ergies above their lowest breakup thresholds is not evident. However, for energies below the lowest breakup threshold in systems for which there are a number of bound states in at least one reaction channel, the BSA will probably be an even better approximation than in the present case, which could explain the successes of the BSA reaction theories. One way of testing this is through the use of three-body model calculations in which the potentials support more than one bound state. An even more interesting system for study might be a model four-body system which features many more arrangement and breakup channels than a three-body system. In either of these systems it may be possible to investigate and justify the use of optical potentials to extend the BSA above the lowest breakup threshold, as in the case of nuclear reaction analyses.[4]

This work was supported in part by the U. S. Department of Energy.

[(a)]Present address: TRIUMF, University of British Columbia, Vancouver, British Columbia V6T 1W5, Canada.

[1]See, e.g., G. Drukarev, The Theory of Electron-Atom Collisions (Academic, New York, 1965); or P. G. Burke, United Kingdom Atomic Energy Authority Report No. AERE-L 170, 1967 (unpublished) (available from H. M. Stationery Office).

[2]See, e.g., K. Takayanagi, in *Physics of Electronic and Atomic Collisions*, edited by J. S. Risley and R. Geballe (Univ. of Washington Press, Seattle, Wash., 1976).

[3]See, e.g., A. Kupperman, in *Physics of Electronic and Atomic Collisions*, edited by J. S. Risley and R. Geballe (Univ. of Washington Press, Seattle, Wash., 1976).

[4]T. Udagawa, H. H. Wolter, and W. R. Coker, Phys. Rev. Lett. 31, 1507 (1973), and references cited therein.

[5]L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960) [Sov. Phys. JETP 12, 1014 (1961)].

[6]A. N. Mitra, Phys. Rev. 139, B1472 (1965).

[7]D. P. Bouldin and F. S. Levin, Phys. Lett. 37B, 145 (1971), and 42B, 167 (1972), and Nucl. Phys. A189, 449 (1972), and in *Few Particle Problems in the Nuclear Interaction*, edited by I. Slaus, S. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North Holland, Amsterdam, 1972), p. 949; see also D. P. Bouldin, Ph.D. thesis, Brown University, 1972 (unpublished).

[8]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[9]D. J. Kouri and F. S. Levin, Nucl. Phys. A250, 127 (1975), and A253, 395 (1975).

[10]The channel-coupling array was first introduced by M. Baer and D. J. Kouri, J. Math. Phys. 14, 1637 (1973).

[11]J. M. Greben and F. S. Levin, Phys. Lett. 71B, 252 (1977), and to be published; F. S. Levin, in Proceedings of the International Symposium on Nuclear Reaction Models, Balatonfured, Hungary, 27 June–1 July 1977 (to be published), p. 25.

[12]F. S. Levin and J. M. Greben, to be published.

[13]P. E. Shanley and R. Aaron, Ann. Phys. (N.Y.) 44, 363 (1967).

[14]A. S. Reiner and A. I. Jaffe, Phys. Rev. 161, 935 (1967).

^{12}C on ^{12}C at 800 MeV/Nucleon: One Fireball or Two?

S. Das Gupta

Physics Department, McGill University, Montreal, Ontario H3A 2T8, Canada,[(a)] *and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720*

(Received 31 July 1978)

I demonstrate that a two-fireball model can explain the proton and pion inclusive spectra for collision between light ions at relativistic energies. For heavier ions, the one-fireball model is recovered.

The fireball model predicts, with a fair degree of success, inclusive spectra for collision of Ne on U, Pb, etc. at relativistic energies. For the basics of this model, relevant to this Letter, see Gosset *et al.*[1] and Kapusta.[2]

Data for lighter ions have become available in recent times. Preliminary data for ^{12}C on ^{12}C at 800 MeV/nucleon have existed for more than a year.[3] Data analysis for Ne on NaF and Ar on Ar is nearing completion.

These data have shown strong deviations from the fireball model as developed in Refs. 1 and 2. From now on I will call this model the one-fireball model. I will show that a simple extension of the one-fireball model idea—two fireballs[5] rather than one—fits the data on lighter ions. When heavier ions are involved, this two-fireball model essentially coalesces into the one-fireball model.

