Validity of Neglecting Continuum Contributions in Two-Body Rearrangement Collisions

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The neglect of n-body breakup contributions, $n \geq 3$, is a widely used but previously untested approximation occurring in the analyses of almost all atomic, molecular, and nuclear collisions. A simple three-body model of deuteron stripping is used to study this approximation. Below the breakup threshold, use of this approximation accounts for the overall shape of the angular distributions and fits the exact transfer cross section at forward angles. With increasing energies, continuum effects render this approximation much less accurate.

Experimental data from almost all atomic, electronic, molecular, and nuclear collisions involving two bodies in both the initial and the final reaction channels are normally analyzed theoretically by means of a bound-state approximation (BSA). Calculations based on this type of approximation have two features in common: First, only reaction channels containing two fragments are retained, and second, only bound states of these fragments are included in the computations. That is, neither virtual nor real breakup states of the fragments enter the calculations. The BSA has taken on a variety of forms depending on how the interfragment, relative-motion wave functions have been treated. These range from the simplest case of plane waves (Born approximation), through forms of the distorted-wave and impulse approximations, to a series of methods for calculating the relative wave functions from a system of coupled equations. Included among the latter are the coupled-states calculations of electron-atom[1] or electron-molecule[2] scattering, the close-coupling calculations of atom-molecule scattering,[3] and the coupled-reaction-channel calculations of nucleon-nucleus or nucleus-nucleus scattering.[4]

As suggested in the preceding, the various forms of the BSA taken together rank as one of if not the most important approximations in the analysis of collision data. This conclusion is strengthened by the successes of the method, which support the belief (generated by experience) that the BSA, supplemented where necessary by the use of empirical optical potentials, provides both a physically and a numerically correct procedure. Nevertheless, the BSA *is* an approximation, and to our knowledge it has never been tested. By this we mean that no multichannel *calculations* ever seem to have been performed that compare exact and BSA results: The successful fitting of data by such an approximation does not constitute a test of its validity.

Lack of such a comparison is not surprising: Exact, multichannel scattering calculations, including the breakup continua, are nontrivial, especially if the coupled integro-differential-equation, wave-function formalisms associated with the BSA are used for this purpose.[1-4] Formal difficulties are considerably reduced if these wave-function formalisms are replaced by a set of integral equations for many-body transition operators, as in the Faddeev-equation formalism for the three-body problem.[5] Even greater simplifications result if a three-body model is used. Although simple, such a model contains both the breakup and rearrangement aspects found in more complex systems. Three-body models are thus obvious first candidates for testing the BSA. In this paper, we present some of the results of recent calculations designed to test this approximation using such a model.

The system we have employed is the three-body Mitra model of deuteron stripping.[6] It was previously used by Bouldin and Levin in their study of the distorted-wave Born approximation (DWBA),[7] and our calculations have been compared with theirs. This model consists of three structureless particles, a "neutron" (n), a "proton" (p), and an infinitely heavy core (C), interacting via attractive, separable, S-wave potentials with Yamaguchi form factors.[8] The n-p interaction was chosen to yield a "deuteron" bound by 2.225 MeV, while the neutron and proton each interact with the core by the same potential, chosen to yield a bound state of 3.3 MeV. Since n and p also have the same mass, the (d,p) and (d,n) amplitudes are identical. The exact, low-energy deuteron elastic and stripping cross sections are similar in shape and magnitude to those obtained experimentally from (d,d) on ^{16}O and (d,p) leading to the $2s_{1/2}$ excited state of ^{17}O, as seen in the original Bouldin-Levin work.[7]

The exact calculations of Bouldin and Levin were based on a set of T-operator equations derived from the spectator wave functions of the Mitra model.[6,7] The present BSA calculations are based on numerical solution of the channel-coupling–array operator equations obtained by first using a Faddeev-Lovelace choice[9] of the channel-coupling array[10] W and then imposing the BSA. (For other applications of the channel-coupling–array formalism to nuclear reaction problems, see Ref. 11.) The amplitudes leading to the DWBA transfer-reaction cross sections were determined[7] by replacing the exact three-body wave function Ψ by the product $\varphi_d u_0$, where φ_0 is the model deuteron ground-state wave function and u_0 is the model deuteron elastic-scattering wave function defined by $u_0 = \langle \varphi_0 | \Psi \rangle$. Hence, this is an exact rather than an optical-model DWBA, and as such contains the breakup contribution present in u_0. By contrast, the elastic-scattering wave function from the BSA contains rearrangement coupling effects but no breakup contributions. The details of our calculations, including the forms of equations solved, will be presented elsewhere.[12] Here we only summarize our results and compare transfer cross sections for two energies, one below and the other above the breakup threshold of 2.225 MeV.

Amplitudes and angular distributions for two processes have been calculated:

$$d + C \to d + C \tag{1}$$

and

$$d + C \to p + (C + n). \tag{2}$$

Note that the reaction channels $n + (C + p)$ and $p + (C + n)$ are identical in the present model. Computations were carried out for incident deuteron energies E_d of 1.78, 6.70, 11.2, and 15.12 MeV; these were originally chosen[7] to compare with the earlier calculations of Shanley and Aaron[13] and Reiner and Jaffe.[14] Comparisons of the exact and the DWBA transfer cross sections[7] with those from the BSA calculations at $E_d = 1.78$ and 6.7 MeV are given in Figs. 1 and 2, respectively.

