PHYSICAL REVIEW LETTERS

VOLUME 41 17 JULY 1978 NUMBER 3

Mass of the Up Quark

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(Received 23 February 1978)

It is shown that $m_u = 0$ does not lead to numerical disagreements and that it implies an improvement of the SU(2)⊗SU(2) symmetry.

The possibility that the mass of the up quark be zero has recently been advanced[1] as a way of avoiding strong CP nonconservation induced by instanton effects.[2,3] Of the known nonunnatural[4,5] alternatives to achieve the suppression of the undesirable effect, one requires the existence of a light pseudoscalar, the axion,[1] and seems to be in very serious difficulty with existing experiment.[6] Another option, the introduction of spontaneous violation of either parity or time-reversal invariance,[4,5] should be postponed until the nontrivial calculation of the suppression becomes available. Last but not least is the alternative of the up quark being massless. In this paper I comment on the arguments that have been used[7] in the past to disregard this possibility. I show that $m_u = 0$ does not lead to numerical disagreements and that it implies that SU(2)⊗SU(2) symmetry is less broken than in the case $m_u \neq 0$. This improvement of the SU(2)⊗SU(2) symmetry emerges at the expense of SU(3).

The present idea[7] about the values of the quark mass parameters, m_u, m_d, and m_s, in the SU(3)⊗SU(3)-breaking Hamiltonian (to zero order in weak and electromagnetic interactions)

$$\mathcal{H}_m = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s\,, \tag{1}$$

where u, d, and s are the Dirac fields of the quarks, relies on the current-algebra relation[8,9] for the masses of the low-lying pseudoscalars (π, K, and η):

$$\delta_{\alpha\beta} f_\alpha^2 m_\alpha^2 = -\langle 0|[Q_A^\alpha, [Q_A^\beta, \mathcal{H}_m(0)]]|0\rangle, \tag{2}$$
$$\alpha, \beta = 1, 2, \ldots, 8,$$

where f_α are the decay constants ($f_\pi = 92$ MeV) and Q_A^α are the axial charges. Equation (2) is valid only to first order in $\mathcal{H}_m$ and, since higher-order effects induce corrections which are too large and model dependent,[9-11] it should not be used without appropriate caution.

Instead of the above-mentioned procedure, where smoothness of a two-point function has to be assumed, one can consider only the relations obtained from the one-boson-to-vacuum matrix elements of the divergence of the axial-vector currents, $\partial^\mu A_\mu^\alpha$, and the (approximate) transformation properties of the symmetry-breaking

Hamiltonian. Let λ^α be the Gell-Mann SU(3) matrices, $\lambda^0=(2/3)^{1/2}$, and

$$q=\begin{pmatrix}u\\d\\s\end{pmatrix},\quad u^\alpha=\bar{q}\lambda^\alpha q,\quad v^\alpha=\bar{q}\lambda^\alpha\gamma_5 q,\quad \alpha=0,1,\dots,8. \tag{3}$$

The nine scalars u^α and nine pseudoscalars v^α transform then according to the $(\underline{3},\underline{3}^*)+(\underline{3}^*,\underline{3})$ representation of SU(3) ⊗ SU(3); in particular,[12]

$$[Q_A{}^\alpha, u^\beta]=id_{\alpha\beta\gamma}v^\gamma. \tag{4}$$

From the one-boson-to-vacuum matrix elements of

$$\partial^\mu A_\mu{}^\alpha=-i[Q_A{}^\alpha,\mathcal{H}_m(0)], \tag{5}$$

it thus follows that

$$m_\pi{}^2 f_\pi=Z_\pi{}^{1/2}(m_u+m_d)/2,\quad \pi=\pi_+,\pi_-,\pi_0, \tag{6a}$$

$$m_{K^+}{}^2 f_K=Z_K{}^{1/2}(m_u+m_s)/2, \tag{6b}$$

$$m_{K^0}{}^2 f_K=Z_K{}^{1/2}(m_d+m_s)/2, \tag{6c}$$

where

$$Z_\pi{}^{1/2}=\langle 0|v^i|\pi\rangle,\quad i=1,2,3,$$

$$Z_K{}^{1/2}=\langle 0|v^i|K\rangle,\quad i=4,5,6,7, \tag{7}$$

and where I have ignored SU(2) breaking in the Z's. Notice also that the values of the masses (of the charged bosons) do not include virtual-photon corrections. From (6) the desired formulas are obtained:

