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Magnetic Moments, Hadron Masses, and Quark Masses

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Two independent quark-model predictions of the Λ magnetic moments, in agreement with one another and with experiment to 1%, are obtained by using two different inputs for SU(3) breaking in quark masses: (1) $(m_s/m_u)=(M_{\Sigma^*}-M_\Sigma)/(M_\Delta-M_N)$; and (2) $m_s-m_u=M_\Lambda-M_N$. I discuss the implications of the surprising sucess of a description of quark magnetic moments as Dirac moments with quark masses related so simply to hadron masses.

The recent value of the Λ magnetic moment[1] is in remarkable agreement with the prediction from a nonrelativistic constituent quark model.[2] The purpose of this Letter is to point out that a recently proposed extension of this model[3] gives two independent predictions, one old and one new, both of which are in agreement with the experiment and with one another at the 1% level. The basic physical assumptions underlying these predictions are (1) that the magnetic moment of a hadron is obtained by adding quark magnetic moments vectorially according to the naive SU(6) recipe, (2) that SU(6) and SU(3) are badly broken because the larger mass of the strange quark reduces its magnetic moment, and (3) that the SU(3)-breaking effect can be calculated using experimental hadron mass splittings as input. The two predictions are obtained by using two different hadron mass splittings.

The old prediction uses hadron spin splittings like the ratio $(M_\Delta-M_N)/(M_{\Sigma^*}-M_\Sigma)$ to define the SU(3)-symmetry breaking. This ratio which is unity in the SU(3) limit is directly related to the ratio of quark magnetic moments under the assumption that the spin splittings come from a "color magnetic" interaction proportional to the color magnetic moments of the quarks which are in turn proportional to electromagnetic moments. The result obtained is

$$\mu_\Lambda=-(\mu_p/3)(M_{\Sigma^{*+}}-M_{\Sigma^+})/(M_{\Delta^+}-M_p)$$
$$=-0.61\mu_N. \qquad (1)$$

The new prediction uses the masses difference $M_\Lambda-M_N$ to define the SU(3)-symmetry breaking and sets this difference equal to the quark mass difference,

$$m_s-m_u=M_\Lambda-M_p. \qquad (2a)$$

This is the new ingredient leading to the second predition. *A priori* there is no reason to choose the Λ-N mass difference for the right-hand side of (2a) rather than Σ-N or Σ^*-Δ. The decuplet mass splitting has commonly been used because the equal mass spacing has been interpreted as indicating that decuplet mass splittings are simpler than octet splittings. However, arguments based on quantum chromodynamics show that the decuplet splitting involves a complicated interplay of both the quark mass difference (2a) and the spin splittings appearing in Eq. (1). The model of Ref. 3 shows that Eq. (2a) with the Λ-N mass difference should be used and eliminates effects of spin splittings.

If the quark magnetic moment is assumed to be the Dirac magnetic moment for quark masses[4] satisfying the relation (2a), the prediction for μ_Λ is

$$\mu_\Lambda = (-\tfrac{1}{3})[(1/\mu_p) + (M_\Lambda - M_p)/M_p\,\mu_N]^{-1} = -0.61\ \mu_N. \tag{2b}$$

Both predictions (1) and (2b) are in remarkable agreement with the new experimental value $\mu_\Lambda = (-0.6138 \pm 0.0047)\mu_N$. That they are also in remarkable agreement with one another suggests a new relation between hadron masses and the proton magnetic moment. Eliminating μ_Λ between (1) and (2b) gives

$$[(M_{\Delta^+} - M_p)/(M_{\Sigma^{*+}} - M_{\Sigma^+})] - 1 = \mu_p(M_\Lambda - M_p)/M_p\,\mu_N. \tag{3}$$

This peculiar relation is in excellent agreement with experiment. The left-hand side is 0.523, the right-hand side is 0.528. This unorthodox combination of hadron mass differences and the proton moment has a simple physical interpretation. The SU(3)-breaking quark mass parameter $(m_s - m_u)/m_u$ is computed in two ways. The left-hand side uses the quark mass *ratio* (m_s/m_u) obtained from hadron spin splittings. The right-hand side uses the quark mass *difference* $(m_s - m_u)$ obtained from hadron strangeness splittings, but needs the proton moment to provide a quark mass scale relating the mass difference to a mass ratio. Thus Eq. (3) says that the quark mass ratio and the quark mass difference determined in two different ways from hadron masses are consistent at the 1% level with the quark mass m_u determined from the proton mass and magnetic moment.

