PHYSICAL REVIEW LETTERS

VOLUME 41 | 4 DECEMBER 1978 | NUMBER 23

Resolution of the $\eta' \to \eta\pi\pi$ Puzzle

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(Received 6 September 1978)

Results from current-algebra calculations with symmetry breaking arising from quark-mass terms are shown not to be inconsistent with the experimental features of $\eta' \to \eta\pi\pi$ decay.

We first summarize the salient features of the decay $\eta' \to \eta\pi\pi$ that emerge from experiment.[1] (i) The $\pi\pi$ spectrum shows only a small derivation from phase space. In terms of the Dalitz variable y and the linear matrix elements $M(1+\alpha y)$, one has $\alpha = -0.08 \pm 0.03$. This value is considerably smaller than previous theoretical predictions. (ii) The $\eta\pi$ spectrum is consistent with phase space with no indication of a $\delta(970)$ resonance. (iii) Although the width of η' is not measured, the branching ratio[2] for $\eta' \to \eta\pi\pi$ is $66.2 \pm 1.7\%$ while for $\eta' \to 2\gamma$ it is $2.0 \pm 0.3\%$. Based on the quark model, we can crudely estimate the decay rate $\Gamma(\eta' \to 2\gamma) = \frac{8}{3}(m_{\eta'}/m_\pi)^3 \times \Gamma(\pi^0 \to 2\gamma) \approx 7.0$ keV. This estimate is also consistent with the $\eta \to 2\gamma$ rate of 300 eV if one allows for an η-η' mixing angle of 10°. The total rate is thus expected to be approximately 350 keV, although this could easily be off by a factor of 2.

We now review briefly the theoretical puzzle in $\eta'(p') \to \eta(p) + \pi(q_1) + \pi(q_2)$ decay. (1) The Adler theorem states that in the limit of one of the pion momentum $q_1 \to 0$, the matrix element should vanish. This effect should show up in the physical region if the "σ" term is neglected. Since the experimental $\pi\pi$ spectrum is consistent with phase space, there is no evidence for Adler zero. (2) Current algebra tells us that in the limit $q_1 \to 0$ and $q_2^2 \to 0$ the matrix element of $\eta \to \eta\pi\pi$ is given by the "σ" term. The magnitude of this term depends on the nature of chiral-symmetry breaking. In the usual quark model with small u and d masses, this term is small and can be estimated reliably. Previous calculations using only this contribution yield an extremely small width[3,4] $\Gamma(\eta' \to \eta\pi\pi) \lesssim 3$ keV. Because of the small value of the experimental slope, it does not help to add a term which is proportional to $q_1 \cdot q_2$ in the matrix element.[4]

To resolve this puzzle various authors[5] have suggested adding terms that transform according to the $(\underline{8}, \underline{8})$ representation of the chiral SU(3) $\otimes$ SU(3) group. This representation enhances the value of the σ term to bring it into agreement with experiment. However, this is completely inconsistent with the present ideas of strong interactions based on quantum chromodynamics, which require the symmetry-breaking terms to transform like quark-mass terms, or the $(\underline{3}^*, \underline{3})$ representation of the chiral group. Thus the resolution of this paradox is important to the development of the theory of strong interactions.

The purpose of this note is to point out a very special kinematical feature of this system that completely accounts for all the features of the decay. We first note that the $\eta\pi$ system is known to couple to $\delta(970)$ resonance. Assuming that δ also couples to the $\eta'\pi$ system with comparable strength, as expected from the quark model, we show that inclusion of this resonance leads to the elimination of the Adler zero from the matrix element, an enhancement of the decay rate, while at the same time accounting for the flat

$\eta\pi$ distribution.

Using standard current-algebra technique and partial conservation of axial-vector current, one can show that the matrix element for off-shell pions satisfies the constraints

$$T(q_1=0;\, q_2^2=m_\pi^2)=T(q_1^2=m_\pi^2;\, q_2\to 0) = 0, \tag{1}$$

$$T(q_1=0;\, q_2^2=0)=-2\sigma_{\eta\eta'}/F_\pi^2, \tag{2}$$

where $\sigma_{\eta\eta'}=\langle\eta'|m_u\bar{u}u+m_d\bar{d}d|\eta\rangle$ and $F_\pi\approx m_\pi$. This contribution can be estimated using SU(3) and the standard Gell-Mann–Oakes–Renner model for chiral-symmetry breaking[3,4]:

$$2\sigma_{\eta\eta'}/F_\pi^2\approx -1. \tag{3}$$

Expanding the off-shell $\eta'\to\eta\pi\pi$ amplitude in powers of q_1 and q_2, we have

