\mathcal{A}^\mu(\lambda)}{\partial\lambda}=f_\alpha\left\{\frac{\partial\mathcal{A}^\mu(\lambda)}{\partial x_\alpha}-\frac{\partial\mathcal{A}^\alpha(\lambda)}{\partial x_\mu}+[\mathcal{A}^\alpha(\lambda),\mathcal{A}^\mu(\lambda)]\right\}, \tag{10}$$

where $\mathcal{A}^\mu(\lambda)$ is the transform of the gauge potential appropriate to a conformal transformation which takes x to $x'(x,\lambda a)$. We now show that the proposed formula, written for this λ-parametrized case,

$$\mathcal{A}^\mu(\lambda)=U^{-1}(\lambda)A^\alpha(z)[\partial z_\alpha(\lambda)/\partial x_\mu]U(\lambda)+U^{-1}(\lambda)\,\partial U(\lambda)/\partial x_\mu, \tag{11a}$$

$$U=(\lambda)=P\exp\{-\textstyle\int_0^\lambda A^\alpha(z)[\partial z_\alpha(\lambda')/\partial\lambda']d\lambda'\}, \tag{11b}$$

satisfies (10). Repeated use of

$$\partial U(\lambda)/\partial\lambda=-A^\alpha(z)[\partial z_\alpha(\lambda)/\partial\lambda]U(\lambda) \tag{12}$$

gives for the left-hand side of (10)

$$U^{-1}(\lambda)\left\{\frac{\partial A^\beta(z)}{\partial z_\alpha}-\frac{\partial A^\alpha(z)}{\partial z_\beta}+\left[A^\alpha(z),A^\beta(z)\right]\right\}U(\lambda)\frac{\partial z_\alpha(\lambda)}{\partial\lambda}\frac{\partial z_\beta(\lambda)}{\partial x_\mu}.$$

Similarly, the right-hand side reads

$$U^{-1}(\lambda)\left\{\frac{\partial A^\beta(z)}{\partial z_\alpha}-\frac{\partial A^\alpha(z)}{\partial z_\beta}+\left[A^\alpha(z),A^\beta(z)\right]\right\}U(\lambda)f_\gamma\frac{\partial z_\alpha(\lambda)}{\partial x_\gamma}\frac{\partial z_\beta(\lambda)}{\partial x_\mu}.$$

The two are equal, since the Lie equation for the conformal transformation of the coordinates is

$$\partial z_\alpha(\lambda)/\partial\lambda=f_\gamma\,\partial z_\alpha(\lambda)/\partial x_\gamma. \tag{13}$$

This establishes that the proposed transformation law properly reduces to representations of Abelian subgroups of the conformal group.

Since (9) was obtained by construction, rather than derivation, it may be possible to find another rule, which has the correct infinitesimal and one-parameter limits. The formula presented here does, however, have a unique property: Although it does not describe a representation of the conformal group, it gives a representation of a much larger group which can be associated with the conformal group—the "group of paths."

Following suggestions of Coleman and Goldstone, we can construct this group. Consider again the manifold of the conformal group, with paths proceeding in every possible way from e to each of the group elements. The members of the group of paths are these paths. Homotopically equivalent ones are not identified, but paths that differ only by an even number of traversals of the same loci are. The identity element comprises the (degenerate) path that stays at e, as well as all other paths that proceed from and return to e along the same loci. Two paths P_1 and P_2 are composed by multiplying P_2 by g_1, the conformal group element on which P_1 ends, and adjoining g_1P_2 to P_1; in other words, the beginning of P_2 is attached to the end of P_1, to form the longer path P_{12} terminating at the element $g_{12}=g_1g_2$. It is now easy to verify that the above construction defines a group—the group of paths—and that the gauge potential transformation law (9) is a representation of this group.

I find it interesting that the physically motivated and attractive infinitesimal transformation law (6) should have such an involved mathematical structure, but the practical consequences of these formal considerations are not apparent to me. The structure of finite supersymmetric transformations, which are a "square root" of gauge-covariant conformal ones, should be explored further to see whether the complexities here exposed are hidden there as well. I benefitted from discussions and suggestions by S. Coleman and J. Goldstone, which I gratefully acknowledge. This research was supported in part by funds provided by the U. S. Department of Energy under Contract No. EY-76-C-02-3069.

Note added.—B. Zumino points out that the gauge-covariant conformal transformation arises in a supersymmetric theory only after auxiliary fields are eliminated, with the help of the Wess-Zumino gauge condition. If they are retained, the conformal transformations are conventional, and no complications are anticipated.

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Energy Dependence of $^4He(p,d)^3He$

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The cross section of $^4He(p,d)^3He$ at $\theta=22.5°$ has been measured between $T_p=200$ and 500 MeV, which together with previous data provides the first excitation function for the (p,d) reaction up to $T_p=770$ MeV and 0.7 GeV/c momentum transfer. The energy variation is found mainly to reflect a strong dependence upon momentum transfer. Using known nuclear wave functions we examine the contributions from ordinary neutron pick-up and certain multinucleon reaction mechanisms.

The (p, d) reaction at intermediate energies can reach large momentum transfer which must be supplied by the nuclear state. The nuclear momentum needed can come from an individual nucleon whose momentum is that kinematically specified by the recoiling nucleus. In such a single-nucleon interaction model, the cross-section dependence in terms of the recoil momentum will reflect the single-nucleon momentum distribution to the extent that there is no rescattering involving other nucleons. Rescattering effects can be important when the momentum distribution varies rapidly and assumes small values. They can be accounted for by means of optical potentials and their effect on the reaction dynamics can be realized in standard distorted-wave Born-approximation calculations. In this application it is the off-shell momenta generated by the optical potential that are tested.[1]

Another way to deal with rescattering effects is by way of reaction diagrams where the multinucleon interactions, that are believed to dominate the reaction dynamics, can be treated implicitly in terms of known subprocesses and specified momentum dependence of the nuclear structure. This approach is of particular interest for assessing the importance of pion production/absorption effects in the (p, d) reaction.