The basic premise of the one-fireball model is the following. For a given impact parameter b there will be an overlap between the target and the projectile. This overlap region forms the fireball. After collision, the overlapping regions of the target and the projectile fuse together and come to rest in the center-of-mass frame of the fireball. The available kinetic energy before the collision goes into creating thermal motion after the collision. Particles are then emitted isotropically in the rest frame of the fireball. Because the c.m. of the fireball is moving in the laboratory, the inclusive spectrum measured in the laboratory will, in general, be anisotropic. For collisions between unequal masses the c.m. velocity of the fireball in the laboratory is generally a function of b. It is possible to include particle production (pions, deltas, etc.) and composites (d, ^{3}He, ^{3}H, etc.) in the formulation using the law of chemical equilibrium.[6] A justification for this procedure has been advanced.[7]

For collision between identical ions, the c.m. of the fireball is independent of the impact parameter and coincides with the c.m. of the target and the projectile. The inclusive spectrum measured in the c.m. is therefore predicted to be isotropic. Figure 1 shows that for ^{12}C on ^{12}C, the measured proton inclusive cross section is strongly anisotropic; at 30° the cross section is approximately five to six times higher than at 90°. Further, although the protons are highly anisotropic, pions are only mildly so.

I find that the following simple extension of the one-fireball model reproduces these characteristics of the data. Let $P_a^{\,i}$ and $E_a^{\,i}$ be the momentum and energy, respectively, of the overlapping part of the projectile before collision in the c.m. of the fireball. After the collision

$$P_a^{\,f} = (1-y)P_a^{\,i}. \tag{1}$$

The factor y is unity in the one-fireball model if the projectile hits the target at all. I think that it is more realistic to consider y as a function of b. The idea is that if the overlapping part of the projectile does not meet a large enough number of nucleons on its way, it will be slowed down but not completely stopped. Similar arguments apply for the overlapping part of the target. A theoretical derivation of y is very difficult and I will not attempt it here. The simple assumption made here is that

$$y = 1 \text{ if } n_{ab} \geqslant n_0, \qquad y = n_{ab}/n_0 \text{ if } n_{ab} < n_0, \tag{2}$$

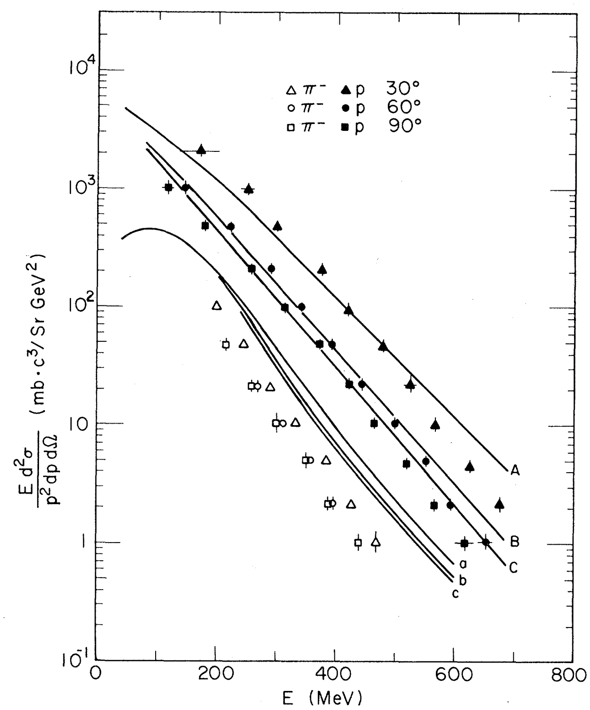

FIG. 1. Theoretical calculations for proton spectrum ($A = 90°$, $B = 60°$, $C = 90°$) and pion spectrum ($a = 90°$, $b = 60°$, $c = 90°$) compared with experimental results for ^{12}C on ^{12}C at 800 MeV/nucleon. All results are in the c.m. system.

where n_{ab} is the initial number of nucleons participating in the collision. My first guess was $n_0 = 16$ and this already gives a reasonable fit. This value is used in the calculation of Fig. 1. For ^{12}C on ^{12}C this choice generates the one-fireball model only for $b < 2$ fm; for higher values of b, the overlapping parts of the projectile and the target continue in their paths after collision but with reduced momentum (two fireballs). Because of symmetry we must have $E_a^{\,f} = E_a^{\,i}$; thus the excess energy after slowing down is dumped as thermal energy. Particles are then emitted isotropically in the rest frame of each fireball.

With this modification, the rest of the calculation proceeds exactly as in Ref. 2. I include the production of pions and deltas using the thermodynamic formulation. The deltas decay into appropriate pions and nucleons. For pion production, a freeze-out density has to be assumed. In accordance with a recent calculation[8] this is taken to be 0.12 fm^{-3}. The reasons for the difference in anisotropy of the protons and the pions are twofold: (a) Pions are lighter, and (b) pions are produced mostly at low values of b.