$$T(q_1,q_2)=A+B(q_1^2+q_2^2)+C(q_1\cdot q_2)+D(q_1\cdot p')(q_2\cdot p)+E(q_1\cdot p)(q_2\cdot p'), \tag{4}$$

where $A=-2\sigma_{\eta\eta'}/F_\pi^2$ and $B=-A/m_\pi^2$. The parameters C, D, and E are functions of the invariants $s=(q_1+q_2)^2$, $t=(p+q_1)^2$, and $u=(p+q_2)^2$. These are not determined by current algebra[6]; however, their magnitude is constrained by the restriction on $\eta\pi$ and $\pi\pi$ system. Although we could write dispersion relations for the functions C, D, and E with subtraction points fixed to yield correct current-algebra restrictions, this procedure is cumbersome and adds no more insight than using an effective Lagrangian which vanishes in the limit $q_1\to 0$ or $q_2\to 0$. We assume ϵ and δ dominance for $\pi\pi$ and $\eta\pi$ final states[7] and write

$$\mathcal{L}=g_{\eta'\delta\pi}\vec{\delta}\cdot\partial_\mu\vec{\pi}\partial^\mu\eta'+g_{\eta\delta\pi}\vec{\delta}\cdot\partial_\mu\vec{\pi}\partial^\mu\eta+g_{\epsilon\eta\eta'}\epsilon\partial_\mu\eta\partial^\mu\eta'+g_{\epsilon\pi\pi}\epsilon\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}. \tag{5}$$

The amplitude T which obeys current-algebra restrictions can be easily written as[8]

$$T=\frac{2\sigma_{\eta\eta'}}{F_\pi^2}\left[-1+(q_1^2+q_2^2)/m_\pi^2\right] - g_{\epsilon\pi\pi}g_{\epsilon\eta\eta'}\frac{(p'\cdot p)(q_1\cdot q_2)}{m_\epsilon^2-s} - g_{\eta'\delta\pi}g_{\eta\delta\pi}\left[\frac{(p'\cdot q_1)(p\cdot q_2)}{m_\delta^2-u-im_\delta\Gamma_\delta}+\frac{(p'\cdot q_2)(p\cdot q_1)}{m_\delta^2-t-im_\delta\Gamma_\delta}\right]. \tag{6}$$

Now although the last term of Eq. (6) satisfies current-algebra restrictions by vanishing in the limit $q_1\to 0$ or $q_2\to 0$, there are large contributions in the physical region. In the rest system of η', we have

$$\frac{4(p'\cdot q_1)(p\cdot q_2)}{m_\delta^2-u-im_\delta\Gamma_\delta}=\frac{2m_{\eta'}E_1(m_{\eta'}^2-m_\eta^2-2m_{\eta'}E_1)}{\Delta^2+2m_{\eta'}E_1-im_\delta\Gamma_\delta}, \tag{7}$$

where $\Delta^2=m_\delta^2-m_{\eta'}^2-m_\pi^2$. Now both Δ^2 and $m_\delta\Gamma_\delta$ are very much smaller than $2m_{\eta'}E_1$. Thus the amplitude is, to a very good approximation,

$$T=\frac{2\sigma_{\eta\eta'}}{F_\pi^2}-\tfrac{1}{4}g_{\epsilon\eta\eta'}g_{\epsilon\pi\pi}(s-2m_\pi^2)\frac{(m_{\eta'}^2+m_\eta^2-s)}{m_\epsilon^2-s}-\tfrac{1}{4}g_{\eta'\delta\pi}g_{\eta\delta\pi}(m_{\eta'}^2-m_\eta^2-s). \tag{8}$$

We see that the effect of δ resonance is to remove the Adler zero from the physical amplitude. At the same time we see that there is no structure in the t and u variables either. We can now estimate the coupling constants from the known $\delta\to\eta\pi$ decay and with use of U(3) symmetry for the coupling constants. This procedure should yield a crude estimate of the amplitude. We find, assuming 10° mixing angle for η-η',

$$g_{\epsilon\pi\pi}g_{\epsilon\eta\eta'}\approx 1.16g_{\eta'\delta\pi}g_{\eta\delta\pi}\approx 1.16g_{\delta\eta\pi}^2.$$

