Relation of α-Decay Rotational Signatures to Nuclear Deformation Changes

Hafez M. A. Radi
Physics Department, Kuwait University, Kuwait, and Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

and

A. A. Shihab-Eldin
Kuwait Institute for Scientific Research, Kuwait, and Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

and

John O. Rasmussen and Luiz F. Oliveira
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 8 September 1978)

It is shown that the rotational signatures for ground-band α decay can be calculated from nuclear β_2 and β_4 deformation values. The α-decay probability function over the nuclear surface is equated with the differences of the parent and daughter surfaces, hence with the changes in deformation parameters. It is suggested that deformation values can be used to infer α wave functions at the nuclear surface for α-transfer reaction theory on rotational signatures.

It has long been recognized that relative intensity patterns to rotational-band members in favored α decay are related to the angular dependence of the α wave function on the nuclear surface.[1] Later, very involved microscopic calculations with many Nilsson orbitals and pairing mixing were made, projecting out α functions on the nuclear surface.[2] These microscopic calculations, using the Fröman matrix approximation[3] to propagate through the barrier, gave general satisfactory α rotational signatures.

The idea that the α surface wave functions could be directly related to deformation changes was advanced some time ago,[4] but not seriously tested until we undertook this work.

It is to be noted that α decay is so very slow compared to other nuclear particle-emission processes that the α wave function just inside the barrier is essentially a standing wave. With time-reversal invariance then, the α wave function just inside the barrier may be treated as purely real, though asymptotically well outside the barrier it goes over to outgoing Coulomb waves.

It seems plausible that the α probability density, $|\psi_\alpha(R_0,\theta)|^2$, at the surface should be proportional to the radial decrease of the surface at θ in going from parent to α-decay daughter. Hence a knowledge of shapes of parent and daughter determines the α wave function, which can then be Legendre expanded to give boundary conditions for the various α partial waves for barrier penetration.

Our starting point is to assume that the squared α-decay surface wave function is equal to the difference between the surface of parent and daughter nuclei:

$$|\psi_\alpha(\theta)|^2 = \text{const}\left(\frac{R_i(\theta) - \xi R_f(\theta)}{R_{0i} - \xi R_{0f}}\right). \qquad (1)$$

In some cases this difference was slightly negative for some values of θ; that is, the surface change at some angles θ would give an unacceptable negative probability density. Physically this arises because the nuclear shape of the daughter at the time of barrier penetration is shape polarized and differs from the ground equilibrium shape value. A definitive treatment then would require a comprehensive consideration of shape polarizabilities and would be an interesting project for future study. For the present we have chosen to avoid the detailed question of shape polarization by introducing a daughter "shrinkage factor," ξ, slightly less than unity and chosen such that the negative surface changes are eliminated with the surface change just going to zero at some part of the surface. We take the nuclear shapes as given by the usual spherical harmonic expansion

$$R(\theta) = r_0 A^{1/3}\left(1 + \sum_{\substack{\nu>0\\ \text{even}}} \beta_\nu Y_{\nu 0}(\theta,\varphi)\right), \qquad (2)$$

and $R_0 = r_0 A^{1/3}$, where $r_0 = 1.2$ fm.

For deformation parameters we have taken in some cases the theoretical equilibrium shape-distortion parameters ϵ and ϵ_4 of Möller, Nilsson, and Nix[5] to get β_2, β_4, β_6, and β_8 using the relations given by Seeger.[6] In other cases we have taken β_2 and β_4 from our unweighted least-squares fitting of the experimental (homogeneous distribution) values of Bemis *et al.*[7] for twelve actinide even-even nuclei. The expressions below fit their data within experimental error except for ^{232}Th, which is slightly outside the limits:

$$\beta_2(Z,N) = 0.6588 - 0.2040Z + 0.001114Z^2 + 0.1166N - 0.000380N^2, \tag{3a}$$

$$\beta_4(Z,N) = -46.322 + 1.1646Z - 0.006276Z^2 - 0.08985N + 0.0002571N^2. \tag{3b}$$

Figure 1 plots the nuclear shape of the two daughter nuclei with the deformation-derived α wave function given by the distance from the surface to the outer curve.

