and 3 modes, and the combination $m = 3$ and $n = 2$ has been detected.[2] In addition, in PLT[3] although $m = 2$ precursor oscillations are observed, the disruption itself often exhibits poloidal asymmetry, presumably corresponding to the generation of odd poloidal mode numbers. Finally, during the disruption $m = 1$ oscillations are often observed within the plasma core; this observation might correspond to the generation of the $m = 1$ mode described here.

The most important feature of the major disruption that any model must explain is the rapid time scale. For the mechanism described here, the characteristic time scale for the development of the magnetic islands is the width of the peak in the 3/2 growth rate. If we define the half-width Γ by $\gamma_{32}(t^M \pm \Gamma) - \gamma_{32} = \Delta\gamma/2$, then $\Gamma \simeq (\gamma_{21}{}^0)^{-1}$. Assuming that the scaling does hold for all values of S, we find that in PLT the predicted value for the disruption time is 230 μsec. If the effect of diamagnetic drifts is included in the expression for $\gamma_{21}{}^0$, then $\Gamma \simeq 440$ μsec. These time scales are consistent with the observed disruption time of 500 μsec. A study of several other machines, including LT-3 and TOSCA for which the preceding analysis rigorously applies as far as the magnitude of the resistivity is concerned, also shows consistency with the observed disruption time (approximately 10 μsec).

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Superfluid Solitons in Helium Films

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It is shown that in monolayer superfluid He^4 films, in addition to third-sound modes, small amplitude effects can lead to the existence of gapless solitons made up of superfluid condensate. These nonlinear excitations can be created by localized perturbations in the superfluid density. The conditions are studied under which such initial disturbance evolves into an ordered string of solitons, and the differences to be expected in thicker films are discussed.

Third-sound phenomena in helium films have provided a wealth of information on the properties of nearly two-dimensional superfluids.[1] Originally proposed by Atkins[2] as the superfluid analog of surface waves, they soon became a precise tool for examining the critical behavior of

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the superfluid density in monolayer films.[3] The most spectacular result that has recently emerged from these studies is the confirmation of the universal discontinuous jump in ρ_s at T_c for two-dimensional superfluids.[4]

Interesting in themselves, third-sound excitations have been investigated with great accuracy in monolayer films by Rutledge *et al.*[5] In these two-dimensional superfluids, where film thickness and healing length lose their meaning, they found positive dispersion for the surface modes, together with a roton branch that becomes excited above 0.6°K. The dispersion relation and temperature dependence of these modes was then derived in the linear regime by using a two-dimensional formulation of Landau's quantum hydrodynamics, the proper approach when dealing with such thin and inhomogeneous films.

Several experiments on third-sound propagation, however, have revealed finite amplitude effects that cannot be explained in terms of the linearized theory. Specifically, it has been reported that in some instances incipient shock behavior develops as the superfluid velocity amplitude increases,[6] whereas in monolayer films undistorted pulse propagation has been observed at very low temperatures.[7] These effects point to the need for extending the above theories in order to include finite amplitude effects and to explore the possibility of novel nonlinear phenomena taking place in superfluid films.

In this Letter it is pointed out that in He^4 films, in addition to third-sound modes, small finite amplitude effects can lead to the existence of gapless solitons made up of superfluid condensate. These nonlinear excitations propagate with a velocity proportional to their amplitude and can be copiously created in experiments where the driving perturbation is localized to a small region of the film. Using inverse-scattering techniques, I study the conditions under which an initial disturbance will evolve into an ordered string of solitons, and discuss the differences to be expected as the film thickness increases.

Ordinary third-sound modes are obtained from a linearized study of the equations of motion for superfluid films.[1] For monolayer films, where both the bulk superfluid density and the thickness are fuzzy concepts, the appropriate formulation has been developed by Rutledge *et al.*[5] Consider a two-dimensional superfluid at temperatures low enough so that the losses to the vapor and substrate can be neglected. The superfluid surface density $\bar{\rho}_s(\vec{r})$ is related to a complex order parameter $|\psi(\vec{r})|$ via $\bar{\rho}_s(\vec{r}) = |\psi(\vec{r})|^2$. The energy is then written as a sum of terms involving the kinetic energy, the van der Walls forces acting on the film, the chemical potential, and the surface energy arising from spatial fluctuations in $\bar{\rho}_s$, i.e.,

$$H = \int d^2r \left(\frac{\hbar^2}{2m} |\nabla\psi|^2 + \frac{A}{2(a+\bar{\rho}_s)^2} - \mu\bar{\rho}_s + \tfrac{1}{2}B(\nabla\bar{\rho}_s)^2 \right), \tag{1}$$

where m is the mass of the helium atom, A and a are constants related to the van der Waals energy ($A \simeq 14$°K and $a \simeq 1.2$ atomic layers), μ is the chemical potential, and B is the surface energy in the thick-film limit. The equation of motion for the order parameter, $i\hbar\, \partial\psi/\partial t = \delta H/\delta\psi$, then becomes

$$i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{A}{(a+|\psi|^2)^3} - \mu\psi - B\psi\nabla^2|\psi|^2. \tag{2}$$

