[14]D. G. Kovar, K. Daneshevar, D. F. Geesaman, W. Henning, F. W. Prosser, K. E. Rehm, J. P. Schiffer, and S. L. Tabor, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, Japan, 1977*, edited by The Organizing Committee (International Academic Printing Co. Ltd., Tokyo, 1977).

New Isovector Collective Modes in Deformed Nuclei

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We study a model of a deformed nucleus in which protons and neutrons are described as interacting rigid rotors with axial symmetry. The nucleus as a whole is no longer axially symmetric. A magnetic-dipole collective state describing rotational oscillations of protons against neutrons is predicted.

The electric dipole giant resonance is an isovector collective excitation existing in all nuclei. It has a semiclassical interpretation as a translational oscillation of protons against neutrons.[1] This two-fluid picture suggests the existence of additional modes of excitation in deformed nuclei. For instance, the neutron and proton deformed fluids might perform rotational oscillations of opposite phase around a common axis,[2] generating an isovector magnetic resonance.[2]

In this paper we study the properties of a deformed nucleus in which protons and neutrons are described as identical rigid rotors with axial symmetry. The orientation of the two rotors is completely specified by the Euler angles α_p, β_p, α_n, and β_n needed to identify their symmetry axes $\vec{\zeta}_p$ and $\vec{\zeta}_n$.

If relative translational motion is excluded, the kinetic energy of the whole system about its center of mass is

$$T=(1/2\mathcal{I}_0)[(I_{\xi_p}^{(p)})^2+(I_{\eta_p}^{(p)})^2+(I_{\xi_n}^{(n)})^2+(I_{\eta_n}^{(n)})^2], \quad (1)$$

where $I_{\xi_p}^{(p)}$, $I_{\eta_p}^{(p)}$, $I_{\xi_n}^{(n)}$, and $I_{\eta_n}^{(n)}$ are the components of the angular momenta of protons and neutrons along their respective principal axes $\vec{\xi}_p$, $\vec{\eta}_p$, $\vec{\xi}_n$, and $\vec{\eta}_n$ (which are arbitrary) and $\mathcal{I}_0$ is their common moment of inertia. Rotations around the symmetry axes are excluded.

We assume the potential to be a function of the angle between the symmetry axes $\vec{\zeta}_p$ and $\vec{\zeta}_n$, which we denote by 2θ. It is then convenient to express T in a form which exhibits its θ dependence. To this end we define the principal axes for the whole nucleus:

$$\vec{\xi}=\frac{1}{\sin(2\theta)}\vec{\zeta}_p\times\vec{\zeta}_n,$$
$$\vec{\eta}=\frac{1}{2\sin\theta}(\vec{\zeta}_p-\vec{\zeta}_n), \quad (2)$$
$$\vec{\zeta}=\frac{1}{2\cos\theta}(\vec{\zeta}_p+\vec{\zeta}_n),$$

and the O(4) generators $\vec{I}=\vec{I}^{(p)}+\vec{I}^{(n)}$ and $\vec{S}=\vec{I}^{(p)}-\vec{I}^{(n)}$. As will be shown in a more detailed presentation of this work, the commutation relations of the components of $\vec{I}$ and $\vec{S}$ are satisfied if we put

$$S_\xi=i\frac{\partial}{\partial\theta},\quad S_\eta=-\cot\theta\, I_\zeta,\quad S_\zeta=-\tan\theta\, I_\eta. \quad (3)$$

We can now replace the four dynamical variables α_p, β_p, α_n, and β_n by the Euler angles identifying the principal axes and θ. The correspondence is one to one if we allow the Euler angles to vary over their full range and θ to vary between zero and $\frac{1}{2}\pi$. In order to express the Hamiltonian in the new variables we observe that the kinetic energy can also be written

$$T=\frac{1}{2\mathcal{I}_0}[(I^{(p)})^2+(I^{(n)})^2]=\frac{1}{4\mathcal{I}_0}(I^2+S^2) \quad (4)$$

with the constraint

$$I_{\zeta_p}^{(p)}=I_{\zeta_n}^{(n)}=0.$$

These constraints are automatically satisfied by the realization (3) of $\vec{S}$. We can therefore write the full Hamiltonian as

$$H=\frac{1}{4\mathcal{G}_0}\left(I^2-\frac{d^2}{d\theta^2}+\cot^2\theta I_\zeta^{\,2}+\tan^2\theta I_\eta^{\,2}\right)+V(\theta). \quad (5)$$

