Azimuthal Correlations in e^+e^- Jets: A Test of Quantum Chromodynamics

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In $e^+e^- \to$ hadrons an azimuthal angular correlation between a plane defined by the hadrons and the plane formed by the beams and the jet axis is possible. In the naive parton model the correlation vanishes. In quantum chromodynamics it does not and with reasonable assumptions may be reliably calculated in perturbation theory for large center-of-mass energy. We calculate it in lowest nontrivial order and discuss its physical origins and experimental observation.

Jets in the hadronic final state of e^+e^- annihilation are a familiar prediction of the parton model.[1] The experimental observation of a $(1+\cos^2\theta)$ angular distribution for the jet axis agrees well with the spin-$\frac{1}{2}$ quark-parton picture.

In the past few years many quark-parton model results have found elegant realization in quantum chromodynamics (QCD). Recently Sterman and Weinberg[2] have proposed that the absence of mass singularities be taken as a criterion for the validity of perturbative calculations in QCD. Observables which satisfy this criterion are calculated as though quarks and gluons were the physical final states, but the result is assumed to describe the real world in which quarks and gluons never appear in isolation. In this sense the Sterman-Weinberg criterion represents a means of extending the parton-model philosophy beyond its familiar limits. Sterman and Weinberg[2] and others[3] defined measures of "jettiness" which satisfy this criterion. The jet structure of e^+e^- annihilation and its angular distribution are to this extent consequences of QCD.

In this note we propose that there is an angular correlation, satisfying the Sterman-Weinberg criterion, between a suitably defined plane of the jets and the plane formed by the e^+e^- beams and the jet axis. This correlation vanishes (like a power of s) in the parton model. It is nonvanishing in order α_s in QCD.

First we will describe precisely in experimental terms the correlation we have in mind. Variations which may be more practical will be discussed below. For each event choose a jet axis according to the prescription of Farhi,[3] choose the axis which maximizes the directed momentum

$$d(\hat{r}) = \sum_a \vec{p}_a \circ \hat{r}\,\theta(\vec{p}_a \cdot \hat{r}), \qquad (1)$$

and call it the thrust[4] ($\hat{T}$) axis. $\hat{T}$ is determined by hadrons on one side of the event [see Fig. 1(a)]. Now go to the other side of the event and look along the thrust direction. Project the hadron momenta on this side [$p_1 \ldots p_4$ in Fig. 1(b)] onto the plane perpendicular to $\hat{T}$ and find the axis ($\hat{C}$) in the plane which minimizes the sum of the absolute values of the transverse momenta

$$\sum_a |\hat{C}\times\vec{p}_{a\perp}| = \min \sum_a |\hat{n}\times\vec{p}_{a\perp}|, \qquad (2)$$

with all $\hat{n}$ in the plane. We shall call this the coplanarity axis. Given $\hat{T}$ and $\hat{C}$ the angle φ is defined in Fig. 2. It is the azimuth of $\hat{C}$ about $\hat{T}$ measured from an x axis defined by the component of the beam momentum perpendicular to the thrust. Note that each jet event yields two independent measurements of φ depending upon which jet is used to determine the thrust.

We propose to study the azimuthal angular distribution $d\sigma/d\varphi$. From the spin of the virtual pho-

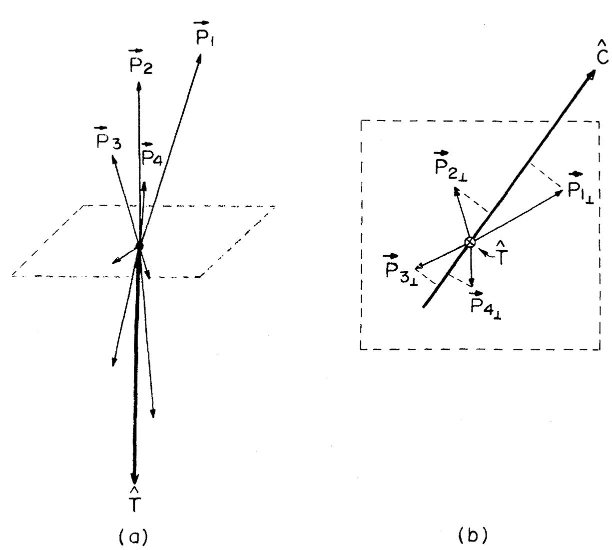

FIG. 1. Definition of thrust ($\hat{T}$) and coplanarity ($\hat{C}$) axes for typical $e^+e^- \to$ hadrons event.

