esu deduced from the TPA data of Ref. 12.

In conclusion, we have presented a new method of nonlinear spectroscopy most suitable for the study of resonances in highly absorbing solids. It has been successfully applied to perform the first active nonlinear scattering on the Γ_1 biexciton of CuCl up to the fourth order. Our preliminary results show that the shape and the position of the resonance as observed by active nonlinear processes are significantly different from those revealed by TPA experiments.

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Collisionless "Current-Channel" Tearing Modes

S. M. Mahajan, R. D. Hazeltine, H. R. Strauss, and D. W. Ross
Fusion Research Center, The University of Texas at Austin, Austin, Texas 78712
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Analytical and numerical studies of collisionless tearing modes that are wider than the "current channel" are presented. The $m=1$ type of inertial mode is shown to be strongly unstable for typical tokamak shear and β_e. Large spatial extension and large growth rate make it a possible candidate for explaining plasma disruption.

We report, in this Letter, analytical and numerical studies of tearing modes in a collisionless plasma. The modes we discuss are characterized by a mode width λ_w greater than the width of the electron layer or the "current channel" $x_e = |\omega/k_\parallel' v_e|$. Here $k_\parallel' = k_y/L_y$, k_y is the azimuthal mode number, L_s is the shear length, v_e is the electron thermal speed, and λ_w measures the region in which the parallel electric field is nonzero. Laval, Pellat, and Vuillemin[1] were the first to present an $m \geq 2$ type of current-channel tearing mode; their result is instructively discussed by Drake and Lee.[2] Later, Chen, Rutherford, and Tang[3] found trapped-particle modification to the mode. Although, these results are easily recovered in the appropriate limit of our dispersion relation, we emphasize here the $m=1$ type of inertial tearing mode first pointed out by Hazeltine and Strauss.[4] Our analytical dispersion relation, which has been verified in detail numerically, modifies the previous results for $\beta_e > m_e/m_i$. We also clarify its relationship to other instabilities. Most importantly, we confirm the potentially rapid growth and wide parameter range for instability of inertial tearing modes with $m=1$ character (large Δ'). Whenever it is consistent to treat the plasma in a collisionless approximation, this mode would be a serious candidate to explain plasma disruption because of its large spatial extension and large growth rate.

The slab geometry formulation of the electromagnetic eigenvalue problem is now standard.[5,6] If the radial wavelength is shorter than the azimuthal wavelength, we have

$$\varphi'' = (\sigma/x_A^2)[(\psi/x) - \varphi], \tag{1}$$

$$\psi'' = (\sigma/x)[(\psi/x) - \varphi], \tag{2}$$

where φ is the electrostatic potential, $\psi = \omega A_\parallel / k_\parallel' c$ is proportional to the vector potential $A_\parallel$, a prime denotes differentiation with respect to the radial coordinate x, $x_A^2 = \omega(\omega + \omega_{i*})/k_\parallel' v_A)^2$, $v_A = (B_0^2/4\pi n_0 m_i)^{1/2}$ is the Alfvén speed, ω_{i*} is the ion diamagnetic drift frequency, and all lengths are normalized to $\rho_s = (T_e/m_i)^{1/2}\Omega_i^{-1}$. In Eqs. (1) and (2), σ is the dimensionless measure of the generalized "conductivity." Although our analytical method can handle fairly complicated models of σ (i.e., with effects of temperature gradients and trapped particles included[3]), we keep here, for illustrative purposes, to the simplest representative collisionless model,

$$\sigma = -x_A^2 \frac{\omega - \omega_{e*}}{\omega + \omega_{i*}}\left[1 + \frac{x_e}{|x|} Z\left(\frac{x_e}{|x|}\right)\right], \tag{3}$$

where $\omega_{e*} = k_y c_s / L_n$ is the electron diagmagnetic drift frequency, $c_s = (T_e/m_i)^{1/2}$ is the sound speed, $L_n = |n_0^{-1}\, \partial n_0/\partial x|$ is the density scale length, and $x_e = \omega/k_\parallel' v_e$ measures the width of the current channel. It is customary, at this stage, to combine Eqs. (1) and (2) to obtain a single second-order differential equation in $E_x = -\partial\varphi/\partial x$.[6] We depart here from the conventional approach, and instead obtain an equivalent equation for $Q = (\psi/x) - \varphi = -(x_A^2 E_x'/\sigma)\alpha E_\parallel/x$,