This Hamiltonian is Hermitian for the wave functions satisfying the condition

$$\lim_{\theta\to(0,\pi/2)}\left(\psi\frac{d}{d\theta}\psi'-\psi'\frac{d}{d\theta}\psi\right)=0. \quad (6)$$

We assume for the potential a harmonic approximation, which in order to be consistent with the geometry of the system must be

$$V(\theta)=\begin{cases}\frac{1}{2}C\theta^2, & 0\leq\theta\leq\frac{1}{4}\pi,\\ \frac{1}{2}C(\frac{1}{2}\pi-\theta)^2, & \frac{1}{4}\pi\leq\theta\leq\frac{1}{2}\pi.\end{cases} \quad (7)$$

Equation (5) shows that the nucleus does not have axial symmetry. The nucleus, however, must be invariant (i) under a rotation of π around $\vec{\xi}$ and (ii) under a separate rotation of π around $\vec{\xi}$ of protons or neutrons. Because of condition (i) the eigenfunctions are superpositions of intrinsic states with even or odd eigenvalues of I_ζ, denoted by K, according to whether they are even ($r_\eta=1$) or odd ($r_\eta=-1$) under a rotation of the whole nucleus about $\vec{\eta}$ through π. In particular, they factorize as

$$\psi_{nIMr_\eta}=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}[D_{MK}{}^I+(-1)^ID_{M-K}{}^I](1+\delta_{K0})^{-1/2}\Phi_{nIK}, \quad (8)$$

for $I=0$, $r_\eta=+1$; $I=1$, $r_\eta=-1$; and $I=2$, $r_\eta=-1$. In the right-hand side K is a function of r_η taking the value zero in the first case and 1 in the others. We will confine ourselves to these states, which must have the symmetry

$$\Phi_{nIK}(\theta)=(-1)^I\Phi_{nIK}(\tfrac{1}{2}\pi-\theta) \quad (9)$$

as a consequence of condition (ii). This symmetry allows us to solve the eigenvalue equation for the Φ's in the interval $0\leq\theta\leq\frac{1}{4}\pi$ only. Expanding the Hamiltonian in this interval up to order θ^2 we get

$$\left(-\frac{1}{4\mathcal{G}_0}\frac{d^2}{d\theta^2}+\frac{K^2}{4\mathcal{G}_0\theta^2}+\frac{1}{2}(2\mathcal{G}_0)\omega^2\theta^2-\epsilon_{nK}\right)\Phi_{nIK}=0, \quad (10)$$

where ϵ_{nK} is the intrinsic excitation energy, and $\omega=(C/2\mathcal{G}_0)^{1/2}$. In Eq. (10) we neglected terms of the order of $1/2\mathcal{G}_0$ as compared to C. This is permissible because $1/2\mathcal{G}_0C=\theta_0^{\,2}$ turns out to be much smaller than 1, as shown by the following numerical estimates. Paralleling the procedure of Goldhaber and Teller[1] we impose the condition

$$\tfrac{1}{2}C\theta_c^{\,2}=\Delta N(\theta_c)v_0, \quad (11)$$

where v_0 is the proton-neutron separation energy and θ_c is the value of θ such that for $\theta>\theta_c$, $\Delta N(\theta_c)$ neutron-proton pairs do not interact. $\Delta N(\theta_c)$ is half the nuclear density times the variation of volume of the nucleus due to the rotation of protons against neutrons by the angle θ_c around $\vec{\xi}$. As a result we get

$$C=(4^3/3\pi^2)ARv_0\delta^2/r_0, \quad (12)$$

where δ is the deformation parameter,[3] r_0 is the range of the neutron-proton interaction, and $R=1.2A^{1/3}$ fm. Assuming the moment of inertia to be[4] $\mathcal{G}_0=\frac{1}{2}\mathcal{G}_{\rm rig}(1+\frac{1}{3}\delta)$ we get $\theta_0^{\,2}\sim1.5\times10^{-3}$ and $\omega\sim6$ MeV, for the values $A=180$, $\delta=0.25$, $v_0=40$ MeV, and $r_0=2$ fm.