By neglect of nonlinear terms, the amplitude and phase fluctuations in ψ away from the ground-state value, ψ_0, determine the behavior of the surface density waves. They correspond to oscillations of the superfluid density $\bar{\rho}_s$, accompanied by temperature waves, while the normal component of the superfluid remains at rest. The dispersion relation is given in the $T \to 0$ limit by

$$\omega^2 = C_3^2k^2 + k^4(\hbar^2 + 4B\bar{\rho}_s^0 m/4m^2), \tag{3}$$

where the first term corresponds to the ordinary dispersionless third-sound modes with an adiabatic velocity given by

$$C_3^2 = 3A\bar{\rho}_s^0/m(a+\bar{\rho}_s^0)^4, \tag{4}$$

and the second term comprises both the surface energy and the single-particle excitations.

In order to go beyond this linearized theory and to incorporate finite amplitude effects, we can proceed in a heuristic fashion. The positive dispersion relation given by Eq. (3) implies that in two-dimensional superfluids the phase velocity behaves, for small k, as

$$V \simeq C_3(1 + k^2/k_0^2), \tag{5}$$

with k_0^2 given by

$$k_0^2 = 8m^2C_3^2/(\hbar^2 + 4Bm\bar{\rho}_s) \tag{6}$$

which experimentally is observed to have a value $k_0 \simeq 0.5$ Å^{-1}. Consider now excitations of the film propagating along the X axis parallel to the substrate. In a coordinate system moving to the left with velocity C_3, Eq. (5) implies that the frequency of the waves is given by $\omega = C_3 k^3/k_0^2$. We can therefore write for the surface superfluid density fluctuations $\bar{\rho}_s(x,t)$ the linear differential equation[8]

$$\frac{\partial \bar{\rho}_s}{\partial t} + \frac{C_3}{k_0^2}\frac{\partial^3 \bar{\rho}_s}{\partial x^3} = 0, \qquad (7)$$

whose solutions are plane waves with phase velocity given by $C_3 k^2/k_0^2$, in agreement with Eq. (6).

Finite amplitude effects[9] can be incorporated into Eq. (7) by writing to lowest order in $\bar{\rho}_s$ the following nonlinear differential equation[10]:

$$\frac{\partial \bar{\rho}_s}{\partial t} + \frac{C_3}{k_0^2}\bar{\rho}_s\frac{\partial \bar{\rho}_s}{\partial x} + \frac{C_3}{k_0^2}\frac{\partial^3 \bar{\rho}_s}{\partial x^3} = 0. \qquad (8)$$

In the absence of boundary conditions, the solutions of this equation were found by Korteweg and de Vries.[11] They correspond to plane waves moving with velocity C_3 (ordinary third sound) plus a soliton mode given by

$$\bar{\rho}_s = A\,\mathrm{sech}^2[(x - Ct)/\Delta], \qquad (9)$$

with a velocity-dependent width, Δ, given by

$$\Delta = 2k_0^{-1}(C_3/C)^{1/2}, \qquad (10)$$

and a phase velocity, C, proportional to the amplitude, A, which, in the rest frame of the substrate, is given by

$$C = C_3(1 - \tfrac{1}{3}A). \qquad (11)$$

Equation (9) corresponds to a single surface hump made up of depleted superfluid condensate,[12] and moving with a velocity smaller than that of ordinary third sound by a factor of $\frac{1}{3}$. It should be noted, however, that its being gapless makes the beahvior of this soliton different from its φ^4 field theory[13] or sine-Gordon counterparts.[14] Whereas the latter require a threshold nonlinearity in order to be generated, these superfluid solitons go continuously over the third-sound modes described by Eq. (7). In fact, by defining a dimensionless nonlinearity parameter σ through

$$\sigma = (A/C_3)^{1/2}k_0\Delta, \qquad (12)$$

it is easy to show that an initial perturbation of the form given by Eq. (9) will continue to propagate with amplitude and width given by Eqs. (10) and (11) provided that $\sigma \geq \sigma_0$, with $\sigma_0 = \sqrt{12}$. For $\sigma < \sigma_0$ this perturbation has a very small amplitude and can be regarded as almost linear. Notice, however, that the crossover from nonlinear third-sound to superfluid soliton behavior occurs continuously as a function of σ.