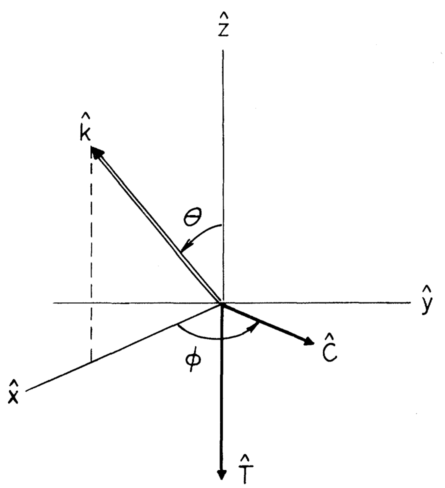

FIG. 2. Definition of azimuthal angle φ for a specific event. $\hat{T}$ and $\hat{C}$ are the experimentally measured thrust and coplanarity axes. $\vec{k}$ is either beam direction. $\hat{T}$ and $\vec{k}$ define the x-z plane.

ton and the usual conservation laws it follows that the most general form is

$$d\sigma/d\varphi = A + B\cos 2\varphi. \qquad (3)$$

To prove this write the squared amplitude as $|M|^2 = l_{\mu\nu}L^{\mu\nu}$, where $l_{\mu\nu}$ is the trace over the lepton currents

$$l_{\mu\nu} \propto (k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1 \cdot k_2) \qquad (4)$$

and $L^{\mu\nu}$ is the analogous trace over hadronic-current matrix elements. Since we observe only $\hat{T}$ and $\hat{C}$ the most general form of $L^{\mu\nu}$ in the center-of-mass frame is

$$L^{00} = \lambda_0, \quad L^{0j} = L^{j0} = \lambda_1 T^j + \lambda_2 C^j,$$
$$L^{ij} = \lambda_3 T^i T^j + \lambda_4 (T^i C^j + T^j C^i) + \lambda_5 C^i C^j \lambda_5 C^i C^j + \lambda_6 \delta^{ij}, \qquad (5)$$

where $\lambda_n (n = 0, 1, \ldots, 6)$ are invariant functions and T^i and C^i are the Cartesian components of $\hat{T}$ and $\hat{C}$. Contracting $l_{\mu\nu}$ with $L^{\mu\nu}$ and using the coordinate system of Fig. 2 we find two sources of φ dependence: Terms proportional to λ_4 yield a $\sin\theta\cos\theta\cos\varphi$ dependence which is odd in $\theta \to \pi - \theta$ and therefore integrates to zero, and terms proportional to λ_5 which yield a term proportional to $\sin^2\theta\cos^2\varphi$. This establishes Eq. (3). Notice that our proof nowhere depends on the definitions of the vectors $\hat{T}$ and $\hat{C}$. We conclude that Eq. (3) is the correlation to be expected between any two orthogonal vectors characterizing the final state of $e^+e^- \to$ hadrons. Thus, for example, the two-particle (p_1, p_2) inclusive distribution (where we associate $\hat{T}$ with $\hat{p}_1$ and $\hat{C}$ with the component of $\vec{p}_2$ normal to $\hat{p}_1$) will also show a $\cos 2\varphi$ dependence, where we are in general unable to calculate the B coefficient in Eq. (3).