The α wave function on the nuclear surface is then expanded in a Legendre series of even order to give a column vector of amplitudes a_0, a_2, a_4, a_6, a_8, higher amplitudes being taken to be zero. The coefficients a_l are calculated by performing a numerical integration in the following simple formula:

$$a_l = \text{const} \times 2\pi \left(\frac{2l+1}{4\pi}\right)^{1/2} \int_{-1}^{+1} P_l(x) \left(\frac{\Delta R_i(x) - \xi b \Delta R_f(x)}{1 - \xi b}\right)^{1/2} dx, \tag{4}$$

where

$$\Delta R(x) = 1 + \sum_{\substack{\nu > 0 \\ \text{even}}} \left(\frac{2\nu+1}{4\pi}\right)^{1/2} \beta_\nu P_\nu(x)$$

and

$$b = (1 - 4/A)^{1/3},$$

and where A is the mass number of the parent.

This surface function is then transformed to the α angular wave function somewhat beyond the barrier by multiplying on the left by a modified Fröman matrix $k_{ll'}$ and by a diagonal matrix representing the differing centrifugal-barrier terms encountered by different partial waves. The equation for the amplitudes in the external region is as follows:

$$C_l = R_0 [v(E_l)]^{1/2} \left(\frac{W_l(E_l)}{G_0(E_l, R_0)}\right) \left|\sum_{l'} k_{ll'}(E_l) a_{l'}\right|, \tag{5}$$

where the W_l/G_0 values are ordinary WKB penetration factors. The amplitude ratios c_l/c_0 can be compared directly with the inverse square root of the hindrance factor as given by the Oak Ridge data group[8] or the Table of Isotopes group.[9]

The Fröman matrix elements were calculated by taking both quadrupole and hexadecapole deformation and electric moments into consideration. The two-dimensional WKB barrier integral $I(r,\theta)$ was evaluated analytically along the lines of constant θ from the nuclear surface to infinity. The matrix elements are then evaluated by numerical integration over θ in the propagation integral

$$k_{ll'}(E_l) = \int_0^{2\pi} \int_0^{\pi} Y_{l0}^*(\theta,\varphi) \exp[I(\infty,\theta)] Y_{l'0}(\theta,\varphi) \sin\theta \, d\theta d\varphi. \tag{6}$$

The integral acquires a small imaginary component, neglected by Fröman, in the integration from the turning point to infinity. This component may be considered a Coulomb excitation component. Unlike Fröman the integral $I(r,\theta)$ is not Legendre expanded but used directly numerically.

In Fig. 2 the calculated values of c_l/c_0 are shown for two theoretical cases along with the reciprocal square roots of experimental hindrance factors.

The agreement is quite satisfactory, somewhat better using the experimental deformation parameters than the theoretical ones. Certainly this α-rotational-signature calculation based on nuclear shape changes in α decay is in as good agreement with experiment as the involved microscopic calculations.[2] The signature is very sensitive to small deformation changes, and so one might use the measured α signatures in inverse calculations to extrapolate the experimental deformations to neighboring nuclear regions.

Another promising application of this approach is for α-transfer reaction theory in the deformed rare-earth region. In this region α decay is not observable, so one would deduce the ground- to ground-band α surface wave function and then use it as a boundary condition in an α-transfer code.

There is a need for further refinements. The penetration of the anisotropic barrier with realistic diffuse nuclear potential needs further study. The Fröman matrix approximation needs study