$$\frac{m_d}{m_u}=\frac{m_{\pi^0}{}^2-(\Delta m_K{}^2)_{u_3}(f_K Z_\pi{}^{1/2}/f_\pi Z_K{}^{1/2})}{m_{\pi^0}{}^2+(\Delta m_K{}^2)_{u_3}(f_K Z_\pi{}^{1/2}/f_\pi Z_K{}^{1/2})}, \tag{8a}$$

$$\frac{m_s}{m_d}=-\frac{(f_K Z_\pi{}^{1/2}/f_\pi Z_K{}^{1/2})[2m_{K^0}{}^2-(\Delta m_K{}^2)_{u_3}]-m_{\pi^0}{}^2}{(f_K Z_\pi{}^{1/2}/f_\pi Z_K{}^{1/2})(\Delta m_K{}^2)_{u_3}-m_{\pi^0}{}^2} \tag{8b}$$

$$\tilde{m}_u-\tilde{m}_d\equiv Z_K{}^{1/2}(m_u-m_d)=2f_K(\Delta m_K{}^2)_{u_3}, \tag{9}$$

where $(\Delta m_K{}^2)_{u_3}$ is the K^+-K^0 mass difference without virtual-photon corrections. In the case $f_K Z_\pi{}^{1/2}/f_\pi Z_K{}^{1/2}=1$, Eqs. (8) reduce to those given by Weinberg.[7]

I will assume that the difference between the experimental K^+-K^0 mass difference and $(\Delta m_K{}^2)_{u_3}$ is given by the Dashen formula[9]

$$(m_{K^+}{}^2-m_{K^0}{}^2)_\gamma=(m_{\pi^+}{}^2-m_{\pi^0}{}^2)_\gamma. \tag{10}$$

Since, according to (6a), the pion mass difference is purely electromagnetic, we have

$$(\Delta m_K{}^2)_{u_3}=m_{K^+}{}^2-m_{K^0}{}^2-m_{\pi^+}{}^2+m_{\pi^0}{}^2=-0.0053\ \text{GeV}^2. \tag{11}$$

The confidence in the relations (10) and (11) relies on the fact that they are in agreement with the magnitude of isospin breaking obtained from the $\Delta I=1$ baryon mass differences in the low-lying octet.[11,13]

Usually[12] symmetry breaking is also related to the parameters ϵ_0, ϵ_3, ϵ_8 and their renormalized version, $\tilde{\epsilon}_i$, defined by

$$\epsilon_0 u^0+\epsilon_3 u^3+\epsilon_8 u^8=m_u\bar{u}u+m_d\bar{d}d+m_s\bar{s}s, \tag{12}$$

$$\epsilon_0=(m_u+m_d+m_s)/\sqrt{6},$$

$$\epsilon_3=(m_u-m_d)/2,$$

$$\epsilon_8=(m_u+m_d-2m_s)/2\sqrt{3}, \tag{13}$$

$$\tilde{\epsilon}_i=Z_K{}^{1/2}\epsilon_i,\quad i=0,3,8. \tag{14}$$

ϵ_3/ϵ_8 and $\sqrt{2}+\epsilon_8/\epsilon_0$ measure isospin breaking and departures from SU(2) ⊗ SU(2), respectively.

For $f_K/f_\pi=1.2$, as given by experiment,[14] (14), (13), (9), and (11) imply

$$\frac{\tilde{\epsilon}_3}{f_\pi}=\frac{f_K}{f_\pi}(\Delta m_K{}^2)_{u_3}=-0.0065\ \text{GeV}^2. \tag{15}$$