We now return to a discussion of Eq. (2). One

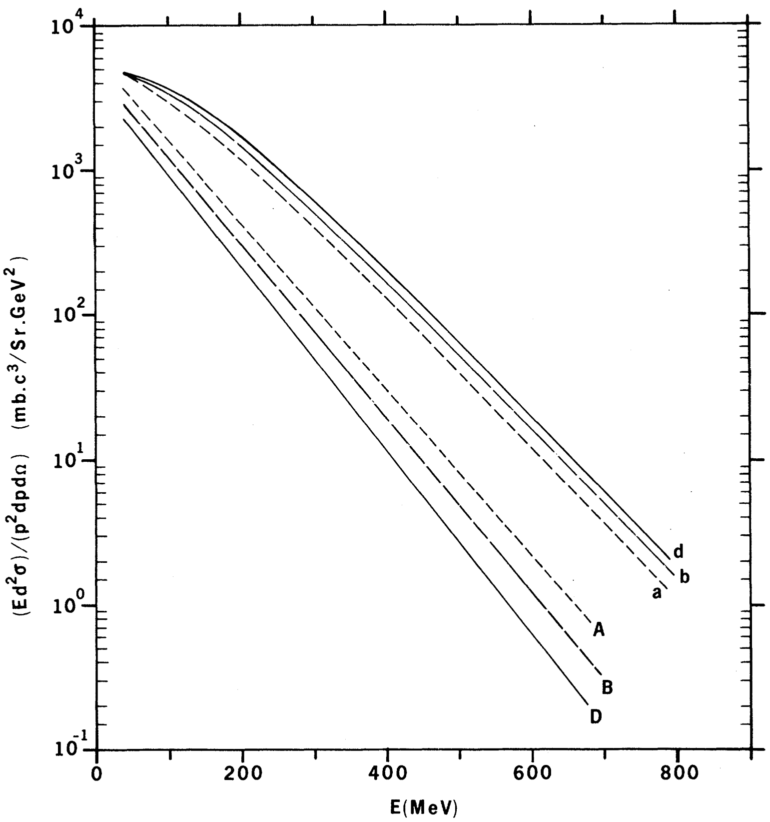

FIG. 2. Calculation of proton spectrum anisotropy for ^{12}C on ^{12}C at 800 MeV/nucleon as a function of n_0 [Eq. (2)]; a, A correspond to spectra at 30° and 90°, respectively, for $n_0=16$; b, B for $n_0=24$; d, D for $n_0=30$. All calculations are performed in the c.m. system.

2 which shows that the constant is determined to better than 25% by existing data. We expect n_0 to be a function of the incident energy but for a given incident energy per nucleon, it should be independent of the mass number of the projectile for the identical projectile-target case. Thus the same value of $n_0 \approx 16$ should reproduce data for ^{20}Ne on ^{20}Ne at 800 MeV/nucleon. The theoretical prediction is compared with experiment in Fig. 3. With heavier projectiles, the anisotropy in the proton spectrum will decrease. For ^{20}Ne on ^{20}Ne at 800 MeV/nucleon, the factor between the proton spectrum at 30° and that at 90° is about 2 to 3. could also try

$$y = (n_{ab}/n_0)^\nu \text{ if } n_{ab} < n_0.$$

The choice of Eq. (2) is thus one of simplicity —one that does an adequate job. If we accept Eq. (2) from this viewpoint, it is obvious that n_0 primarily determines the anisotropy of the proton spectrum. Larger n_0 will lead to more anisotropic proton spectra. This is demonstrated in Fig. For Ar on Ar my calculation predicts that the fac-

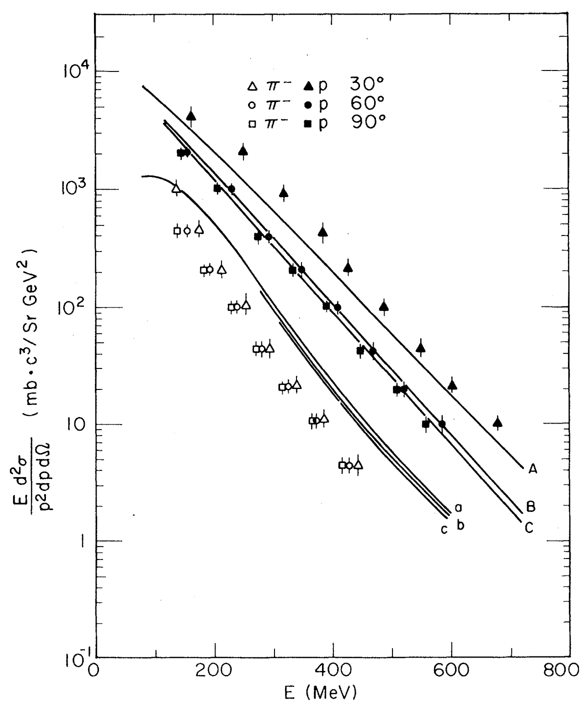

FIG. 3. Same as in Fig. 1 except for Ne on NaF.

tor will drop to less than 2.