The success of these relations suggests a review of the underlying physics and its implications for hadron models. The first prediction (1) is equivalent to a similar prediction obtained by De Rújula, Georgi, and Glashow (DGG)[2] using expressions involving quark mass ratios. Our derivation shows that explicit reference to quark masses is unnecessary; proportionality between electromagnetic and color magnetic moments is sufficient. The second prediction (2b) and the relation (3) require the explicit assumption that quark magnetic moments depend upon masses like Dirac moments and that the relevant quark mass difference is given by Eq. (2a). This much more serious assumption is generally not valid in conventional models. In the DGG model[2] Eq. (2a) does not hold because hadron masses include additional terms like kinetic energies which are inversely proportional to quark masses and do not cancel in the difference (2a). The model of Ref. 3 avoids these terms by the use of scaling properties of the Quigg-Rosner[5] logarithmic potential model. In this model kinetic energies and mass splittings in the hadron spectrum are independent of the quark mass and cancel out of mass differences like (2a).

However, it is still a big step further to use the quark mass difference of Eq. (2a) as the mass parameter in the magnetic moments and to obtain results valid to a few percent. The success of the relations (2) and (3) at this level indicate that the "quasinuclear colored quark model" of Ref. 3 and the three basic assumptions above should be taken more seriously than indicated by their crude derivations. The underlying physics is that the *same quark mass parameter* appears in the simplest possible way in the electromagnetic moments, the color magnetic moments, and the hadron mass splittings. That electromagnetic and color moments should depend upon the same mass parameter is not surprising. But the value of the magnetic moment is not expected to be determined to 1% by the mass parameter which enters hadron mass splittings and includes binding energies as well as quark masses.

The magnetic moment of a Dirac particle bound in an external potential depends upon a mass parameter which is a function of the Lorentz character of the potential.[6] For a Lorentz scalar potential this mass parameter is indeed the total energy of the bound state, including the binding energy. But for a Lorentz vector potential the magnetic moment is not affected by the binding (the magnetic moment of an electron strongly bound in the electrostatic field of a Van de Graaff accelerator is the same as that of a free electron). The results (2b) and (3) suggest that the dominant binding potential for quarks in hadrons is Lorentz scalar rather than the Lorentz four-vector of a Coulomb or a one-gluon-exchange potential. But such an argument is not expected to hold to 1%. Note that Lorentz scalar confinement is implicit in bag models[7] which use Lorentz scalar bags as the principal confining mechanism and have only weak effects due to gluon exchange.

All the above leads to a deeper questioning of what indeed is the meaning of the quark mass. This mass appears as a parameter in many quark-model calculations of observable hadron properties, but very different values are used in different calculations, varying from zero to infinity. There are "current quarks" which have nearly zero mass, bound "constituent quarks" whose mass is of the order of hadron masses, and free quarks, which have a very heavy mass or an infinite mass if quarks are permanently confined.

An intuitive picture of quark masses motivated by quantum chromodynamics shows that an isolated quark has a strong color field at large distances and strong long-range forces if there are no other quarks nearby to cut off the color lines of force and confine color. The mass of an isolated quark must include all the energy in the associated color field at large distances, since this field must move with the quark and contribute to its inertial mass. In models with quark confinement, the energy in the field of an isolated quark is infinite and quarks have infinite mass and are unobservable.

Quarks bound in color-singlet hadrons do not have the large color field at large distances and therefore do not have a large inertial mass. The mass parameter associated with the motion of these bound quarks inside hadrons and with their magnetic moments must be simply related to the energy in the color field which moves with each quark. This may determine the value of the quark mass successfully used in constituent quark models and in the relations (2) and (3) of this paper. In scattering processes the mass to be used for the quark should depend upon how much of the associated color field recoils with the quark. At very high momentum transfers the quark may have received a kick which moves it so fast that its color field does not move with it. This would account for the small quark masses used for current quarks or quark partons, and the necessity to treat the color field separately as a "gluon component" in the hadron wave function for deep inelastic processes.

Within this continuum of quark mass values from zero to infinity used for different processes there seems to be an intermediate region relevant to hadron spectroscopy where each valence quark has an inertia roughly given by its share of the hadron mass and only valence quarks need be considered.[8] These "constituent quark masses" determine the scales of mass splittings in the hadron spectrum and of hadron magnetic moments. There is as yet no rigorous derivation of these properties of constituent quarks from quantum chromodynamics, but the remarkable success and precision of nonrelativistic quark-model predictions in describing the experimental spectrum suggest that a more fundamental derivation must exist.

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[1]L. Schachinger *et al.*, Phys. Rev. Lett. 41, 1348 (1978).

[2]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[3]H. J. Lipkin, Phys. Lett. 74B, 399 (1978).

[4]O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).

[5]C. Quigg and J. L. Rosner, Phys. Lett. 71B, 153 (1977).

[6]H. J. Lipkin and A. Tavkhelidze, Phys. Lett. 17, 331 (1965).

[7]T. De Grand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[8]G. Altarelli, N. Cabibbo, L. Maiani, and R. Petronzio, Nucl. Phys. B69, 531 (1974).