The BSA works well below breakup: Not only does it reporduce the overall trend of the exact cross section, it also give a quantitative fit up to 35°. Results from the DWBA are qualitative at best and are a factor 0.67 too low in the forward direction. We conclude that the rearrangement coupling effects, which are taken into account in the BSA, are very important below breakup. The continuum effects, which are not taken

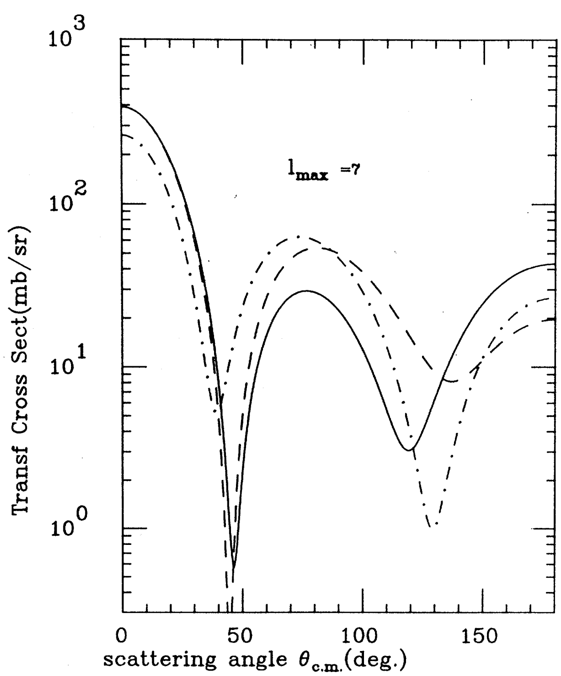

FIG. 1. Calculated (d,p) angular distributions at $E_d = 1.78$ MeV. The solid curve is the exact cross section; the dash-dotted curve, the DWBA; and the dashed curve, the BSA, while l_{max} denotes the highest partial wave entering the calculation.

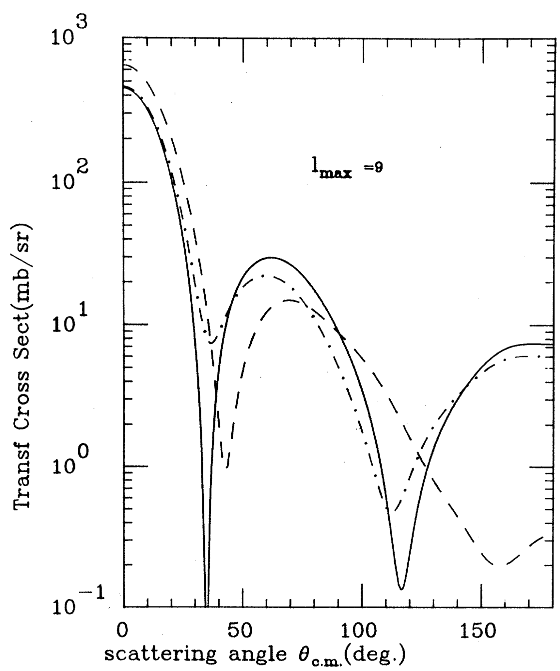

FIG. 2. Calculated (d,p) angular distributions at $E_d = 6.7$ MeV. The three curves have the same meaning as in Fig. 1.

into account in the BSA but are included in the DWBA, are obviously less important here.

Above breakup, the situation is reversed. There, the DWBA gives a very good fit to the exact results up to 30° and reproduces the overall trend of the cross section for higher angles. The BSA, on the other hand, gives a qualitative fit to the exact results only to about 90° and is a factor of 1.5 too large in the forward direction. Obviously, the rearrangement effects included in the BSA become less important than the continuum effects included in the DWBA through the exact distorted wave function u_0.

As we shall show elsewhere,[12] the BSA becomes worse as E_d increases; this must be a consequence of the increasing role of the breakup continuum. Similar results also hold for the elastic cross sections: The ability of the BSA to reproduce the trend of the exact calculations decreases with increasing E_d, and is best for $E_d = 1.78$ MeV.