Let us now look into possible mechanisms through which well-defined solitons can be generated by externally perturbing a helium film. Consider an initially localized change in the superfluid surface density. Its time development can be understood qualitatively by considering Eq. (8). For very short times the first two terms of the nonlinear equation dominate and the perturbation will steepen in the regions where it has a negative slope. Were it not for the dispersive term, a discontinuity would then ensue. However, as steepening progresses, the third term becomes important and stabilizes the sharp edge, leading to the formation of oscillations of short wavelength which become the incipient pulses.[15] Those pulses with $\sigma < \sigma_0$ will propagate as a wave train, spatially leading the solitons, which correspond to values of $\sigma > \sigma_0$.[16]

The asymptotic behavior of the initial perturbation can be studied exactly through the use of inverse-scattering techniques.[17] If the initial localized amplitude is described by $\bar{\rho}_s^{\,i} = \bar{\rho}_s(x, t=0)$, the solution of Eq. (8) for long times can be written as

$$\bar{\rho}_s(x,t) = \sum_{n=1}^{N} 2E_n\,\mathrm{sech}^2[(E_n)^{1/2}(x - 4E_n t) - C], \qquad (13)$$

where E_n is the nth eigenvalue of a particle in a potential well described by $\bar{\rho}_s^{\,i}(x)$, C is a constant, and N is the number of solitons, which is given in the large-$\bar{\rho}_s^{\,i}$ limit by

$$N = \pi^{-1/2}\int_{-\infty}^{\infty}|\bar{\rho}_s^{\,i}(x)|^{1/2}dx + 1. \qquad (14)$$

Since the energies of the bound states are ordered (i.e., $E_1 > E_2 > E_3 > \ldots$) and the solitons move with a velocity proportional to E_n, Eq. (13) implies that the initial disturbance will evolve into a series of superfluid solitons spatially ordered in such a way that the leading and trailing ones will have the smallest and largest amplitude, respectively. Several points deserve comment at this stage. (1) Bound states for the initial eigenvalue problem will appear only for $\bar{\rho}_s^{\,i} > 0$, i.e., the localized superfluid surface density is negative with respect to its ground-state value, $\bar{\rho}_{s_0}$. If $\bar{\rho}_s^{\,i} < 0$, its final evolution will correspond to a nonlinear period wave train. (2) For negative-dispersion systems solitons emerge for $\bar{\rho}_s^{\,i} < 0$, i.e., for a localized enhancement of $\bar{\rho}_s^{\,i}$ above $\bar{\rho}_{s_0}$. (3) In the small-$\bar{\rho}_s^{\,i}$ limit and if $\bar{\rho}_s^{\,i} > 0$ there will always exist one bound state corresponding to a single soliton emerging at long times.

If the initial depletion in the surface superfluid density has a Gaussian shape, with amplitude $\bar{\rho}_s^{\,i}$ and width Δ, the number of solitons emerging will be given according to Eq. (14) as

$$N \simeq 0.5(\bar{\rho}_s^{\,i}\Delta)^{1/2}+1. \tag{15}$$

It should be noticed that for the case of a δ function one still obtains one bound-state eigenvalue ($E_1 = \bar{\rho}_s^{\,i}/2$) so that a soliton with that amplitude will propagate behind the normal third-sound mode.

In two-dimensional superfluids it should therefore be possible to create superfluid solitons by applying heat pulses to a localized region of the film, thereby depressing the superfluid density. These objects could then be detected by exploiting their two main properties: (1) They can go transparently through each other, and (2) their phase velocities are proportional to their amplitudes. The approximations made here in the derivation of their main properties imply working at temperatures below 0.4°K.

The above considerations can also be applied to thicker films, but with one important proviso. As the coverage increases and crossover to three-dimensional behavior takes place, the ordinary linearized theory of third sound becomes relevant. As one extends it into the nonlinear regime in the spirit of the present approach, the dispersion introduced by finite thickness can overcome the surface terms of Eq. (3) and eventually become negative. As I have mentioned, for negative-dispersion systems the inverse-scattering method makes the existence of bound states depend on having an initial perturbation of opposite sign to the one discussed here. If such crossover would occur, the equivalent of a cooling pulse should be applied in order to create superfluid solitons. The resulting string of enhanced $\bar{\rho}_s$ humps would then lead the normal third-sound wave in a mirror situation of the scenario described for the monolayer limit.