Because of the small value of θ_0 we can extend to ∞ the range of θ in Eq. (10). The $I=0$ solutions of Eq. (10) are

$$\Phi_{n00}=[2^{2n-1}(2n)!\pi^{1/2}\theta_0]^{-1/2}H_2(\theta/\theta_0)\exp(-\theta^2/2\theta_0^{\,2}) \quad (13)$$

with eigenvalues

$$\epsilon_{n0}=(2n+\tfrac{1}{2})\omega. \quad (14)$$

It should be noted that only even Hermite polynomials appear because of the Hermiticity conditions [Eq. (6)].

The $r_\eta=-1$, $I=1,2$ intrinsic eigenfunctions are

$$\Phi_{nI1}=\left[\frac{2n!}{\Gamma(n+\rho+1)}\frac{1}{\theta_0}\right]^{1/2}\left(\frac{\theta}{\theta_0}\right)^{\rho+1/2}L_n{}^\rho\left(\frac{\theta^2}{\theta_0^{\,2}}\right)\exp(-\theta^2/2\theta_0^{\,2}),\qquad \rho=\tfrac{1}{2}(1+K^2)^{1/2}. \quad (15)$$

where L_n^ρ are the generalized Laguerre polynominals.[5] The corresponding eigenvalues are

$$\epsilon_{nI} = (2n + \rho + 1)\omega. \qquad (16)$$

The lowest-lying excited states determined are Φ_{100}, Φ_{0I1}, with intrinsic excitation energies $\epsilon_{10} \sim 12$ MeV and $\epsilon_{01} \sim 10$ MeV for the values of the parameters previously given. The $I=0$ state is not coupled to the ground state by electromagnetic transitions. The $I=1$ state is excited by the isovector component of $M1$ radiation with a strength probability $B(M1) \sim 9$ times the single-particle Weisskopf estimate. The $I=2$ state is excited by the isovector component of $E2$ radiation with a transition probability $B(E2) \sim 3$ times the single-particle Weisskopf estimate. All isoscalar electromagnetic probabilities are zero.

A microscopic calculation[6] based on a schematic model predicts an isovector $K=1$ state which has been interpreted as the microscopic counterpart to our $r_\eta = -1$ state. The state of Ref. 6, however, has an excitation energy of 2.3 MeV and a $B(E2)$ strength of 0.5 Weisskopf units (for $A=180$ and $\delta=0.25$), which are considerably lower than our values. On the other hand, the present approach is so different from the one adopted in Ref. 6 that the correspondence between the two states is not obvious to us. In particular, the centrifugal force, which is responsible for the high excitation energy of $K \neq 0$ states, plays a fundamental role in the present model unlike the case in the usual models. The importance of the centrifugal force becomes apparent in the classical analysis of the present model. The simplest classical motions of our system are either rotational oscillations of protons against neutrons or rotations of the nucleus as a whole while the proton and neutron symmetry axes stay at a fixed angle as determined by the equilibrium between the centrifugal force and the restoring force.

Our model differs from the one of Ref 6 in another major respect. Our states, in fact, including the ground state, although characterized by a single value of K do not have axially symmetric deformation. This is because the relative motion of protons and neutrons is simultaneously localized around the $\vec{\xi}$ and $\vec{\eta}$ axes. Were the motion localized only around a single axis, axial symmetry would be approximately retained because of the small value of θ_0. As an effect of this symmetry breaking the intrinsic quadrupole moment of the whole nucleus in its ground state is one-half of the value we would obtain if axial symmetry were preserved.

In view of the above considerations we think that a detailed microscopic analysis of our model would be highly desirable. A microscopic calculation[6] based on the vibrating potential model predicts a state which could be interpreted as a rotational oscillation. In such a calculation, however, axial symmetry is assumed.

[1]M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).

[2]The possible occurrence of such an excitation mode was discussed by one of us (F. P.) at the Institute of Theoretical and Experimental Physics of Moscow in March 1975 and by R. Hilton in Dubna in June 1976. No quantitative treatment, however, has been done till now as far as we know.

[3]See, for instance, A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II, p. 47.

[4]See Ref. 3, p. 75.

[5]I. S. Gradshteyn and I. Ryzhik, *Tables of Integrals, Series and Products* (Academic, New York, 1965).

[6]T. Suzuki and D. J. Rowe, Nucl. Phys. A289, 461 (1977).