$$\frac{d}{dx}\left(\frac{x_A^2 x^2\, dQ/dx}{x^2 - x_A^2}\right) + \sigma Q = -\frac{2x E_0 x_A^2}{(x^2 - x_A^2)^2}, \tag{4}$$

where E_0 is related to Δ', the stability parameter of the kink-tearing mode theory,[5,7] by

$$E_0 = -(\Delta')^{-1}\int_{-\infty}^{+\infty}[\sigma Q/x]\,dx. \tag{5}$$

We have chosen to write our equation in terms of Q ($E_\parallel/x$) instead of E_x, because it is the behavior of $E_\parallel$ which determines whether the mode has a current channel or not. We notice from Eq. (1) that while the $E_\parallel$ profile can be much broader than the σ profile, i.e., when $\lambda_w > |x_e|$, E_x is constrained to follow the σ profile. This makes E_x an unsuitable variable for the study of current-channel modes; and presumably explains the less accurate results of Ref. 4.

We now solve Eqs. (4) and (5) by setting up a variational principle for Q. Following standard methods,[5] we can show that the functional ($\langle\cdots\rangle$ denotes $\int_{-\infty}^{+\infty}\cdots dx$)

$$S = \left[\Delta' + \frac{\epsilon\pi i}{x_A}\right]\left\{\left\langle Q\frac{d}{dx}\left[\frac{x^2 x_A^2\, dQ/dx}{x^2 - x_A^2}\right]\right\rangle + \langle\sigma Q^2\rangle\right\} + 2x_A^2\left\langle\frac{Qx}{x^2 - x_A^2}\right\rangle^2 \tag{6}$$

is variational, in that $\delta S = 0$, generates Eq. (4) with the constraint Eq. (5). To do that we need to use the relation

$$(\Delta' + \epsilon\pi i/x_A)E_0 = 2x_A^2\int_{-\infty}^{+\infty}[Qx/(x^2 - x_A^2)^2]\,dx,$$

which is obtained by multiplying Eq. (4) with $(1/x)$ and integrating over all x. In Eq. (6), $\epsilon = 1$ if $\mathrm{Im}\,x_A > 0$, and $\epsilon = -1$ if $\mathrm{Im}\,x_A < 0$. The extremal value of $S \equiv S^* = 0$ yields the dispersion relation. Since we are looking for localized solutions of $E_\parallel$ which tear the magnetic surfaces, the appropriate trial function should be even in $E_\parallel$. Recalling that $Q = E_\parallel/x$, we choose the trial function

$$Q = x^{-1}\exp(-\alpha x^2/2), \quad \mathrm{Re}\,\alpha > 0, \tag{7}$$

where α is a variational parameter, to evaluate S. All the integrals involved can be evaluated exactly.[8] A general analysis of S will be presented elsewhere. In this Letter, however, we are going to concentrate on recovering the modes which are much wider than the current channel, i.e., for which $|x_e\alpha^{1/2}| < 1$. For further simplification, we also assume that $|x_A\alpha^{1/2}| < 1$. With these approximations, and for $\mathrm{Im}\,x_A > 0$, we can write

$$S \propto S_0 + S_1\alpha^{1/2} + S_2\alpha, \tag{8}$$

where

$$\begin{aligned} S_0 &= \Delta'\left[-\frac{i\pi}{x_A} - \frac{x_A^2}{x_e}\frac{\omega - \omega_{e*}}{\omega + \omega_{i*}}\, i\sqrt{\pi}\right] + \pi^{3/2}\frac{\omega - \omega_{e*}}{\omega + \omega_{i*}}\frac{x_A}{x_e}, \\ S_1 &= \left(\Delta' + \frac{i\pi}{x_A}\right)2\sqrt{\pi}\, x_A^2\frac{\omega - \omega_{e*}}{\omega + \omega_{i*}}, \\ S_2 &= -2i\pi x_A\Delta'. \end{aligned} \tag{9}$$