Keeping $f_K/f_\pi=1.2$, I will now consider two extreme cases in Eqs. (8): (i) $Z_\pi=Z_K$ [small SU(3) breaking], and (ii) $m_u=0$. In the first case we obtain

$$m_d/m_u=2.1,\quad m_s/m_d=23.5, \tag{16}$$

$$\epsilon_3/\epsilon_8=0.02,\quad \epsilon_8/\epsilon_0=-1.29\ \text{GeV}^2, \tag{17}$$

which is interpreted as an indication that SU(2) and SU(2) ⊗ SU(2) are good symmetries with the former being broken less than the latter. On the other hand if we set $m_u=0$ then (6) implies the sum rule

$$(\Delta m_K{}^2)_{u_3}f_K Z_\pi{}^{1/2}=-m_{\pi^0}{}^2 f_\pi Z_K{}^{1/2}, \tag{18a}$$

while from (13) one gets[15]

$$\sqrt{2}\,\epsilon_0+\epsilon_8+\sqrt{3}\,\epsilon_3=0. \tag{18b}$$

From (18a) we have, using (11) and (8b),

$$Z_K{}^{1/2}/Z_\pi{}^{1/2}=0.36, \tag{19}$$

$$\frac{m_s}{m_d}=-1-\frac{m_{K^0}{}^2}{(\Delta m_K{}^2)_{u_3}}=46.8. \tag{20}$$

Notice that for $m_u=0$ SU(2) ⊗SU(2) becomes exact in the limit $m_s/m_d=\infty$ and thus the higher value given in (20) (twice that of the previous case) is welcome. In fact, directly from (13) and (20) it follows that

$$\frac{\epsilon_8}{\epsilon_0} = -\sqrt{2}\left[1-\tfrac{3}{2}\left(1+\frac{m_s}{m_d}\right)^{-1/2}\right] = -1.37 \qquad (21)$$

which is *closer* to $-\sqrt{2}$ than in the $Z_\pi = Z_K$ case. We conclude that SU(2) ⊗SU(2) is a better symmetry if $m_u=0$. On the other hand the value of ϵ_3/ϵ_8 changes only by 6%,

$$\epsilon_3/\epsilon_8 = \sqrt{3}\,(2m_s/m_d - 1)^{-1} = 0.019\,, \qquad (22)$$

and thus the $\Delta I = 1$ baryon mass differences in the low-lying octet remain practically unaltered.

Finally I would like to comment on Eq. (2). It is true that with

$$-\langle 0|\bar{u}u|0\rangle - \langle 0|\bar{d}d|0\rangle = f_\pi Z_\pi^{1/2},$$
$$-\langle 0|\bar{s}s|0\rangle - (\langle 0|\bar{u}u|0\rangle + \langle 0|\bar{d}d|0\rangle)/2 = f_K Z_K^{1/2}, \qquad (23)$$

it also implies Eq. (8) and (9). On the other hand, the exact relation of which (2) is an approximation is (α indices suppressed)

$$f^2m^2 + \int_{m_0^2}^{\infty} \frac{dq^2}{q^2}\,\rho(q^2) = -\langle 0|[Q_A,[Q_A,\mathcal{H}_m]]|0\rangle\,, \qquad (24)$$

where

$$\rho(q^2) = \sum_n (2\pi)^3\delta^4(p_n - q)\,|\langle 0|\partial^\mu A_\mu|n\rangle|^2\,, \qquad (25)$$

and where m_0^2 is the threshold of the continuum. The positivity of $\rho(q^2)$ renders questionable approximations (2) and (23) in which the integral over the continuum is neglected, particularly if there is an enhancement such as that which has been proposed[16] as a possibility to explain the deviations from the Goldberger-Treiman relation and which has in fact been observed[17] in the kaon case. Equation (19) reflects also the fact that (at least if $m_u=0$) smoothness of two-point functions is a bad approximation.

I conclude that the possibility that the mass of the up quark is zero is not ruled out, in which case SU(2) ⊗SU(2) becomes a better symmetry [at the expense of SU(3)] than in the case $m_u \neq 0$.

I am indebted to Professor M. A. B. Bég for stimulating discussions and encouragement. A discussion with Professor H. Pagels is also acknowledged. Finally, I wish to thank the members of the Physics Department of the Rockefeller University, where this work was done, for their kind hospitality. This work was supported in part by National Science Foundation Latin American exchange program and by CONACyT (Mexico) under Contract No. 540-B.

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