However, when $\hat{T}$ is the thrust axis and $\hat{C}$ the coplanarity axis, $d\sigma/d\varphi$ satisfies the criterion of Sterman and Weinberg and, if they are right, can be reliably calculated in perturbation theory. This is because our definition of φ does not distinguish between an event with a quark of momentum $\vec{p}$ and energy E on the one hand, and an event with a quark momentum $\vec{p}_1$, and energy E_1, accompanied by a gluon with momentum $\vec{p}_2$ and energy E_2 such that $\vec{p}_1 + \vec{p}_2 = \vec{p}$ and $E_1 + E_2 = E$ on the other. The same is true for a gluon with momentum $\vec{p}$ and a quark-antiquark pair with momenta $\vec{p}_1$ and $\vec{p}_2$ such that again $\vec{p}_1 + \vec{p}_2 = \vec{p}$ and $E_1 + E_2 = E$. Provided, then, that higher-order corrections to $d\sigma/d\varphi$ contain no infrared divergences, the only possible Q^2 dependence will be associated with $\alpha_s(Q^2)$, where $\alpha_s(Q^2)$ $[\equiv g^2(Q^2)/4\pi]$ is the renormalization-group running charge of QCD. Therefore, $d\sigma/d\varphi$ will be given by a power series in $\alpha_s(Q^2)$,

$$Q^2 \frac{d\sigma}{d\varphi} = \sum_{n=0}^{\infty} \left[\frac{\alpha_s(Q^2)}{\pi}\right]^n (A_n + B_n \cos 2\varphi), \qquad (6)$$

where the A_n and B_n coefficients are finite constants. We have calculated the coefficient B_1. It is given by

$$B_1 = (Q^2\sigma_0/2\pi)(\tfrac{16}{3}\ln\tfrac{3}{2} - 2), \qquad (7)$$

where σ_0 is the lowest-order e^+e^- annihilation cross section into hadrons. Therefore,

$$\frac{2\pi}{\sigma_0}\frac{d\sigma}{d\varphi} = 1 + O(\alpha_s(Q^2)) + \frac{\alpha_s(Q^2)}{\pi}(\tfrac{16}{3}\ln\tfrac{3}{2} - 2)\cos 2\varphi + O(\alpha_s^2(Q^2)). \qquad (8)$$

Equation (7) is obtained from the diagrams shown in Fig. 3. To first order in α_s the final $q\bar{q}G$ state defines a plane

$$e^+(\vec{k}_1) + e^-(\vec{k}_2) \to q(\vec{p}_1) + \bar{q}(\vec{p}_2) + G(\vec{p}_3). \qquad (9)$$

The angle φ is determined once the $\hat{T}$ and $\hat{C}$ axes are identified. The maximum directed momentum [Eq. (1)] is simply the momentum of the most energetic particle i. Therefore the thrust axis will be in the direction of $\vec{p}_i$ $(\hat{T} \equiv \hat{p}_i)$. Note that any of the three final quanta may be most

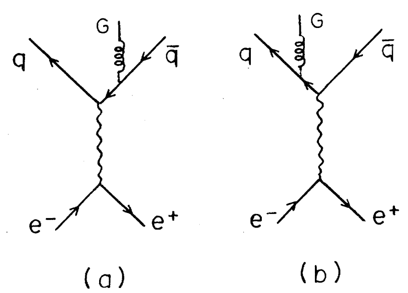

FIG. 3. QCD diagrams for $e\bar{e} \to q\bar{q}G$.

energetic. The coplanarity axis is given by $\hat{p}_{j\perp}$ $(j \neq i)$, the momentum component of either of the other particles transverse to the thrust axis. Given $\hat{T}$ and $\hat{C}$ the coordinate system of Fig. 2 is constructed and φ is determined. The calculation leading to Eq. (7) is completely conventional except that the coordinate system shifts depending upon which particle is most energetic.