We note that these theoretical predictions for pion spectra are too high compared to experiments by about a factor of 2 to 3. The numerical value for the freeze-out density[8] used in the calculation affects the normalization of the pion spectrum most directly. A higher value would yield better agreement for pions while the proton spectrum would remain essentially unchanged. However, many modifications of the model proposed here are possible which will affect details of agreement.

The firestreak model[9,8] applied to light-ion reactions also produces an anisotropic proton spectrum. However, the fit to the data is much worse than what is achieved in the two-fireball model. Lastly, it is possible to formulate a hydrodynamical description[10] which will produce anisotropy in proton and pion spectra.

I am very much indebted to S. Nagamiya for informing me of experimental results prior to publication. I wish to thank G. Westfall for making available to me the firestreak-model predictions. I also wish to thank P. Siemens, J. I. Kapusta, M. Gyulassy, Y. Karant, R. Landau, and N. K. Glendenning for discussion. This work was carried out under the auspices of the U. S. Department of Energy and is also supported in part by the National Research Council of Canada and the Quebec Department of Education.

(a)Permanent address.

[1]J. Gosset, H. H. Gutbrod, W. G. Meyer, A. M. Poskanzer, A. Sandoval, R. Stock, and G. D. Westfall, Phys. Rev. C 16, 629 (1977).
[2]J. I. Kapusta, Phys. Rev. C 16, 1493 (1977).
[3]S. Nagamiya, I. Tanihata, S. R. Schnetzer, W. Brückner, L. Anderson, G. Shapiro, H. Steiner, and O. Chamberlain, Lawrence Berkeley Laboratory Nuclear Science Annual Report No. LBL-6575, 1977 (unpublished), p. 150, and J. Phys. Soc. Jpn., Suppl. 44, 378 (1978).
[4]S. Nagamiya, private communication.
[5]R. C. Hwa and C. S. Lam, Phys. Rev. Lett. 27, 1098 (1971); R. K. Adair, Phys. Rev. D 5, 1105 (1972); M. Jacob and R. Slansky, Phys. Rev. D 5, 1847 (1972).
[6]A. J. Mekjian, Phys. Rev. Lett. 38, 640 (1977).
[7]S. Das Gupta, Lawrence Berkeley Laboratory Report No. LBL-7734, 1978 (unpublished).
[8]J. Gosset, J. I. Kapusta, and G. D. Westfall, Phys. Rev. C 18, 844 (1978).
[9]W. D. Myers, Nucl. Phys. A296, 177 (1978).
[10]Y. Kitazoe, M. Sano, and K. Yamamoto, J. Phys. Soc. Jpn., Suppl. 44, 386 (1978).

Nuclear Structure and (e,e') Reactions: The Significance of High-Momentum-Transfer Data and Meson-Exchange Currents

J. Dubach[(a)]
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

W. C. Haxton
Theory Division, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
(Received 12 September 1978)

We show that accurate high-momentum-transfer electron-scattering data now being obtained can provide essential constraints on analyses of nulcear structure. However, the usefulness of these data is seriously impaired if meson-exchange current effects are ignored.

A new generation of intermediate-energy accelerators and high-resolution spectrometers is beginning to produce accurate electron-scattering data at high momentum transfers. The potential usefulness of such data in analyses of nuclear structure as well as the special problems inherent in extracting information from high-momentum-transfer (e,e') data have been at best vaguely addressed. Indeed, procedures for treating structure in several well-studied nuclear transitions remain popular despite predicting form factors in radical disagreement with such data. In this Letter we treat one such transition, excitation of the 15.11-MeV 1^+1 level in ^{12}C, in order to show what *can* and *cannot* be learned about nuclear structure from (e,e') experiments. This choice is motivated by the high-quality data recently obtained by Flanz *et al.*[1] which complement a considerable body of older data concentrated at lower momentum transfers.[2] We are