A simultaneous solution of $\partial s/\partial\alpha = 0$, $S^* = 0$, gives $\alpha^{1/2} = -S_1/2S_2$ and leads to the dispersion relation

$$S_0 = S_2\alpha = S_1^2/4S_2 \tag{10}$$

which must be solved for the eigenvalue ω. Of course, the acceptable solution must satisfy the consistency criteria $\mathrm{Re}\,\alpha > 0$, $|x_e\alpha^{1/2}| < 1$, $|x_A\alpha^{1/2}| < 1$, and $\mathrm{Im}\,x_A > 0$. The first of these assures a localized solution, the second, a solution with a current channel, and the third is simply for convenience, and is not essential to the analysis. To make further progress, we consider the infinite-mode-width limit, $\alpha \to 0$, which implies that $E_{\parallel}$ is essentially constant. Since $E_{\parallel} = \psi - x\varphi$, $\alpha \to 0$ is equivalent to the constant ψ approximation if $x\varphi$ is neglected. Indeed, we recover, in the zeroth-order dispersion relation $S = 0$, the mode of Laval, Pellat and Vuillemin[1] which was derived making use of the constant-ψ approximation. However, we also find an unstable mode when $(\Delta')^{-1}$ is zero. We use $(\Delta')^{-1} = 0$ as a definition of $m=1$ modes. For simplicity, let us put $\omega_{i*}=0$, then $S=0$ leads to

$$(\omega - \omega_{e*})[1 - (i\Delta' \omega/\pi k_{\parallel}' v_{A})] = i\gamma_T , \tag{11}$$

where $\gamma_T = (\Delta' k_{\parallel}' v_A^2/\sqrt{\pi} v_e)$. Notice that for $\omega_{i*} = 0$, the constraint $\mathrm{Im} x_A > 0$ is satisfied for any growing mode. For small Δ', the second term in the square brackets in Eq. (11) is small, and the resulting dispersion relation $\omega = \omega_{e*} + i\gamma_T$ describes the collisionless tearing mode of Ref. 1. For $(\Delta')^{-1}=0$, Eq. (11) is the dispersion relation for $m=1$ type of modes, and the solution is

$$\omega = \frac{\omega_{e*}}{2} \pm i\left[k_{\parallel}'^2 v_A^2 \frac{v_A}{v_e}\sqrt{\pi} - \frac{\omega_{e*}^2}{4}\right]^{1/2} \tag{12}$$

which has a growing root if $\beta_e^{3/2} > 4(L_n/L_s)^2(\pi m_e/m_i)^{1/2}$, where expressions for v_A and ω_{e*} have been used. This restriction is a consequence of evaluating the zeroth-order dispersion relation, and also of expanding the integrals for small $|x_A \alpha^{1/2}|$. A numerical solution of the variational dispersion relation allows us to handle large values of β_e. For $|x_A \alpha^{1/2}| < 1$, the current-channel condition $|x_e \alpha^{1/2}| < 1$ simply requires that $\beta_e > m_e/m_i$. It can be easily seen that for the growing root of Eq. (12), all the consistency conditions are readily satisfied. Since $\beta_e > m_e/m_i$ is required, it is clear that the "current-channel" inertial tearing mode has no electrostatic limit.

We have also solved the Eqs. (1)–(3) by direct numerical integration using a code developed by Miner.[9] This code carries out a finite-element Galerkin procedure employing basic cubic splines. The boundary conditions are $\psi'(0) = \varphi(0) = 0$, and $\psi'(x_b) = \psi(x_b)(x_b + 2/\Delta')^{-1}$, $\varphi(x_b) = \psi(x_b)/x_b$. Since the latter condition is $E_{\parallel} = 0$, the boundary point is chosen outside the $E_{\parallel}$ layer, $|\alpha^{1/2} x_b| \gg 1$.

An analysis of Eq. (11) reveals that a positive nonzero $(\Delta')^{-1}$ decreases the growth rate of the $m=1$ mode. As $(\Delta')^{-1}$ is increased further, the mode smoothly goes over to the Laval mode, which has a smaller growth rate than the $m=1$ mode. A negative $(\Delta')^{-1}$, on the other hand, enhances the growth rate of the mode, and as Δ' becomes small, γ approaches the large magnetohydrodynamic growth rate $\pi k_{\parallel}' v_A/|\Delta'|$. The same circumstance pertains in collision-dominated regimes.[6] All of the above-mentioned features of the dispersion relation Eq. (11) have been verified in detail by carrying out extensive numerical experiments in various parameter regimes. Thus we have verified the mode of Laval, Pellat, and Vuillemin which is relevant for small Δ'. We present here the comparison