The $\cos 2\varphi$ dependence originates in helicity conservation at the $e\bar{e}\gamma$ vertex. The quark and antiquark created at the virtual-photon vertex have opposite helicity. The pair therefore has angular momentum component $J_\xi = \pm 1$ along the axis ($\hat{\xi}$) defined by their momenta. A $\cos 2\varphi_\xi$ dependence (φ_ξ is the azimuth about the $\hat{\xi}$ axis) arises from the interference of the $J_\xi = 1$ and $J_\xi = -1$ amplitudes. In the parton model the quark and antiquark are assumed to develop independently into the final hadrons. Suppose the coplanarity is measured on the antiquark jet, then completeness allows us to replace the quark jet by the quark. The $J_\xi = \pm 1$ amplitudes involve quarks of opposite helicity and are therefore orthogonal. Thus there is no $\cos 2\varphi$ dependence in the parton model. The same argument applies trivially to the zeroth-order diagram in QCD. In first-order QCD a $\cos 2\varphi$ dependence can arise in two ways. The first is from interference between diagrams 3(a) and 3(b). The trick is that the $\hat{\xi}$ axis is different in (a) and (b). It is along the quark momentum in (a) and along the antiquark momentum in (b). The helicity $+1$ amplitude in (a) contains both $J_\xi = +1$ and $J_\xi = -1$ relative to the $\hat{\xi}$ axis of (b), and will interfere with the helicity $+1$ amplitude in (b). The second source of $\cos 2\varphi$ dependence is merely the possibility that the $\hat{\xi}$ and $\hat{T}$ axes are not the same in the square of (a) or (b). Consider helicity $+1$ in (a) but suppose the antiquark is most energetic. $\hat{\xi}$ and $\hat{T}$ are not parallel. φ_ξ is not the measured φ and a $\cos 2\varphi$ dependence develops from the pure helicity amplitude.[5]

From this discussion it is evident that a $\cos 2\varphi$ dependence is a signal of either communication between opposite-side jets or systematic deviation of the jet axis from the original $q\bar{q}$ axis. Clearly a $\cos 2\varphi$ dependence is interesting regardless of the model which predicts it.

Finally we turn to some experimental considerations. As noted above, each event yields two measurements of φ. From Fig. 3 it is clear that to $O(\alpha_s)$ only the jet on one side is correlated (the side opposite the most energetic particle). The other jet is uncorrelated: The hadrons into which it evolves will to this order show no $\cos 2\varphi$ dependence. All $O(\alpha_s)$ effects are on one side. Thus, for example, we expect the correlated jet to have a larger oblateness than the uncorrelated one. (Oblateness is the maximum directed momentum in the plane perpendicular to the thrust and is a measure of the jet.) The correlation may therefore be enhanced in the data if for each event only the jet with the larger oblateness is used to determine φ.

Experimentalists prefer to discuss jets in terms of the momentum ellipsoid

$$M_{ij} = \sum_a (\delta_{ij} |p_a|^2 - p_{ai} p_{aj}) \qquad (10)$$

rather than in terms of maximum directed momentum. Theorists eschew M_{ij} because it is not (in general) free of mass singularities.[2] We believe φ correlations are sufficiently interesting that they should be looked for in M_{ij} if thrust and coplanarity measurements are not yet possible. To do so replace $\hat{T}$ by the largest axis of the ellipsoid and $\hat{C}$ by the second largest in the entire discussion. The cross section $d\sigma/d\tilde{\varphi}$ with $\tilde{\varphi}$ defined from the momentum ellipsoid does not follow simply from QCD. Nevertheless a naive calculation to order α_s would yield the same cross section as in Eq. (7).