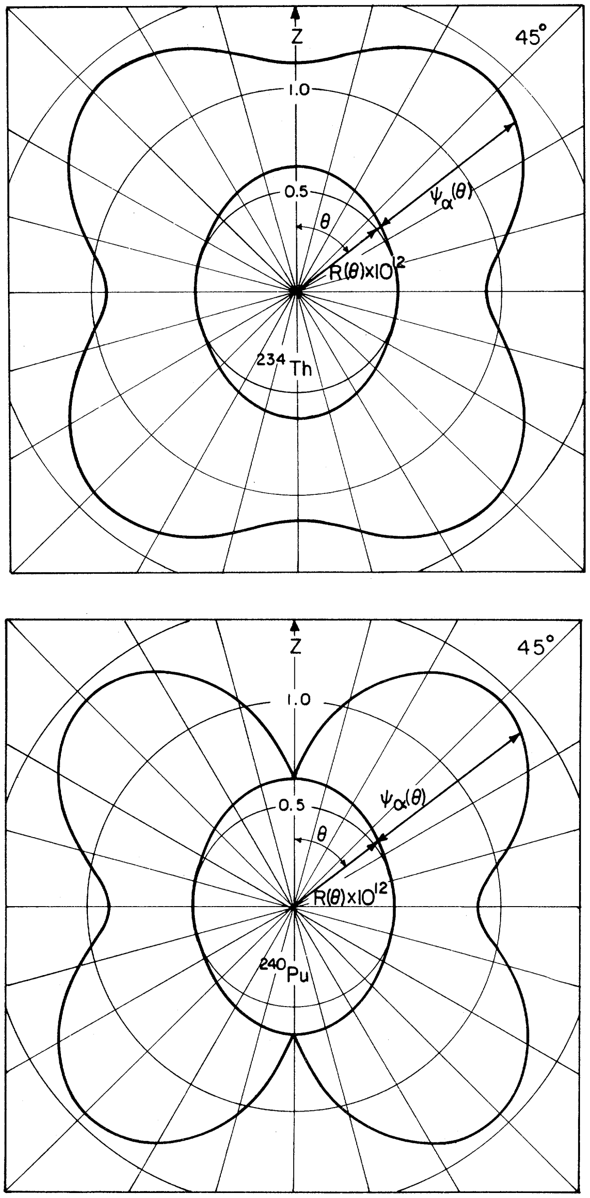

FIG. 1. Plot of the α wave function on the nuclear surface obtained from Eq. (1) for ^{234}Th and ^{240}Pu.

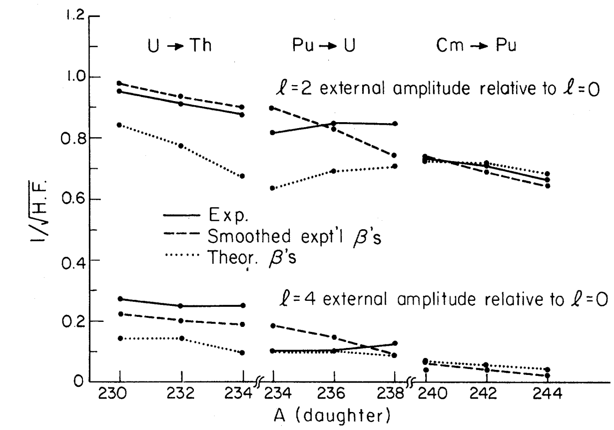

FIG. 2. Plot of hindrance-factor values against the mass number of the daughter. The solid line gives the experimental values (Ref. 9), the dashed line is for the smoothed experimental β_2 and β_4 obtained by Eqs. (3a) and (3b), and the dotted line represents values obtained for the theoretical β_2, β_4, $\cdots$, β_8 (Ref. 5).

with a diffuse potential. Older coupled-channels studies did not include hexadecapole terms and diffuse nuclear potentials. The need for a "shrinkage factor" to avoid negative differences between parent and daughter surfaces is of some concern and suggests that some deeper relationship involving shape polarizabilities should be sought.

One of the authors (H.M.A.R.) would like to acknowledge the support of the Kuwait Institute for Scientific Research and the hospitality of the Nuclear Science Division of Lawrence Berkeley Laboratory. Another one of us (L.F.O.) gratefully acknowledges support from Comissão Nacional de Energia Nuclear, Brazil.

This work was done under the general auspices of the Nuclear Physics Division of the U. S. Department of Energy.

[1]J. O. Rasmussen and B. Segall, Phys. Rev. 103, 1298 (1956).

[2]H. J. Mang and J. O. Rasmussen, K. Dan. Vidensk. Selsk., Mat.-Fys. Skr. 2, No. 3 (1962); J. K. Poggenburg, H. J. Mang, and J. O. Rasmussen, Phys. Rev. 181, 1697 (1969).

[3]P. O. Fröman, K. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 1, No. 3 (1957).

[4]J. O. Rasmussen, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland, Amsterdam, 1965), Vol. 1, p. 701.

[5]P. Möller, S. G. Nilsson, and J. R. Nix, Los Alamos Scientific Laboratory Report No. LA-UR-74-462, 1974 (unpublished).

[6]P. A. Seeger, in *Proceedings of the International Conference on Properties of Nuclei Far off the Region of Beta-Stability, Leysin, Switzerland, 1970* (CERN Scientific Information Center, Geneva, Switzerland, 1970), Vol. 1, p. 217.

[7]C. E. Bemis, K. McGowan, J. L. C. Ford, W. T. Milner, P. H. Stelson, and R. L. Robinson, Phys. Rev. C 8, 1466 (1973).

[8]Y. A. Ellis and M. R. Schmorak, Nucl. Data B 8, 345 (1972).

[9]C. M. Lederer, V. S. Shirley, E. Browne, J. M. Dairiki, R. E. Doebler, A. A. Shihab-Eldin, L. J. Jardine, J. K. Tuli, and A. B. Byron, *Table of Isotopes* (Wiley, New York, 1978), 7th ed.