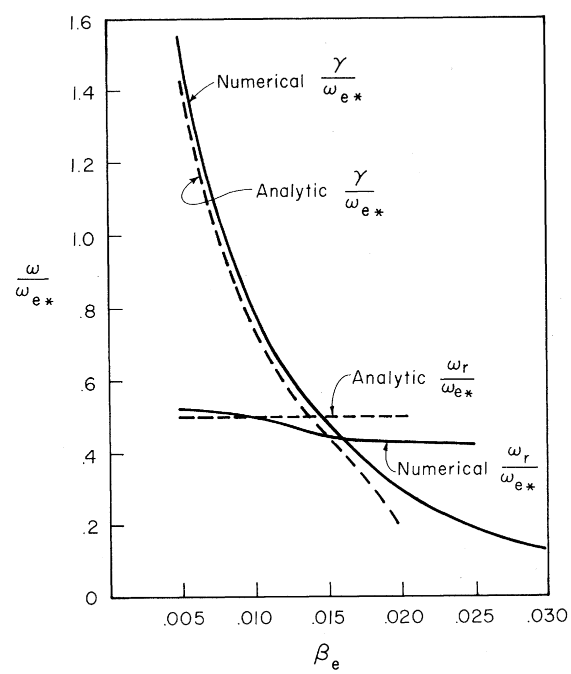

FIG. 1. Normalized eigenfrequency ω/ω_{e*} vs β_e for $\omega_{i*}=0$, $\Delta'=\infty$, and $L_n/L_s = 0.1$.

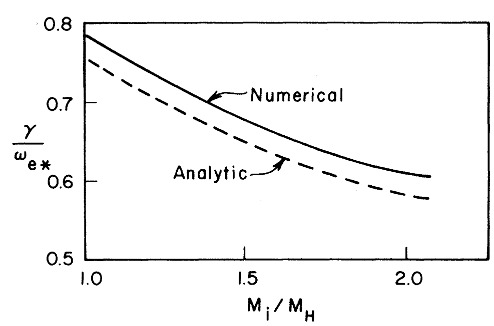

FIG. 2. Normalized growth rate γ/ω_{e*} vs the ion mass m_i/m_H, where $m_H = 1836 m_e$, for $\beta_e = 0.01$ and $L_n/L_s = 0.1$.

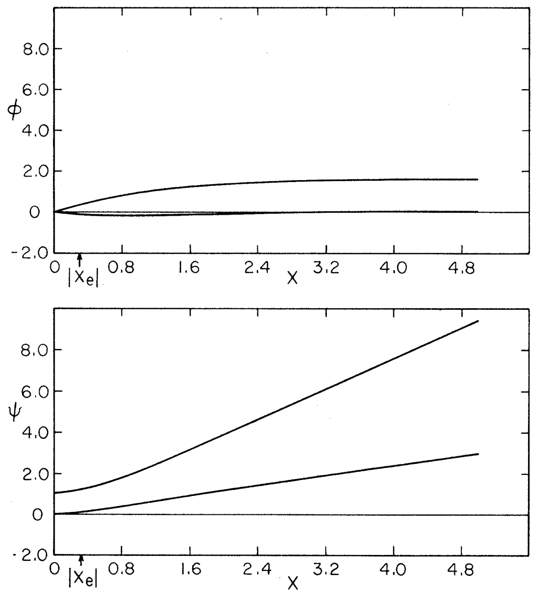

FIG. 3. φ and ψ vs x/ρ_s for $\beta_e=0.005$, $L_n/L_s=0.1$, $\omega_{i*}=0$, and $\omega=0.52+1.54i$. $|x_e|$ denotes the current-channel width.

between our analytical and numerical results for the $m=1$ ($\Delta'=\infty$) inertial tearing mode. In Fig. 1, we plot the real part of the frequency ω_r and the growth rate γ as a function of β_e. The analytical [Eq. (12)] and numerical results are clearly in excellent agreement in the limit of validity of Eq. (12). The numerical solution has been extended to higher values of β_e, and the instability persists. To check the scaling of the growth rate further, we plot γ as a function of the ion mass in Fig. 2, and the agreement is again excellent. In Fig. 3, we show a typical plot of φ and ψ as a function of x/ρ_s. The mode width, i.e., the region in which $E_\parallel$ remains finite is $\simeq 2.5\rho_s$, while the current-channel width is $\simeq 0.25\rho_s$. Thus the mode is indeed much wider than the current channel.

Therefore, we have demonstrated the existence of an $m=1$ ($\Delta'=\infty$) "current-channel" tearing mode in a collisionless plasma by analytical and numerical methods. The mode has a growth rate $\sim\omega_{e*}$, which makes the growth time comparable to typical disruption time. The mode remains unstable for realistic shear $L_n/L_s\sim 0.1$, and for a wide range of β_e: greater than m_e/m_i and up to a few percent. Its large growth rate coupled with its large spatial extension make this mode very important for high-temperature, moderate-density plasmas which can be treated in a collisionless approximation.

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