A quantitative estimate of the role of the continuum can be obtained by using the discontinuity relation satisfied by the (on-shell) partial-wave amplitudes below the breakup threshold[13]:

$$\mathrm{Im}T_{\mathrm{el}}^{(l)} = -\pi\rho_{\mathrm{el}}|T_{\mathrm{el}}^{(l)}|^2 - 2\pi\rho_{\mathrm{tr}}|T_{\mathrm{tr}}^{(l)}|^2, \tag{3}$$

where $T_{\mathrm{el}}^{(l)}$ and $T_{\mathrm{tr}}^{(l)}$ are the lth partial-wave on-shell elastic and stripping amplitudes, respectively, and ρ_{el} and ρ_{tr} are the densities of states for the elastic and stripping channels. The elastic amplitude may always be written as

$$T_{\mathrm{el}}^{(l)} = [1 - \eta_l \exp(2i\delta_l)]/2\pi i\rho_{\mathrm{el}}, \tag{4}$$

where η_l is the (real) absorption coefficient. Substituting (4) into (3) leads to the relation

$$\eta_l^2 = 1 - 8\pi^2\rho_{\mathrm{el}}\rho_{\mathrm{tr}}|T_{\mathrm{tr}}^{(l)}|^2. \tag{5}$$

Since (3) holds only below the breakup threshold, values of η_l calculated from (4) and (5) will agree only for $E_d < 2.225$ MeV. For energies above the breakup threshold, we use (5) to define a new absorption parameter $\tilde{\eta}_l$, which will be larger than η_l because the latter accounts for inelastic continuum effects. Below the breakup threshold, the deviation of $\tilde{\eta}_l$ from η_l is a measure of the accuracy of the Bouldin-Levin calculation[7]: We find that they agree to better than 1%. Above breakup, the deviation between η_l and $\tilde{\eta}_l$ is a partial measure of the neglect of the continuum. In Table I we show the values of η_l for $E_d = 1.78$ MeV and also compare η_l and $\tilde{\eta}_l$ for $E_d = 6.70$ MeV. The deviation of η_l from unity is a measure of the full inelasticity. We see from the table that at 6.7 MeV, most of the inelasticity is due to

TABLE I. Values of η_l below the breakup threshold and comparisons of η_l and $\tilde{\eta}_l$ above the breakup threshold.

l	$E_d = 1.78$ MeV η_l	$E_d = 6.7$ MeV η_l	$\tilde{\eta}_l$
0	0.859	0.656	0.704
1	0.995	0.967	0.974
2	0.641	0.049	0.341
3	0.960	0.736	0.758
4	0.996	0.935	0.940
5	1	0.984	0.985
6	1	0.996	0.996

rearrangement coupling effects rather than to the breakup continnum. This should not be taken to mean that a BSA calculation would yield results in good agreement with the exact ones, since it does not at the larger angles (Fig. 1), a feature we shall discuss in Ref. 12. The point here is that for $E_d = 6.7$ MeV, both $T_{\mathrm{el}}^{(l)}$ and $T_{\mathrm{tr}}^{(l)}$ already include effects due to three-body breakup in intermediate states, effects which are not contained in the BSA. Therefore the relatively small differences between η_l and $\tilde{\eta}_l$ for almost all l simply mean that most of the effects of breakup are in the two-body amplitudes, so that the neglected breakup contribution to the discontinuity relation is small. Hence, a dispersion-type calculation which includes only two-particle unitarity would be successful if one would use reliable input for the dynamical singularities. For higher energies, however, the role of the physical breakup channel is much more important.[12]

We may now ask if these results justify the use of the BSA: In particular, do the results at 1.78 MeV, for which we would expect the BSA to be most accurate, provide such a justification? Our answer is yes: The ability of the BSA to account for the overall shape of the transfer (and elastic[12]) cross sections and to fit the exact transfer results in the forward direction *using only the two bound states of the system* is much more significant than the failure to fit the exact results over the entire angular range. We have also found that as the energy increases, the accuracy of the BSA decreases, thus clearly limiting its validity in this model to energies below the breakup threshold. This is because of the physical breakup continuum becoming comparable to rearrangement coupling effects.

The extrapolation of these results to justify the BSA for realistic scattering systems at energies above their lowest breakup thresholds is not evident. However, for energies below the lowest breakup threshold in systems for which there are a number of bound states in at least one reaction channel, the BSA will probably be an even better approximation than in the present case, which could explain the successes of the BSA reaction theories. One way of testing this is through the use of three-body model calculations in which the potentials support more than one bound state. An even more interesting system for study might be a model four-body system which features many more arrangement and breakup channels than a three-body system. In either of these systems it may be possible to investigate and justify the use of optical potentials to extend the BSA above the lowest breakup threshold, as in the case of nuclear reaction analyses.[4]

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^{12}C on ^{12}C at 800 MeV/Nucleon: One Fireball or Two?

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I demonstrate that a two-fireball model can explain the proton and pion inclusive spectra for collision between light ions at relativistic energies. For heavier ions, the one-fireball model is recovered.

The fireball model predicts, with a fair degree of success, inclusive spectra for collision of Ne on U, Pb, etc. at relativistic energies. For the basics of this model, relevant to this Letter, see Gosset *et al.*[1] and Kapusta.[2]

Data for lighter ions have become available in recent times. Preliminary data for ^{12}C on ^{12}C at 800 MeV/nucleon have existed for more than a year.[3] Data analysis for Ne on NaF and Ar on Ar is nearing completion.

These data have shown strong deviations from the fireball model as developed in Refs. 1 and 2. From now on I will call this model the one-fireball model. I will show that a simple extension of the one-fireball model idea—two fireballs[5] rather than one—fits the data on lighter ions. When heavier ions are involved, this two-fireball model essentially coalesces into the one-fireball model.

The basic premise of the one-fireball model is