Finally we realize that the cylindrical geometry of e^+e^- colliding-ring detectors will introduce a φ bias. To minimize this and to enhance the signal relative to the background we suggest doing the experiment in a moderate angular range symmetric about $\theta = \pi/2$. The cross section relevant to this experiment is

$$2\pi\left(\frac{d\sigma}{d\varphi\, d\cos\theta}\right)\left(\frac{d\sigma_0}{d\cos\theta}\right)^{-1} = 1 + O(\alpha_s(Q^2)) + \frac{2\sin^2\theta}{1+\cos^2\theta}\,\frac{\alpha_s(Q^2)}{\pi}\left(\tfrac{16}{3}\ln\tfrac{3}{2} - 2\right)\cos 2\varphi + \text{terms odd in } \theta \to (\pi - \theta). \qquad (11)$$

If the experiment is not symmetric in $\cos\theta$, additional correlations of the form $\sin\theta\cos\theta\cos\varphi$ will be present (and could be calculated).

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[1]S. D. Drell, D. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1959), and Phys. Rev. D 1, 1617 (1970); R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[2]G. Sterman and S. Weinberg, Phys. Rev. Lett. 39, 1436 (1977).

[3]E. Farhi, Phys. Rev. Lett. 39, 1587 (1977); H. Georgi and M. Machacek, Phys. Rev. Lett. 39, 1237 (1977); S. S. Shei, to be published.

[4]A. De Rújula, J. Ellis, E. G. Floratos, and M. K. Gaillard, CERN Report No. Ref. TH. 2455, 1978 (to be published).

[5]Our argument assumes the QED vertex precedes the QCD vertex. In the opposite time ordering a $\cos\varphi$ dependence can arise but this vanishes when integrated over the remaining final-state phase space.

Upper Limit on Parity Mixing in ^{21}Ne

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The parity-nonconserving circular polarization of γ rays from the $2.789 \rightarrow 0.0$ MeV transition in ^{21}Ne is found to be $(-9 \pm 51)\times 10^{-4}$, which corresponds to a parity-mixing matrix element $|\langle H_{\rm PNC}\rangle| = 0.009 \pm 0.054$ eV between the 2.80-MeV, $J^\pi = \frac{1}{2}^\pm$ levels. This value is considerably smaller than the measured parity mixing in the $J=\frac{1}{2}$ doublet in ^{19}F.

As yet we know very little about the parity-nonconserving (PNC) interaction between two nucleons. In particular the relative strengths of the $\Delta T = 0$, 1, and 2 components of the PNC interaction, which provide information on the basic hadronic weak interaction,[1] are not yet determined. These isospin properties are best found from experiments in light nuclei. However, nonzero effects have been seen only in three cases: $n+p$,[2] ^{16}O,[3] and ^{19}F.[4] These are not sufficient to determine the effective PNC N-N potential, even if one includes the precise upper limits obtained for $p+p$, $p+d$,[5] ^{18}F,[6] and the many circular polarization (P_γ) results in heavy nuclei.[7]

A particularly interesting system for studying nuclear parity mixing occurs at $E_x = 2.8$ MeV in ^{21}Ne where a $J^\pi = \frac{1}{2}^+$ and a $J^\pi = \frac{1}{2}^-$ level are separated by only 7.6 ± 0.7 keV (see Millener *et al.*[8]). We have chosen to examine this system for two reasons which we discuss more fully below: First, our measurement in ^{21}Ne is the first in an odd-N, even-Z nucleus, where the PNC matrix element connecting two levels of opposite parity can be inferred directly from a measurement of the pseudoscalar observable; and second, the ^{21}Ne system is unusually sensitive—a very small PNC matrix element produces relatively large experimental effects.

All measurements which show definite PNC effects in nuclei with odd $A = N + Z$ have been in odd-Z nuclei. In the single-particle approximation the PNC effects in all these odd-Z nuclei measure nearly the same linear combination of the basic PNC N-N amplitudes. By studying an odd-N nucleus one probes a different linear combination. For example, consider a schematic model where the $J^\pi = \frac{1}{2}^-$ level in ^{19}F (odd Z) consists of a proton hole in the $T=0$ ^{20}Ne core while the $\frac{1}{2}^-$ level in ^{21}Ne (odd N) consists mainly of a nucleon hole in the $A=22$, $T=1$ core. From sim-