Finally, I would like to comment on the effects of dissipation processes on the propagation properties of solitons in helium films. If viscous damping would take place, its effect could be incorporated into the formalism by the addition of a term $\mu\,\partial^2\bar{\rho}_s/\partial x^2$ to the right-hand side of Eq. (8), with μ an effective viscosity of the medium. For low values of μ the soliton string would still propagate undistorted, but as the viscous term overtakes the dispersive term, shock waves with an oscillatory structure could then be generated.[18] At low enough temperatures, however, it is expected that superfluid solitons will become the dominant modes of excitation.

I wish to thank I. Rudnick for making me aware of the existence of possible nonlinear effects in third-sound experiments and for many useful conversations. The low-temperature physics group of the University of Illinois kindly provided me with a preprint of Ref. 5. It was a pleasure to share these insights with M. L. Cohen, S. Doniach, J. R. Schrieffer, and L. Turkevich. Their comments and encouragement are appreciated.

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[8]Notice that Eq. (7) is invariant under the transformation $x \to -x$, $\bar{\rho}_s \to -\bar{\rho}_s$, and so it applies to positive-dispersion systems by realizing that $\bar{\rho}_s > 0$ implies a depletion of the surface superfluid density.

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Observation of Edge Dislocations in Smectic Liquid Crystals

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Isolated single-layer edge dislocations in smectic liquid crystals are observed by polarization microscopy. The modification of the smectic-A–smectic-C transition temperature by the strain field of the dislocations is used to make them visible. Observations of periodic arrays in thin samples, by various polarization-contrast mechanisms, and the measurement of the Burgers vector confirm the nature of the defects observed.

We report here the first direct observation of elementary edge dislocations in smectic liquid crystals. Theory for the structure and properties of such defects has been developed,[1] and their presence has been invoked as an explanation of various observations,[2] but they have never before been seen as individual defects in an otherwise nearly perfect sample.[3] We have developed an observational technique that takes advantage of the large susceptibility associated with a second-order phase change. Near the critical temperature, T_c, for the smectic-A–smectic-C phase change, the strains associated with a single dislocation modify the structure of the sample in a way that is made visible by using polarized-light microscopy. We describe first the principle underlying the experiment, second, the basic observations and the evidence that we are seeing dislocations, and third, the determination of the Burgers vector of the dislocations.

The smectic-A phase is a one-dimensional crystal in which the rodlike molecules are oriented normal to the molecular layers [Fig. 1(a), left-hand sides]. In the smectic-C phase the long molecular axis is tilted by a polar angle θ with respect to the layer normal [Fig. 1(a), right-hand sides]. This tilting is accompanied by a decrease of layer thickness, which in a simple rigid-rod model should vary as $\cos\theta$. A compressive stress normal to the layers favors the C phase and raises the transition temperature, while a dilative stress lowers it, an effect already studied.[4]

To utilize this effect to make dislocations visible, we prepare thin single-crystal samples between slightly nonparallel glass slides [Fig. 1(a)]. The smectic layers are anchored parallel to the glass surfaces by treatment with a surfactant, typically hexadecyl trimethyl ammonium bromide.[5] Some distribution of edge dislocations must exist, and for a small enough wedge angle ($\lesssim 10^{-3}$ rad) elementary edge dislocations should be separated enough for optical resolution. Upon crossing a dislocation, the abrupt change, Δm, in the number of smectic layers, m, contained in the sample thickness produces an abrupt change, $\Delta\epsilon$, in the component of strain normal to the layers: $\Delta\epsilon = \Delta m/m$. Since the glass is about 1000 times more rigid elastically than the liquid crystal, this strain must be accomodated in the liquid crystal [Fig. 1(b)]. This nonuniform strain produces a spatial modulation of T_c, so that the dislocations become visible as phase boundaries [Fig. 1(c)].

To see this more precisely, we write the free

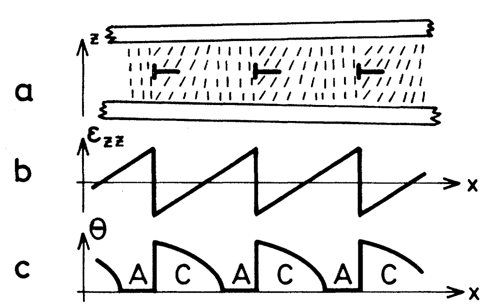

FIG. 1. (a) Schematic cross section of a sample containing a dislocation array, with plots of (b) strain vs position and (c) accompanying variation of tilt angle vs position.

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