Using $\Gamma(\delta\to\eta\pi)\approx 50$ MeV, we find $g_{\delta\eta\pi}^2/4\pi=(5.6\times 10^{-2})m_\pi^{-2}$. The matrix element is best written in terms of the Dalitz variable

$$y=[(m_\eta+2m_\pi)/m_\pi]T_\eta/Q-1,$$

where T_η is the kinetic energy of η and $Q=m_{\eta'}-m_\eta-2m_\pi$. The relation between s and y is given by $s=[6.77-2.4y]m_\pi^2$. Substituting the values of $\delta(970)$ and $\epsilon(700)$ we find for $T=M(1+\alpha y)$ the values

$$|M|=8.5,\quad \alpha=-0.12. \tag{9}$$

The partial width is then given by[9]

$$\Gamma(\eta' \to \eta\pi\pi) = 3|M|^2[1+0.24\alpha+0.27\alpha^2] = 210 \text{ keV}. \quad (10)$$

We thus find a width enhanced by almost a factor of 70 from the naive current-algebra result. The dominant contribution to the decay rate arises from δ while the ϵ term yields the correct slope. To summarize our results, the inclusion of δ and ϵ resonances satisfactorily accounts for the slope parameter and the decay rate for $\eta' \to \eta\pi\pi$. The reason for no deviation from background in the $\eta\pi$ distribution and the absence of Adler zero are understood from the accidental cancellation of the zero with the δ pole term.

The authors would like to acknowledge the hospitality accorded them by the Theory Group at Los Alamos Scientific Laboratory. This work was supported by the U. S. Department of Energy.

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[6]The solution $B=-A/m_\pi^2=C/2$, $D=E=O$ yields the Schwinger model; see J. Schwinger, Phys. Rev. 167, 1432 (1968). This model predicts a large slope $\alpha \approx -0.44$. With the value of A given in Eq. (3) and with C arbitrary, a large slope is required to increase the theoretical estimate, as discussed in Refs. 3 and 4.

[7]The effect of scalar mesons δ and ϵ on the η' decay was first considered in an effective-Lagragian framework by J. Schechter and Y. Ueda, Phys. Rev. D 3, 2874 (1971), and 9, 987 (1973). The main difference between that work and the present approach involves the use of derivative couplings and the numerical estimates of the coupling constants.

[8]An attempt to resolve the $\eta' \to \eta\pi\pi$ puzzle by considering a purely phenomenogical amplitude dominated by the δ pole was made by C. Singh and J. Pasupathy, Phys. Rev. Lett. 35, 1193 (1975). Their amplitude does not manifestly satisfy the current-algebra constraints, and consequently predicts enhancement in the η-π distribution at large invariant mass, a feature totally absent from the data (see Ref. 1).

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Observables for the Analysis of Event Shapes in e^+e^- Annihilation and Other Processes

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(Received 18 September 1978)

We present a set of rotationally invariant observables which characterizes the "shapes" of events, and is calculable in quantum-chromodynamics perturbation theory for final states consisting of quarks and gluons (G). We include the effects of fragmentation to hadrons in comparing the shapes of events from the processes $e^+e^- \to q\bar{q}$, $e^+e^- \to q\bar{q}G$, and $e^+e^- \to$ heavy resonance $\to GGG$, and from heavy-quark and lepton production. We indicate how our analysis may be extended to deep-elastic lepton-hadron interactions and hadron-hadron collisions involving large transverse momenta.

Experiments[1] have shown that at high center-of-mass energies ($\sqrt{s}$) the final states in $e^+e^- \to$ hadrons usually consist predominantly of two jets of hadrons presumably resulting from the process $e^+e^- \to q\bar{q}$. Quantum chromodynamics (QCD) explains this basic two-jet structure,[2] but predicts that one of the outgoing quarks should sometimes emit a gluon (G), tending to lead to three-jet final states.

Previous attempts[3] to discriminate between two- and three-jet events concentrated on finding a "jet axis" by minimization, and then measuring the collimation of particles with respect to it. Instead, one may use observables which directly characterize the "shape" of each event. Since there is no natural axis defined in the final state of e^+e^- annihilation, it is convenient to consider rotationally invariant observables. A set of such observables is given by [$Y_l^m(\Omega)$ are the usual spherical harmonics and $P_l(\cos\varphi)$ the Legendre polynomials]

$$H_l \equiv \left(\frac{4\pi}{2l+1}\right)\sum_{m=-l}^{+l}\left|\sum_i Y_l^m(\Omega_i)\frac{|\vec{p}_i|}{\sqrt{s}}\right|^2 = \sum_{i,j}\frac{|\vec{p}_i||\vec{p}_j|}{s}P_l(\cos\varphi_{ij}), \quad (1)$$